INTERMEDIATE DYNAMICS FOR ENGINEERS

This book has sufficient material for two full-length semester courses in intermediate engineering dynamics. For the first course a Newton–Euler approach is used, followed by a Lagrangian approach in the second. Using some ideas from differential geometry, the equivalence of these two approaches is illuminated throughout the text. In addition, this book contains comprehensive treatments of the kinematics and dynamics of particles and rigid bodies. The subject matter is illuminated by numerous highly structured examples and exercises featuring a wide range of applications and numerical simulations.

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Intermediate Dynamics for Engineers

A UNIFIED TREATMENT OF NEWTON–EULER AND LAGRANGIAN MECHANICS

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This book is dedicated to my adventurous daughter, Anna

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Oliver M. O'Reilly
Frontmatter
More information

Contents

Preface

The writing of this book started more than a decade ago when I was first given the assignment of teaching two courses on rigid body dynamics. One of these courses featured Lagrange's equations of motion, and the other featured the Newton–Euler equations. I had long struggled to resolve these two approaches to formulating the equations of motion of mechanical systems. Luckily, at this time, one of my colleagues, Jim Casey, was examining the elegant works [205, 207, 208] of Synge and his co-workers on this topic. There, he found a partial resolution to the equivalence of the Lagrangian and Newton–Euler approaches. He then went further and showed how the governing equations for a rigid body formulated by use of both approaches were equivalent [27, 28]. Shades of this result could be seen in an earlier work by Greenwood [79], but Casey's work established the equivalence in an unequivocal fashion. As is evident from this book, I subsequently adapted and expanded on Casey's treatment in my courses. My treatment of dynamics presented in this book is also heavily influenced by the texts of Papastavridis [169] and Rosenberg [182]. It has also benefited from my graduate studies in dynamical systems at Cornell in the late 1980s. There, under the guidance of Philip Holmes, Frank Moon, Richard Rand, and Andy Ruina, I was shown how the equations governing the motion of (often simple) mechanical systems featuring particles and rigid bodies could display surprisingly rich behavior.

There are several manners in which this book differs from a traditional text on engineering dynamics. First, I demonstrate explicitly how the equations of motion obtained by using Lagrange's equations and the Newton–Euler equations are equivalent. To achieve this, my discussion of geometry and curvilinear coordinates is far more detailed than is normally found in textbooks at this level. The second difference is that I use tensors extensively when discussing the rotation of a rigid body. Here, I am following related developments in continuum mechanics, and I believe that this enables a far clearer derivation of many of the fundamental results in the kinematics of rigid bodies.

I have distributed as many examples as possible throughout this book and have attempted to cite up-to-date references to them and related systems as far as feasible. However, I have not approached the exhaustive treatments by Papastavridis

[169] nor its classical counterpart by Routh [184, 185]. I hope that sufficient citations to these and several other wonderful texts on dynamics have been placed throughout the text so that the interested reader has ample opportunity to explore this rewarding subject.

Using This Text

This book has been written so that it provides sufficient material for two full-length semester courses in engineering dynamics. As such it contains two tracks (which overlap in places). For the first course, in which a Newton–Euler approach is used, the following chapters can be covered:

1. Kinematics of a Particle (Section 1.5 can be omitted)
2. Kinetics of a Particle
   Appendix on Tensors
6. Rotation Tensors
7. Kinematics of Rigid Bodies
8. Constraints on and Potentials for Rigid Bodies
9. Kinetics of a Rigid Body
11. Multibody Systems

The second course, in which a Lagrangian approach is used, could be based on the following chapters:

1. Kinematics of a Particle
2. Kinetics of a Particle
3. Lagrange's Equations of Motion for a Single Particle
4. Lagrange's Equations of Motion for a System of Particles
5. Dynamics of Systems of Particles
   Appendix on Tensors
6. Rotation Tensors (with particular emphasis on Section 6.8)
7. Kinematics of Rigid Bodies
8. Constraints on and Potentials for Rigid Bodies
9. Kinetics of a Rigid Body
10. Lagrange's Equations of Motion for a Single Rigid Body
11. Multibody Systems

In discussing rotations for the second course, time constraints permit a detailed discussion of only the Euler angle parameterization of a rotation tensor from Chapter 6 and a brief mention of the examples on rigid body dynamics discussed in Chapter 9.

Most of the exercises at the end of each chapter are highly structured and are intended as a self-study aid. As I don't intend to publish or distribute a solutions manual, I have tailored the problems to provide answers that can be validated. Some of the exercises feature numerical simulations that can be performed with MATLAB or MATHEMATICA. Completing these exercises is invaluable both in terms of comprehending why obtaining a set of differential equations for a system is important and for visualizing the behavior of the system predicted by the model. I also strongly recommend semester projects for the students during which they can delve into a specific problem, such as the dynamics of a wobblestone, the flight of a Frisbee, or the reorientation of a dual-spin satellite, in considerable detail. In my courses, these projects feature simulations and animations and are usually performed by students working in pairs who start working together after 7 weeks of a 15-week semester.

Image Credit

The portrait of William R. Hamilton in Figure 4.6 in Subsection 4.11.3 is from the Royal Irish Academy in Dublin, Ireland. I am grateful to Pauric Dempsey, the Head of Communications and Public Affairs of this institution, for providing the image.

Acknowledgments

This book is based on my class notes and exercises for two courses on dynamics, ME170, ENGINEERING MECHANICS III, and ME175, INTERMEDIATE DYNAMICS, which I have taught at the Department of Mechanical Engineering at the University of California at Berkeley over the past decade. Some of the aims of these courses are to give senior undergraduate and first-year graduate students in mechanical engineering requisite skills in the area of dynamics of rigid bodies. The book is also intended to be a sequel to my book *Engineering Dynamics: A Primer*, which was published by Springer-Verlag in 2001.

I have been blessed with the insights and questions of many remarkable students and the help of several dedicated teaching assistants. Space precludes mention of all of these students and assistants, but it is nice to have the opportunity to acknowledge some of them here: Joshua P. Coaplen, Nur Adila Faruk Senan, David Gulick, Moneer Helu, Eva Kanso, Patch Kessler, Nathan Kinkaid, Todd Lauderdale, Henry Lopez, David Moody, Tom Nordenholz, Jeun Jye Ong, Sebastién Payen, Brian Spears, Philip J. Stephanou, Meng How Tan, Peter C. Varadi, and Stéphane Verguet. I am also grateful to Chet Vignes for his careful reading of an earlier draft of the book.

Many other scholars helped me with specific aspects of and topics in this book. Figure 9.1 was composed by Patch Kessler. Henry Lopez (B.E. 2006) helped me with the roller-coaster model and simulations of its equations of motion. Professor Chris Hall of Virginia Tech pointed out reference [118] on Lagrange's solution of a satellite dynamics problem. Professor Richard Montgomery of the University of California at Santa Cruz discussed the remarkable figure-eight solutions to the three-body problem with me, Professor Glen Niebur of the University of Notre Dame provided valuable references on Codman's paradox, Professor Harold Soodak of the City College of New York provided valuable comments on the tippe top, and Professors Donald Greenwood and John Papastavridis carefully read a penultimate draft

of this book and generously provided many constructive comments and corrections for which I am most grateful.

Most of this book was written during the past 10 years at the University of California at Berkeley. The remarkable library of this institution has been an invaluable resource in my quest to distill more than 300 years of work on the subject matter in this book. I am most grateful to the library staff for their assistance and the taxpayers for their support of the University of California.

Throughout this book, several references to my own research on rigid body dynamics can be found. In addition to the students mentioned earlier, I have had the good fortune to work with Jim Casey and Arun Srinivasa on several aspects of the equations of motion for rigid bodies. The numerous citations to their works are a reflection of my gratitude to them.

This book would not have been published without the help and encouragement of Peter Gordon at Cambridge University Press and would contain far more errors were it not for the editorial help of Victoria Danahy. Despite the assistance of several other proofreaders, it is unavoidable that some typographical and technical errors have crept into this book, and they are my unpleasant responsibility alone. If you find some on your journey through these pages, I would be pleased if you could bring them to my attention.