'Musica est exercitium arithmeticae occultum nescientis se numerare animi.' 'Music is a hidden arithmetical exercise of the soul, which does not know that it is dealing with numbers.'

(Gottfried Wilhelm Leibniz, 1646–1716)

1.1 The origin of music

For thousands of years, music has played an essential role in social interactions, rituals and ceremonies, although its exact origins are shrouded in mystery. From an evolutionary viewpoint, the desire to produce musical sounds is not unique to man. We are all familiar with the sound of birds singing. Some non-human primates also sing; monkeys in the rainforests of Asia and gibbons in the jungles of Thailand produce haunting musical calls: their duets probably serve to strengthen pair bonding, but singing may also serve to alarm other group members (Chung and Geissmann, 2000; Geissmann, 2002). If musical sounds are important in primate communication, they must also have been essential in early human communication. Perhaps as suggested by Charles Darwin (Darwin, 2004), singing may even have preceded speech and might have been the primary method of human communication.

One can only speculate as to why our ancestors produced their first musical instruments; it may have been an attempt by early man to imitate natural sounds, such as the wind blowing through a hollow reed or the singing of a bird. The earliest known instruments are flutes, made from bone or mammoth ivory, dating back tens of thousands of years to the Palaeolithic Age. For example, in the Geißenklösterle cave near Blaubeuren in Germany, archaeologists uncovered the oldest known musical instrument – a primitive, but carefully constructed, swan-bone flute estimated to be at least 35 000 years old (Fig 1.1; Hahn and Münzel, 1995; Münzel *et al.*, 2002). Similar flutes have been found at other locations in Europe, suggesting that music was certainly a part of Stone Age life (see also Section 5.5).

But more sophisticated forms of music also have ancient roots. We know that lyres with many strings were in use in ancient Sumeria almost 5000 years ago. The ancient Egyptians included complex forms of music and dance in their everyday life and religious worship. Paintings in their tombs and temples show professional musicians and a wide variety of instruments like lyres, pipes, lutes and drums. Many of these instruments have survived and may be regarded as predecessors of our modern instruments.



Fig. 1.1 A 35 000-year-old swan-bone flute from the Geißenklösterle cave in Germany (Conard *et al.*, 2009). Its three holes allow playing harmonic tones, thereby supporting the idea that our harmonic sense must have biological roots.

1.2 The power of music and harmony

We know that music has been an important part of every culture and civilization throughout history, but what exactly it represents to us is still a mystery. At some time we have all experienced the inexplicable and magical power that some forms of music have over us, but the origin of this power remains elusive. The Austrian author Robert Musil expresses this enigma in the following way: 'The human mystery of music is not that it is music, but that with the help of a dried sheep gut it succeeds in bringing us nearer to God' (Musil, 1982).

It is often said that music 'speaks to our unconscious mind' and, indeed, it may be more subtle and obscure than all other forms of art. As a rule, music does not provide us with an identifiable portrayal or, at least, an association to our world, as is often the case with paintings or sculptures. It is abstract. Yet, when we listen to certain musical compositions the combinations of melody, harmony and rhythm may evoke profound, sometimes almost unworldly, emotions within us: 'in its physical effect, in the way it grabs one by the head and shoulders, it is one of those manifestations of beauty that border on the 'heavenly' and of which only music and no other art is capable' (Mann, 1997).

Thomas Mann is expressing the ecstatic feeling of 'musical chills' or 'shivers', a manifestation of the intense feelings of pleasure which most of us have experienced when listening to certain pieces of music (Gabrielsson, 2012; see also Section 12.3.2). Somehow, we are able to transcend reality, to be transported into another world. Modern imaging techniques have recently been used to map brain activity during these experiences, and have shown that brain areas associated with reward or euphoria (intense feelings of happiness) are especially strongly activated (Zatorre, 2003; see also Section 10.6).

Music also has an exceptionally strong ability to revive old memories. As we listen to our favourite symphonies, arias or pop songs, the emotions that are aroused may become united with vivid memories and our mood or even our behaviour may change. It is this mysterious, almost magical power that has made music a ubiquitous and fundamental expression of human life. In Chapter 12 I will discuss how this effect of emotional memory relates to the dynamics and oscillatory activity of our brain.

Music as a universal language

3

1.3 Music as a universal language

In a way music, and the harmony within it, could be considered a universal language, a form of communication stretching beyond the barriers of speech. Music has evolved in many different forms throughout the world and, as we all know, even within the same cultural environment musical tastes can be drastically different. Nature, how-ever, has provided us with the perceptual means to understand, and often enjoy, the varied musical traditions of different cultures. Although some aspects of unfamiliar forms of music may sound strange to our ears, all music seems to contain some crucial elements that we can immediately recognize and relate to. A major reason for this is that all humans share not only a similar sense of rhythm and pitch, but also a similar appreciation for musical harmony – that is, for certain harmonic intervals (Stumpf, 1890).

Our perception of these intervals is based on mathematical rules (see also Section 11.8) and, consequently, must be universal. A vivid depiction of this is the final scene of Steven Spielberg's movie *Close Encounters of the Third Kind*. In a clash between civilizations the American army faces an extraterrestrial spaceship at the foot of the Devil's Tower in Wyoming, and the question arises of whether and how these two radically different civilizations might be able to speak to each other.

The seemingly unsolvable communication gap is bypassed by the use of musical signals. The musical language 'Solresol', which bridged the chasm in the movie, was invented in the nineteenth century by Jean-François Sudre. It is based on a seven-note scale where each note acts as an element of language. Indeed, musical scales – the foundation of most musical compositions – are astonishingly similar in all cultures. Although the number of notes may differ, almost all scales are based on two major harmonic intervals: octaves and fifths.

We are able to recognize symmetry, proportion and balance in music, just as we can in visual art. In terms of music, harmony refers to certain relationships between tones of the same (prime) or different pitches. The pitch of a tone may be regarded as a subjective measure of its relative highness or lowness and characterizes its position in the complete range of tones. In our standard Western (diatonic) scale an interval of eight notes (an octave) represents the highest degree of musical harmony or consonance (after the prime). There is an indisputable 'sameness' about two tones an octave apart; they actually sound so similar that they are often confused with primes. For example, men and women singing together 'in unison' are usually singing the same notes an octave apart, although they may not be aware of it (something similar applies to communication signals of certain species of electric fish and mosquitoes; see Section 9.1). Correspondingly, musical systems everywhere, ancient and modern, use the interval of the octave as their indispensable cornerstone.

An interval of five notes (the fifth) is usually considered to be the most consonant interval after the octave and, consequently, is also a universal component of musical systems. The two notes in a fifth do not have the same degree of similarity as those in an octave, but do seem to fuse in a very natural and pleasing way (fusion; Stumpf, 1890).

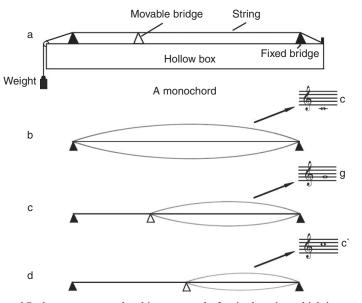
Harmonically related tones are perceived as consonant when played simultaneously, but they can also be combined in sequences to form melodies. It is clear that we are able to easily recognize such subsequent, harmonically related tones and find them aesthetically pleasing. One may ask what is different or unique about such harmonious sequences of tones and what exactly it is that we are receiving, processing and responding to (see Section 11.7). As far as we know, the first person who addressed these questions was the great mathematician and philosopher Pythagoras, about 2500 years ago.

1.4 Musical harmony and whole numbers

Pythagoras was born on the Greek island of Samos in the middle of the sixth century BC. According to Boethius (AD 480–524), who recorded the story more than 1000 years later (Boethius, 1989), Pythagoras noticed the musical and harmonious sounds that were sometimes produced by a blacksmith and his assistants as they pounded with their hammers on an anvil (Fig. 1.2a). His investigations showed that the sounds of the hammers had a pitch which depended on their weight, the heaviest hammer producing a lower pitch than the lighter ones. However, only some of the hammers produced a pleasant harmonious ringing when struck together. His conclusion was that harmonious sounds rang out only when the weights of the hammers being struck together were in



Fig. 1.2 The four drawings show Pythagoras entering the blacksmith's shop (a) and exploring the physical basis of pitch and harmony with bells and water-filled tumblers (b), a string instrument (c) and with different flutes (d) (from *Theorica musice* by F. Gaffurius (1492).



Musical harmony and whole numbers

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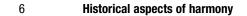
Fig. 1.3 (a) The pictured Pythagorean monochord is composed of a single string which is stretched by a weight and supported by three bridges, two fixed and one movable. (b) Plucking the whole string produces a certain tone. When the string length is shortened, for example to two-thirds or one-half of its original length, tones a fifth (c) or an octave above (d) are produced.

simple ratios. Pythagoras had discovered the association between the ratios of small whole numbers and musical consonance.

In order to test his theory, Pythagoras – so the story goes – invented a simple onestringed instrument known as a monochord (Fig. 1.3). Its movable bridge could be used to change the length of the string and thus the pitch of the tone that was generated. By dividing the string in half (ratio 1:2), the pitch increased by an octave. In addition, Pythagoras found two lesser consonances, the fifth (ratio 2:3) and the fourth (ratio 3:4). Today we know that the pitch of the tone produced is determined by the frequency (periodicity) of vibrations, which changes both with the length and the tension of a string.

Pythagoras realized that musical intervals can be arranged in a hierarchy; those with the ratios of small integers (so-called 'perfect' intervals such as the octave, fifth and fourth) are the most consonant. As the ratio becomes more complex, involving larger numbers, the sound becomes more dissonant. Simpler numerical relationships are thus associated with more pleasant (euphonious) combinations of sounds, revealing a connection between mathematics and aesthetics.

The musical ratios accepted by the Pythagoreans as pleasant (1:2, 2:3 and 3:4) involved only the first four integers. To them, these numbers developed their own mystical meaning; together they form the *tetraktys* (meaning 'fourness'), which was looked upon as a numerical representation of the orderly perfection of the cosmos



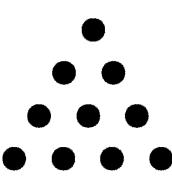


Fig. 1.4 The tetraktys, the symbol of harmony in nature and music, represents the four smallest integers which add up to 10.

(Fig. 1.4). Pythagoreans made an oath on this figure: 'By him who gave to our soul the tetraktys, the source and root of ever-flowing nature.' One can see that all of the three consonance ratios are included in this triangle as adjacent pairs of lines, starting from any of its vertices. Moreover, the sum of the first four integers is 10, which was considered to be of particular significance, a perfect number and a symbol of completeness (wikipedia.org/wiki/Tetraktys).

1.5 Universal harmony

The legend has it that from his pioneering experiments with the vibrating strings of his monochord, Pythagoras had succeeded in establishing a mathematical basis of melodic intonation. In his mind the relationship of whole numbers to musical consonances was seen as a mystery of such great significance that he went on to build an entire philosophical system based on these findings. In his metaphysical concept of 'universal harmony' Pythagoras attempted to explain the whole of creation in terms of numbers and mathematics. He was convinced that the entire universe, including the heavens as well as our minds, is based on a harmony expressed by the same integer relationships as the musical consonances.

Accordingly, the theory of mathematical proportions was a cornerstone of Pythagorean philosophy. It consisted of four pillars – arithmetic, geometry, astronomy and music. Astronomy was interpreted as numbers in motion, geometry as numbers at rest, arithmetic as numbers absolute and music as numbers applied. Of these four disciplines, music was considered to be the only one that directly involves our senses and thus has the potential to influence our behaviour. It is no wonder that the Pythagoreans saw music as a bridge between the transitory world of our physical experience and the eternal laws of nature – a way in which we can subconsciously appreciate the beauty of the mathematical order of the universe.

Harmony of the Spheres

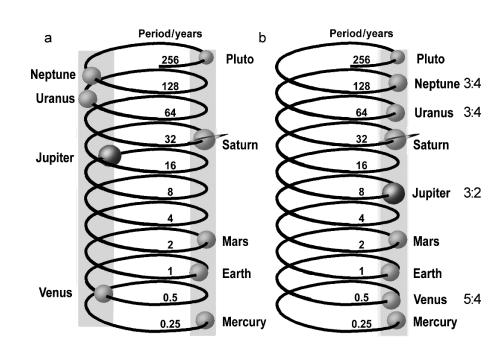
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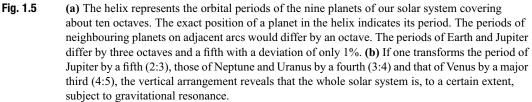
However, one should not fail to notice that Pythagorean musical theory also had its critics. As time progressed, pure mathematical ideals had become ever more entrenched in the minds of the Pythagoreans and the realities of hearing were increasingly neglected. Practising musicians, in particular, felt that the Pythagoreans were too interested in reducing musical sounds to numerical relationships, and were ignoring what was actually heard; musical harmony had become something to be judged by the intellect, not by the human ear.

Pythagorean number mysticism reached its pinnacle in the metaphysical ideals of Plato (428–347 BC). He stated that human perception must be at fault if musicians did not perform according to mathematical principles. Naturally, such extremely theoretical views provoked antagonism to Pythagorean musical theory. The most vocal critic of Plato's ideas about harmony was the musician Aristoxenus of Tarentum, born in the fourth century BC, and a pupil of Aristotle. His musical philosophy was in stark contrast to Plato's; he felt that a science of music should not be based on physics and mathematics but on the sounds the ear really hears. He dismissed their emphasis on arithmetical relationships as unimportant in terms of music itself: 'harmonic or musical properties attach only to what is heard' (Barker, 2012). In Chapter 11 we will conclude that this may be only an apparent contradiction because in a way the harmonic processing of our hearing system corresponds to mathematical principles.

1.6 Harmony of the Spheres

Pythagoras' idea of a musical-mathematical harmony permeating the universe led to the concept of the 'Harmony of the Spheres', an appealing idea that united music with astronomy. Pythagoras thought of the cosmos as a huge lyre with crystal spheres instead of strings. In this system, the Earth was in the centre and the sun, moon and planets were attached to the crystal spheres. The distances of the planets from Earth were ordered as in a musical scale, with the interval between the Earth and an outer sphere of fixed stars being equal to an octave. The relative speeds of the planets, in accordance with their distances from Earth, were thus in the same proportions as the musical consonances (compare Fig. 1.5). As the planets revolved they each emitted a tone and a wonderfully harmonious sound was produced, the Musica Mundana, or 'music of the spheres'. This 'cosmic music', which, as his disciples believed, only Pythagoras could hear, was thought to reveal the sound of the harmony of the creation of the universe (see below). The theme of a harmonically constructed universe was revisited several more times in the ensuing centuries, first by Plato and then most notably in the late sixteenth century by Johannes Kepler (Helmholtz, 1863: p. 375). More recently, Paul Hindemith's opera 'The Harmony of the World' about Kepler focuses on the same idea. In Hindemith's theories of musical harmony, a tonal centre can be identified with the sun and other tones with planets at varying distances from the sun.





1.7 Harmony in modern astrophysics

Pythagorean number mysticism may seem to us today to be completely metaphysical. It must be emphasized, however, that apparently never before had anyone attempted to describe natural phenomena using mathematics. Pythagoras' emphasis on the importance of whole numbers in the order of nature transformed both science and music theory, and his observations can thus be seen as pioneering the study of natural science as we now know it. As a result of his discoveries, mathematics became inextricably linked with science and philosophy, and science became joined with music. Until the seventeenth century music was considered a branch of science, and scientific experimentation was often directed at solving musical problems.

Nowadays we have become used to scientific reasoning based on experimental observation and it therefore appears remarkable that the Pythagoreans constructed their view of a mathematically governed universe with scarcely any actual evidence. It was not until more than 2000 years later that Sir Isaac Newton formulated his laws of motion and gravitation, and finally showed that the universe could indeed be explained in terms of mathematics.

Harmony in modern astrophysics

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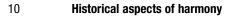
On the other hand, today we also know that planets do indeed tend to synchronize their travels around the sun. For example, the relations of the periods of Uranus, Neptune and Pluto are quite close to 1:2:3 (84, 164, 248 years). Also, the moons of Jupiter have orbital periods in nearly octave relationships, namely 1, 2, 4 and about 8 times 1.8 days (Murray and Dermott, 1999).

It has been known for centuries that the planetary orbits of our solar system are regularly spaced – this was described by an empirical rule, the 'Titius–Bode law'. Astrophysicists believe that this empirical law is most likely based on resonance (the preference for a certain oscillation frequency), or near resonance, of the periodic movements of the planets around the sun (Dermott, 1973; Torbett *et al.*, 1982).

The extent to which our solar system is resonating may be visualized by means of a helix with ten turns: the helix in Fig. 1.5a covers periods from 0.24 to 256 years, where each turn represents a certain octave. Symbols for planets are shown at locations corresponding to their periods. As the figure shows, planets whose periods differ by about one or more octaves $(1:2^n, n = 1, 2, 3...)$ line up roughly along vertical chains. There are two chains located on opposite sides of the helix; the underlying grey bars on opposite sides are separated by about a fifth in addition to the vertical octave. However, the periods of all planets (with the exception of Neptune and Pluto) differ by larger intervals including additional octaves, fifths, fourths and thirds. More precisely, if one ignores octaves a Jupiter is separated from the Earth-group (Mercury, Earth, Mars, Saturn, Pluto) by a fifth (3:2), Neptune and Uranus by a fourth (4:3) and Venus by a major third (5:4). If one transforms the periods of Jupiter, Neptune, Uranus and Venus, they differ by about one or several octaves and, accordingly, all symbols of planets line up roughly vertically (Fig. 1.5b). Obviously, to a first approximation, the whole solar system from Mercury to Pluto is subject to gravitational resonances and, as a consequence, to some extent obeys simple harmonic rules.

Furthermore, the 2006 Nobel Prize for Physics was awarded to John Mather and George Smoot for their measurements of the 'cosmic microwave background radiation'. Extremely precise measurements of the temperature (to one part in 10^5) of this cosmic radiation have shown that 'the early universe resounded with harmonious acoustic oscillations' (Hu and White, 2004). The harmonic sound waves were connected with cosmic radiation, which could escape only 400 000 years after the Big Bang, when neutral atoms were built. Since they could not absorb the electromagnetic waves the harmonic echo of the Big Bang can still be measured today in the sky (Fig. 1.6).

Many scientific developments join that of the cosmic symphony. For instance, the theory of strings postulates that the smallest ultimate particles of our world resemble vibrating strings. Obviously, the idea of cosmic harmony, including the role of resonance and ratios of small integers, is still alive. In this respect, Pythagoras was obviously right. But what about the complementary part of his philosophy: is there indeed a particular role for resonance and small integers in our mind? If we could understand the neuronal basis of pitch perception, would it clarify the role of numbers and ratios for the perception of harmony? Could the association between consonance and simple ratios



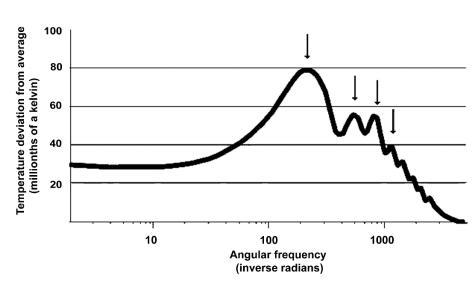


Fig. 1.6 Tiny temperature differences of cosmic microwave radiation across the sky reveal the harmonic acoustic vibrations of the universe 'shortly' after the Big Bang. The first peak in the power spectrum is caused by the fundamental frequency of the 'echo' of the early universe; the subsequent peaks show the effects of the overtones (see arrows; modified from Hu and White, 2004).

be explained by the way nerve cells in our auditory system process pitch information and by the way they react to harmonic sounds? And, moreover, is there also a possible role of resonance and harmony in non-auditory systems of the brain?

These are the final questions that are addressed in Chapters 11 and 12 of this book. But first, some historic facts and basic knowledge about sound is presented in Chapter 2, while Chapters 3 and 4 are devoted to key historical developments and discoveries which have formed the foundation of modern theories of pitch perception. Chapter 5 will offer some new evidence for an auditory time constant and its role in animal communication, speech and music. Moreover, after an introduction to the neurobiology of our hearing system (Chapter 6) the subsequent chapters provide essential research results and theories concerning the neuronal processing mechanisms (Chapters 7–9) and the spatial representation (Chapter 10) of pitch.