THEORY OF ORBIT DETERMINATION

Determining orbits of natural and artificial celestial bodies is an essential step in the exploration and understanding of the Solar System. However, recent progress in the quality and quantity of data from astronomical observations and spacecraft-tracking has generated orbit determination problems which cannot be handled by classical algorithms. This book presents new algorithms capable of handling the millions of bodies which could be observed by next-generation surveys, and which can fully exploit tracking data with state-of-the-art levels of accuracy.

After a general mathematical background and summary of classical algorithms, the new algorithms are introduced using the latest mathematical tools and results, to which the authors have personally contributed. Case studies based on actual astronomical surveys and space missions are provided, with applications of these new methods. Intended for graduate students and researchers in applied mathematics, physics, astronomy, and aerospace engineering, this book is also of interest to non-professional astronomers.

ANDREA MILANI is Full Professor of Mathematical Physics in the Department of Mathematics, University of Pisa. His areas of research include the *N*-body problem, the stability of the Solar System, asteroid dynamics and families, satellite geodesy, planetary exploration, orbit determination, and asteroid impact risk.

GIOVANNI F. GRONCHI is a Researcher of Mathematical Physics in the Department of Mathematics, University of Pisa. His research is on Solar System body dynamics, perturbation theory, orbit determination, singularities, and periodic orbits of the *N*-body problem.

COVER ILLUSTRATION: The orbits of eight potentially hazardous asteroids (PHA); they have a minimum intersection distance with the orbit of the Earth of less than 0.05 astronomical units. Together with many more smaller objects, they form a swarm surrounding the orbit of our planet (represented, not to scale, in green, orbit in yellow), are observable with either telescopes or radar, and provide a good example of an orbit determination problem. The objects in this figure are the brightest PHA, with diameters larger than 2 km; thus an impact with the Earth would result in a global catastrophe. There has been interesting recent progress in the theory of orbit determination, to which the authors of this book have contributed. New algorithms have been developed to exclude the possibility that any of these objects have the possibility of impacting the Earth, at least in the next 100 years. The same result also applies to somewhat smaller PHA, but the impact of either a much smaller known asteroid or an asteroid still to be discovered is still possible; thus the orbit determination work must go on. The orbit diagram is superimposed on an actual image of the sky (courtesy of G. Rhemann, Astrostudio, Vienna) which includes a Solar System body: a comet discovered in 2008 by A. Boattini, showing its coma.

Cambridge University Press & Assessment
978-0-521-87389-5 — Theory of Orbit Determination
Andrea Milani, Giovanni Gronchi
Frontmatter
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ANDREA MILANI AND
GIOVANNI F. GRONCHI
Department of Mathematics, University of Pisa

CAMBRIDGE
UNIVERSITY PRESS

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Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

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a department of the University of Cambridge.

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www.cambridge.org
Information on this title: www.cambridge.org/9780521873895

First published 2010

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data
Milani, Andrea.
Theory of orbit determination / Andrea Milani, Giovanni Gronchi.
p. cm.
ISBN 978-0-521-87389-5 (hardback)
1. Orbit determination. 2. Celestial mechanics. I. Gronchi, Giovanni (Giovanni
Federico) II. Title.
QB355.M55 2009
521´.3 – dc22 2009034270

ISBN 978-0-521-87389-5 Hardback

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PREFACE

This book is a tool for our own teaching and an opportunity to rethink and reorganize the results of our own research. However, I think such a book can be useful to others, for two main reasons. First, spaceflight is no longer the privilege of the few superpowers, but is becoming available to many nations and agencies. Orbit determination is an essential knowhow, both in the planning phase of mission analysis and in the operations of space missions. Thus its mathematical tools need to become widely available.

Second, the knowledge and skill used in orbit determination, for both natural and artificial celestial bodies, was available only among a restricted group of specialists. The prevailing attitude was a proprietary one: the knowledge and the software were protected by formal copyright and/or by secrecy, although protecting in this way the pure mathematical theory is, in the long run, impossible. This attitude might have been justified under the conditions of the world of 30–40 years ago, in the critical phases of the competition to achieve *space firsts*. Now it is time to teach and disseminate this knowledge, allowing the formation of a wider group of specialists.

I know that many of the *rules of thumb* and practical advice contained in this book will be rated as well known, even obvious, by the few experts, but this is not the point. Even well-known results may need to be presented in a rational, rigorous, and didactically effective new way, together with the outcome of recent innovative research. On the other hand, this book does not have the intent of providing a comprehensive review of all that has been done in this field, because the size would become impractical. This book is about making widely available the outcome of the research done by my group over many years, and includes methods for which there are rigorous mathematical arguments and which have been fully tested by us first hand, and found to be effective. In the last $\simeq 15$ years there has been enormous progress in this field, and several other research groups have given important contributions: we are in no way claiming that their methods would not work, we are just giving a list of methods which we know to work.

The above arguments may not be enough for the approval of all the people in this field, but I do think that to state the mathematical foundations and rules of orbit determination, thus removing a vague flavor of craftmanship, can also benefit the already existing specialists. The orbit determination expert, in the very competitive environment in which space missions and large astronomical projects are selected today, is too often under pressure to endorse claims of wonderful results to be achieved with very limited means. By ignoring the rules of good practice it is possible to claim illusory precision and/or completeness for the solution, including the orbits and other parameters which can be operationally, technologically, and scientifically relevant. Maybe being able to cite a textbook stating clearly what is appropriate and what is illusory can help in relieving this improper pressure.

This book is based on the experience accumulated in $\simeq 30$ years of research with my coworkers of the former *Space Mechanics Group* (now *Celestial Mechanics Group*) at the Department of Mathematics, University of Pisa. Thus it contains, besides the formal mathematical theory and the teaching examples, a number of "case studies" based upon actual research projects. They are about space missions and about natural objects: one of the goals is to stress the common mathematics used in satellite geodesy and in dynamical astronomy, and at the same time to present clearly the main differences.

The preparation of this book has been made possible by the collaboration of my younger colleague, Dr. Giovanni F. Gronchi. Besides classical material and original results by myself and Gronchi, this book contains the output of research done by the members of our group and by either regular or occasional external coworkers. Thus I would like to include a long, but still possibly incomplete, list of coworkers whose contributions have to be acknowledged: L. Anselmo, O. Arratia, S. Baccili, A. Boattini, C. Bonanno, M. Carpino, G. Catastini, L. Cattaneo, S.R. Chesley, S. Cicalò, L. Denneau, L. Dimare, P. Farinella, D. Farnocchia, Z. Knežević, L. Iess, R. Jedicke, A. La Spina, M. de' Michieli Vitturi, A.M. Nobili, A. Rossi, M.E. Sansaturio, G. Tommei, G.B. Valsecchi, D. Villani, D. Vokrouhlický.

This book is dedicated to two good friends and valuable coworkers: Paolo Farinella and Steve Chesley. They could have been among the authors of this book, but they both left in the year 2000, when the book project was immature. Steve went back to his home country, from where he can still advise me on these subjects. Paolo went where he can give me neither his essential scientific insight nor the warmth of his friendship. Thus I would like to thank both of them for what I learned with them and from them.

Andrea Milani Comparetti, Pisa, December 2008