Every day thousands of decisions are made by all kinds of committees, parliaments, councils and boards by a ‘yes–no’ voting process. Sometimes a committee can only accept or reject the proposals submitted to it for a decision. On other occasions, committee members have the possibility of modifying the proposal and bargaining an agreement prior to the vote. In either case, what rule should be used if each member acts on behalf of a different-sized group? It seems intuitively clear that if the groups are of different sizes then a symmetric rule (e.g. the simple majority or unanimity) is not suitable. The question then arises of what voting rule should be used. *Voting and Collective Decision-Making* addresses this and other issues through a study of the theory of bargaining and voting power, showing how it applies to real decision-making contexts.

**Annick Laruelle** is Professor of Economics at the University of Caen, Lower Normandie.

**Federico Valenciano** is Professor of Mathematics and Game Theory at the University of the Basque Country, Bilbao.
To my parents
Annick

For loved ones gone and those still around
Federico
Contents

List of figures x
Preface xi

1 Preliminaries 1
  1.1 Basic set-theoretic notation 1
  1.2 Some combinatorics 2
    1.2.1 Permutations and combinations 2
    1.2.2 Some useful approximations 3
  1.3 Voting rules 4
    1.3.1 Dichotomous voting rules 5
    1.3.2 Some particular voting rules 7
  1.4 Expected utility theory 10
    1.4.1 Players, games and game theory 10
    1.4.2 Preferences and utility 10
    1.4.3 Lotteries and expected utility 11
    1.4.4 Expected utility preferences 13
  1.5 Some basic game theory notions 18
    1.5.1 Equilibrium 19
    1.5.2 Cooperative and non-cooperative game theory 20
    1.5.3 Subgame perfect equilibrium 21
    1.5.4 Basic cooperative models 24
  1.6 Exercises 26

2 Seminal papers, seminal ambiguities 30
  2.1 Seminal papers and seminal ambiguities 30
    2.1.1 Nash (1950): The bargaining problem 30
    2.1.2 Shapley (1953): The value of a TU game 34
    2.1.3 Shapley–Shubik (1954): A power index 37
    2.1.4 Banzhaf (1965): Power as decisiveness 39
    2.1.5 Penrose (1946), Rae (1969) and Coleman (1971) 41
## Contents

2.1.6 Through the axiomatic glasses: Dubey (1975), Dubey–Shapley (1979) 41  
2.2 Clear-cut models to dissipate ambiguity 44  
2.3 Further reading 46  
  2.3.1 Axiomatic approach 47  
  2.3.2 Probabilistic approach 48  
2.4 Exercises 48  

3 ‘Take-it-or-leave-it’ committees 52  
  3.1 The take-it-or-leave-it scenario 52  
  3.2 Success and decisiveness in a vote 54  
  3.3 Preferences, behaviour and probabilities 55  
  3.4 Success and decisiveness ex ante 57  
  3.5 A priori assessments based on the voting rule 60  
    3.5.1 Rae index 62  
    3.5.2 Banzhaf(–Penrose) index 62  
    3.5.3 Coleman indices 63  
    3.5.4 König and Bräuninger’s inclusiveness index 65  
    3.5.5 Summary and remarks 65  
  3.6 Success versus decisiveness 67  
    3.6.1 Success is the issue in a take-it-or-leave-it scenario 67  
    3.6.2 Conditional success 69  
    3.6.3 Summary 70  
  3.7 The choice of voting rule: egalitarianism and utilitarianism 71  
    3.7.1 Egalitarianism 74  
    3.7.2 Utilitarianism 74  
  3.8 The choice of voting rule in a committee of representatives 77  
    3.8.1 An ideal two-stage decision procedure 78  
    3.8.2 Egalitarianism in a committee of representatives 81  
    3.8.3 Utilitarianism in a committee of representatives 87  
  3.9 Exercises 97  
  3.10 Appendix 99
4 Bargaining committees

4.1 The bargaining scenario 106

4.2 A model of a bargaining committee: voting rule and voters’ preferences 107

4.3 Cooperative game-theoretic approach 109
   4.3.1 Rationality conditions 110
   4.3.2 Axiomatic characterizations 112
   4.3.3 Discussion 116

4.4 A non-cooperative model of a bargaining committee 117
   4.4.1 Probabilistic protocols 119
   4.4.2 Bargaining protocols under a voting rule 123
   4.4.3 Discussion 127

4.5 Egalitarianism and utilitarianism in a bargaining committee 128

4.6 The neutral voting rule in a committee of representatives 130

4.7 Exercises 134

5 Application to the European Union

5.1 Voting rules in the European Council 136

5.2 The Council as a take-it-or-leave-it committee 142
   5.2.1 Criteria based on probabilities 143
   5.2.2 Criteria based on utilities 156

5.3 The Council as a bargaining committee 163

5.4 Exercises 170

Conclusions 172

References 176

Index 182
Figures

1.1 Battle of the sexes in sequential form  
2.1 A bargaining problem:  
   (a) Classical à la Nash model  
   (b) Assuming ‘free disposal’  
2.2 The Nash bargaining solution  
2.3 A three-person bargaining problem  
4.1 Continuation payoffs after the choice of proposer in a three-person problem
Preface

The important changes that have taken place in the European Union as a result of the latest enlargements have made it necessary to redesign decision-making procedures again and again. This has contributed to a renewal of interest in issues related to the choice and design of dichotomous voting procedures in recent years, to a conspicuous increase in the number of academic papers, both theoretical and applied, related in one way or another to these issues and to heated debates within the scientific community. As a result of this ‘fever’ there have been various movements within this community that have gone beyond the academic realm, including press articles and explicit attempts to influence politicians or their advisers on the choice of voting rule for the EU Council of Ministers. At the basis of some of these recommendations is what is called, perhaps a little ostentatiously, ‘a priori voting power theory’. The main purpose of this book is to provide a critical revision of the foundations of this theory and of the recommendations that stem from it, based on more than ten years of joint research on the subject.

Prior to this collaboration, the first author of this book was preparing her Ph.D. One of the chapters of her thesis sets out the application to the EU Council of the two-stage model of the decision-making process in committees of representatives\(^1\). This model assumes that each representative follows the will of the majority in his/her constituency on every issue. Then, assuming that each citizen votes ‘yes’ or ‘no’ independently with probability 1/2, one can calculate the probability of a citizen being crucial or decisive for a given voting rule in the committee. This (usually very small) probability is interpreted as the ‘a priori voting power’ of the citizen and is known as the citizen’s ‘Banzhaf index’.

If this interpretation is accepted, egalitarianism recommends choosing a voting rule that gives equal Banzhaf indices to all citizens whatever their constituency. This recommendation is known as the (first) ‘square

\(^1\) Joint work with Mika Widgrén [52].
root rule’ because it entails choosing the rule for which each representative in the committee has a Banzhaf index proportional to the square root of the population that he/she represents.

This model was also the starting point of our joint research. Since then our views have changed considerably, to the extent that we refused to sign a letter addressed to the EU Governments and supported by a group of scientists endorsing the square root rule as the choice of voting rule for the EU Council\(^2\). Ten years of work lie behind this shift of views.

As shown in [52], citizens of different countries had different Banzhaf indices for the qualified majority rule in the fifteen-member EU. Our first endeavour was to seek a measure of inequality in this context [41, 43], but we soon turned our attention to the foundations. Why the Banzhaf index? Why not the Shapley–Shubik index, apparently preferred by game theorists, or any other ‘power index’? We first addressed the question of the axiomatic foundations of power indices in the framework of simple games [39, 40, 42] only to honestly conclude that there were no conclusive arguments for the superiority of any of them on these grounds alone. We then turned our attention to the probabilistic approach [37, 44, 45, 46]. In this approach voters’ behaviour is described by a probability distribution over vote configurations, and power indices are interpreted as probabilities either of being decisive or of obtaining one’s preferred outcome. This point of view led us to adhere for a while to the Banzhaf index as the best-founded index, but we soon grew increasingly dubious about its consistency. One of the factors that contributed to these doubts was our critical examination of the so-called ‘postulates and paradoxes’ so popular in the literature on power indices, their inconsistencies and their lack of real discriminating capacity [47]. To our surprise, the notion of success or satisfaction, i.e. the likelihood of obtaining one’s preferred outcome, which is inextricably intermingled with decisiveness in any pre-conceptual notion of voting power, behaved even better than decisiveness with respect to some postulates. This sparked doubts concerning the soundness of the notion of voting power as the likelihood of being decisive, and led us to consider the notion of success or satisfaction as the relevant issue in certain voting situations [38]. On the other hand, a most inspiring interview in 2002 with David Galloway, who had twenty years

\(^2\) Available at www.esi2.us.es/mbilbao/pdffiles.letter.pdf
Preface

of experience working for the Council of Ministers of the European Union, made it clear to us that bargaining was a (if not ‘the’) crucial ingredient in the workings of the Council. Bargaining is a genuine game situation that calls for a game-theoretic approach. We thus considered an alternative model, one of whose primitives was the preference profile over the feasible agreements [48, 51]. It thus seemed clear that the analysis of voting situations required a preliminary description of the voting environment: a small committee does not make the same use of a voting rule as a Parliament. The model cannot include the voting rule as the unique ingredient, it must be enriched to describe the specificity of the voting environment.

In this way, a gradual process of accumulative reflection drove us finally to a radical change in our way of looking at several basic issues. This book presents our proposal for new foundations along with a systematic presentation of the changes that this entails in the whole theoretic edifice.

To begin with, a clear distinction must be drawn at the level of the environment between two extreme types of collective decision-making bodies or committees: committees with the capacity solely to accept or reject proposals submitted to it, and committees with the capacity to bargain among feasible agreements. This at first sight obvious distinction proves rich in conceptual consequences. First, it clarifies what one is talking about, something which has been established only vaguely from the outset in the voting power tradition, where the voting rule is the only clearly specified ingredient. Second, each type of situation requires a different model and a separate analysis. This neat distinction also clarifies the different issues posed by each type of decision-making environment. It is worth remarking here that the question of power is not the primary or basic issue in either case. Moreover, in the first type of committee (which we call ‘take-it-or-leave-it committees’), where behaviour immediately follows preferences, the notion of voting power does not even make sense. In contradistinction, in a ‘bargaining committee’ the notion of bargaining power in a genuinely game-theoretic sense emerges as related to the likelihood of being decisive.

The normative recommendations that stem from this approach differ conspicuously from those based on the traditional approach, particularly for the choice of voting rule in a committee of representatives whose members act on behalf of groups of different sizes. The square root rule recommendation alluded to above appears in this light as
correct but distorting and ill-founded if the goal is to obtain the preferred outcome. It can be re-founded in expected utility terms, but its possible validity (as well as that of the so-called ‘second square root rule’) is restricted to ‘take-it-or-leave-it committees’, where the very notion of voting power is irrelevant. By contrast, in the case of bargaining committees of representatives, the model yields completely different recommendations. But this is not the place to anticipate our conclusions in detail (impatient readers may skip to the Conclusions section at the end of the book).

We hope that by this point the readers will have a clear idea of what this book is about, and will perhaps understand how hard it was for us to find a title that was clear and concise enough. We were reluctant to include the words ‘voting power’ in the title, in spite of the fact that it is precisely those interested in voting power issues who will probably be most interested in the book. We believe that the ‘sex-appeal’ of these words is responsible to some extent for the obscurities that have survived for so long at the root of the topic. The feeling of importance that comes with the use of the word ‘power’ only makes a humble, rigorous and detached analysis more difficult (just the opposite of ‘game theory’, a frivolous name for an ambitious research programme).

The monograph most closely related to this book is The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes by Felsenthal and Machover [22], published in 1998. It was a valuable attempt to conduct a critical revision of the foundations of traditional voting power theory, a tradition in which inertia, disregard of its inconsistencies and obscurities and the mechanical application of different indices on no clear grounds were the rule. These authors stress a distinction between two notions of voting power: ‘I-power’, or power to influence the outcome, and ‘P-power’, the expected share in a fixed prize. They hold that there are several points which support this distinction. For instance, I-power is a probabilistic notion related to ‘policy seeking’, while P-power is a game-theoretic notion related to ‘office seeking’. The Banzhaf index is considered as the right measure of a priori I-power, and other candidates are rejected on the basis that they violate some I-power ‘postulates’ (i.e. supposedly desirable properties whose violations are referred to as ‘paradoxes’). As to the second type of power, the Shapley–Shubik is presented with reserves as the most serious known candidate for a measure, but doubts are explicitly cast on the coherence of the very notion of P-power.
When their book appeared, we agreed with their critical views and found the intuition behind their I/P-power distinction to be basically correct. However, we felt that the distinction was too vague and insufficient. Moreover, as time passed, we felt that accepting it as a satisfactory remedy for the obscurities at the level of the foundations was only a conformist way of hindering a real progress in resolving the lack of clarity at a deeper level. We also found the foundations of the P-power notion unconvincing, to the extent that, as stated above, we adhered for a while the notion of voting power as influence as the only coherent notion of voting power, and discarded the Shapley–Shubik index. Interested readers will find a brief account of the different implications of the their approach and ours in the Conclusions section.

Another related book is Morriss’ *Power: A Philosophical Analysis* [58, 59]. This author is also critical with the frequently unjustified applications of power indices. He conducts a careful discussion about the semantics of the word ‘power’, distinguishing between ‘power-as-ability,’ and ‘power-as-abledness’. Indeed the book is intended to be more philosophical and less formal than ours. It may provide interesting additional reading (especially Part IV).

Our purpose is not to survey all the huge amount of material on the topic published over more than fifty years, though we do, of course, pay attention to what we consider the most significant contributions in the field. A few seminal contributions are presented in some detail.

As is clear from its title, in this book we consider only dichotomous voting rules that specify collective acceptance or collective rejection for each possible yes–no vote profile. We do not consider the possibilities of abstention or not showing up. These conditions preclude the difficulties evidenced by Arrow’s [2] impossibility theorem when more than two alternatives are involved, and the possibility of ‘strategic’ voting [27]. Consequently, the copious social choice literature on these issues is orthogonal to this book.

We hope the book may be of interest and of use to students and researchers alike in political and social science, as well as in game theory and economics, especially the public economics, public choice, and social choice families. We try to present results and, especially, normative recommendations in an honest, humble, precise ‘if . . . then’ form, in which the ‘if’ part is explicit and transparent. This requires a formal formulation. However, to make the book accessible to as wide an audience as possible, we have tried to keep the level of formalization low.
enough not to discourage readers with less mathematical backgrounds but at the same time high enough to be precise. The meaning of formal statements is always expressed in plain words. Some proofs, especially in the case of technically complex published results, have been omitted. Chapter 4 may perhaps be the most difficult for those readers not familiar with game theory. Nevertheless, it is our hope that readers with less mathematical backgrounds can get a grasp of the main ideas presented in the book by skipping the mathematical details and just reading the rest.

The book is organized as follows. Chapter 1 presents the basic set-theoretic notation and some combinatorics, along with the notation and terminology on dichotomous (acceptance/rejection) voting rules used throughout the book. This chapter also contains two brief sections devoted to the basics of expected utility theory and a summary overview of a few basic concepts of game theory. In Chapter 2 a few seminal papers are briefly and critically reviewed, and the basic distinction in this book between ‘take-it-or-leave-it’ and ‘bargaining’ committees is introduced. Chapter 3 is devoted to ‘take-it-or-leave-it’ committees, a probabilistic model of which allows us: (i) to address the question of the conceptual and analytical distinction between the notions of success and decisiveness; (ii) to provide a common perspective in which several ‘power indices’ from the literature can be seen as variations of two basic ideas; and (iii) to address the question of the optimal voting rule in a take-it-or-leave-it committee from two points of view: egalitarianism and utilitarianism. Chapter 4 deals with bargaining committees. A game-theoretic model is proposed, and the question of the players’ expectations is addressed first from a cooperative-axiomatic point of view, and then from a non-cooperative point of view. This chapter concludes with a recommendation for bargaining committees of representatives. Finally, in Chapter 5 the different rules used in the EU Council and some more recent proposals are examined from the point of view of the models presented in Chapters 3 and 4. A section with exercises at the end of each chapter is intended to provide readers with a means to check how well they have understood the chapters.

We would like to end this preface by expressing our gratitude to some people and institutions. We would like to thank people of various fields such as Fuad Aleskerov, Steve Brams, Nimrod Megiddo, Hans Peters, and Stef Tijs, who independently suggested to us the stimulating
notion of writing a book on this subject before the idea had crossed our minds.

We also thank Dan Felsenthal and Moshé Machover, our main scientific opponents in the last years. In spite of our sometimes overheated scientific arguments (particularly in their interesting and controversial ‘petit comité’ ‘VPP’s’ meetings, to which they have never failed to invite us every year since the first meeting in 2001), our disagreements have always been stimulating and inspiring.

We also thank Jon Benito, Arri Chamorro, Elena Iñarra, Jean Lainé, Vincent Merlin, Maria Montero, Stefan Napel and Norma Olaizola who read some chapters and made valuable suggestions on how to improve them. Thanks also go to William Thomson, who taught the second author to make drawings, and to Chris Pellow who did his best to make our English sound better. It goes without saying that all mistakes and defects are entirely our responsibility.

Finally, we thank Chris Harrison and Philip Good, of Cambridge University Press. Chris encouraged us to present our project to Cambridge, and Philip has been in charge of in the latter stages.

We are also grateful for the financial support received since we committed ourselves to this project, from the Spanish Ministerio de Educación y Ciencia under projects BEC2003-08182 and SEJ2006-05455, the latter co-funded by the ERDF, from Acción Integrada HF-2006-0021, and from the French Government under the EGIDE-Picasso project. The first author also acknowledges financial support from the Spanish M.E.C. under the Ramón y Cajal Program at the earliest stages of this project.

Though less apparent, support from colleagues, friends and families was also important and is warmly acknowledged.

March 2008

Annick Laruelle
Federico Valenciano