

## **An Introduction to Description Logic**

Description logic (DL) has a long tradition in computer science and knowledge representation, being designed so that domain knowledge can be described and so that computers can reason about this knowledge. DL has recently gained increased importance since it forms the logical basis of widely used ontology languages, in particular the web ontology language OWL.

Written by four renowned experts, this is the first textbook on Description Logic. It is suitable for self-study by graduates and as the basis for a university course. Starting from a basic DL, the book introduces the reader to its syntax, semantics, reasoning problems and model theory, and discusses the computational complexity of these reasoning problems and algorithms to solve them. It then explores a variety of different description logics, reasoning techniques, knowledge-based applications and tools, and describes the relationship between DLs and OWL.

# An Introduction to Description Logic

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