POTENTIAL FLOWS OF VISCOUS AND VISCOELASTIC FLUIDS

The goal of this book is to show how potential flows enter into the general theory of motions of viscous and viscoelastic fluids. Traditionally, the theory of potential flow is presented as a subject called "potential flow of an inviscid fluid"; when the fluid is incompressible, these fluids are, curiously, said to be "perfect" or "ideal." This type of presentation is widespread; it can be found in every book and in all university courses on fluid mechanics, but it is deeply flawed. It is never necessary and typically not useful to put the viscosity of fluids in potential (irrotational) flow to zero. The dimensionless description of potential flows of fluids with a nonzero viscosity depends on the Reynolds number, and the theory of potential flow of an inviscid fluid can be said to rise as the Reynolds number tends to infinity. The theory given here can be described as the theory of potential flows at finite and even small Reynolds numbers.

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CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

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www.cambridge.org
Information on this title: www.cambridge.org/9780521873376

First published 2007

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data
Joseph, Daniel D.
Potential flows of viscous and viscoelastic fluids / Daniel D. Joseph, Toshio Funda, Jing Wang.
p. cm. – (Cambridge aerospace series ; 21)
Includes bibliographical references and index.
ISBN-13: 978-0-521-87337-6 (hardback)
ISBN-10: 0-521-87337-1 (hardback)
1. Viscous flow. 2. Viscoelasticity. I. Funada, Toshio, 1948–
II. Wang, Jing, 1979– III. Title. IV. Series.
QA929.J67 2007
532.0533–dc22 2006039193

ISBN 978-0-521-87337-6 Hardback

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Preface

Potential flows of incompressible fluids with constant properties are irrotational solutions of the Navier–Stokes equations that satisfy Laplace's equation. How do these solutions enter into the general problem of viscous fluid mechanics? Under certain conditions, the Helmholtz decomposition says that solutions of the Navier–Stokes equations can be decomposed into a rotational part and an irrotational part satisfying Laplace's equation. The irrotational part is required for satisfying the boundary conditions; in general, the boundary conditions cannot be satisfied by the rotational velocity, and they cannot be satisfied by the irrotational velocity; the rotational and irrotational velocities are both required and they are tightly coupled at the boundary. For example, the no-slip condition for Stokes flow over a sphere cannot be satisfied by the rotational velocity; harmonic functions that satisfy Laplace's equation subject to a Robin boundary condition in which the irrotational normal and tangential velocities enter in equal proportions are required.

The literature that focuses on the computation of layers of vorticity in flows that are elsewhere irrotational describes boundary-layer solutions in the Helmholtz decomposed forms. These kinds of solutions require small viscosity and, in the case of gas–liquid flows, are said to give rise to weak viscous damping. It is true that viscous effects arising from these layers are weak, but the main effects of viscosity in so many of these flows are purely irrotational, and they are not weak.

The theory of purely irrotational flows of a viscous fluid is an approximate theory that works well especially in gas–liquid flows of liquids of high viscosity at low Reynolds numbers. The theory of purely irrotational flows of a viscous fluid can be seen as a very successful competitor to the theory of purely irrotational flows of an inviscid fluid. We have come to regard every solution of free-surface problems in an inviscid liquid as an opportunity for a new study. There are hundreds of such opportunities that are still available.

The theory of irrotational flows of viscous and viscoelastic liquids that is developed here is embedded in a variety of fluid mechanics problems ranging from cavitation, capillary breakup and rupture, Rayleigh–Taylor and Kelvin–Helmholtz instabilities, irrotational Faraday waves on a viscous fluid, flow-induced structure of particles in viscous and viscoelastic fluids, boundary-layer theory for flow over rigid solids, rising bubbles, and other topics. The theory of stability of free-surface problems developed here is a great improvement of what was available previously and could be used as supplemental text in courses on hydrodynamic stability.

We have tried to assemble here all the literature bearing on the irrotational flow of viscous liquids. For sure, it is not a large literature, but it is likely that despite an honest effort we missed some good works.

We are happy to acknowledge the contributions of persons who have helped us. Terrence Liao made very important contributions to our early work on this subject in the early 1990's. More recently, Juan Carlos Padrino joined our group and has made truly outstanding contributions to problems described here. In a sense, Juan Carlos could be considered to be an author of this book and we are lucky that he came along. We are indebted to G. I. Barenblatt and to K. R. Sreenivasan for their support and help in promoting viscous potential flow as a topic at the foundation of fluid mechanics. The National Science Foundation has supported our work from the beginning.

We worked day and night on this research; Funada in his day and our night and Joseph and Wang in their day and his night. The whole effort was a great pleasure.

List of Abbreviations

2D	two-dimensional
3D	three-demensional
BEM	boundary-element method
BU	Benjamin and Unsell
C/A	convective–absolute
c.c.	complex conjugate
DM	dissipation method
ES	exact solution
FHS	fully hydrodynamic system
FVF	fully viscous flow
IPF	inviscid potential flow
JBB	Joseph, Belanger, and Beavers
JBF	Joseph, Beavers, and Funada
KH	Kelvin–Helmholtz
KT	Kumar and Tuckerman
LHC	Longuet-Higgins and Cokelet
MVK	Miksis, Vanden-Broeck, and Keller
ODE	ordinary differential equation
PAA	polyacrylamide
PDE	partial differential equation
PISO	pressure implicit with splitting of operators
PNSCC	principal normal stress cavitation criterion
PO or PEO	polyox or polyethylene oxide
QUICK	quadratic upwind interpolation for convective kinematics (scheme)
RT	Rayleigh–Taylor
TVF	Taylor vortex flow
VCVPF	viscous correction of VPF
VPF	viscous potential flow