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Iterative Receiver Design

Iterative processing is an important technique with applications in many fields, including digital communications, image processing, and machine learning. By exploiting the power of factor graphs, this detailed survey provides a general framework for systematically developing iterative algorithms for digital receivers, and highlights connections between important algorithms. Starting with basic concepts in digital communications, progressively more complex ideas are presented and integrated, resulting in the development of cutting-edge algorithms for iterative receivers. Real-world applications are covered in detail, including decoding for turbo and LDPC codes, and detection for multi-antenna and multi-user systems. This accessible framework will allow the reader to apply factor graphs to practical problems, leading to the design of new algorithms in applications beyond digital receivers.

With many examples and algorithms in pseudo-code, this book is an invaluable resource for graduate students and researchers in electrical engineering and computer science, and for practitioners in the communications industry.

Additional resources for this title are available online at www.cambridge.org/9780521873154.

HENK WYMEERSCH is a postdoctoral associate in the Laboratory for Information and Decision Systems at the Massachusetts Institute of Technology. He obtained his PhD in Electrical Engineering from Ghent University, Belgium, in 2005. In 2006 he received the Alcatel Bell Scientific Award for “an original study of Information & Communication Technology, Concepts and Multimedia applications.”

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Preface

In early 2002, I was absent-mindedly surfing the Internet, vaguely looking for a tutorial on turbo codes. My PhD advisor at Ghent University, Marc Moeneclaey, thought it wise for his new students to become familiar with these powerful error-correcting codes. Although I finally settled on W. E. Ryan's "A turbo code tutorial," my search led me (serendipitously?) to the PhD thesis of Niclas Wiberg. This thesis shows how to describe codes by means of a (factor) graph, and how to decode them by passing messages on this graph. Although interesting, the idea seemed a bit far-fetched, and I didn't fully appreciate or understand its significance. Nevertheless, Wiberg's thesis stayed in the back of my mind (or at least, I'd like to think so now).

During 2002 and 2003, I worked mainly on synchronization and estimation algorithms for turbo and LDPC codes. A colleague of mine, Justin Dauwels, who was at that time a PhD student of Andy Loeliger at the ETH in Zürich, had developed a remarkable synchronization algorithm for LDPC codes, based on Wiberg's factor graphs. Justin was interested in comparing his synchronization algorithm with ours, and became a visiting researcher in our lab for the first two months of 2004. I fondly remember many hours spent in the department lunchroom, with Justin (painstakingly) explaining the intricacies of factor graphs to me. His discussions motivated me to re-write my source code for decoding turbo and LDPC codes using the factor-graph framework. To my surprise, writing the code was an almost trivial process! Understanding factor graphs made understanding and programming algorithms so much easier. Why didn't more people use these graphs? As I became more engrossed in the topic, I realized that, despite the presence of some excellent tutorials on factor graphs, many researchers still felt intimidated by them. It would be very useful for someone to write an accessible tutorial on factor graphs, a "Factor graphs for dummies," as it were. By the time I felt I had a reasonable amount of experience on factor graphs, I had to write my PhD thesis and could not pursue this idea any further.

Late 2005, I was a postdoctoral fellow at MIT trying to survive my first winter in Boston. For some reason (perhaps tiredness, perhaps cerebral frostbite), I finally bit the bullet and decided to write a book on factor graphs in the context of digital communications. I was lucky enough to have a postdoc advisor, Moe Win, who graciously allowed me to work full-time on this project. The book you are holding now is pretty much the book I set out to write. Despite being somewhat longer than I originally intended, this book is limited in many ways, primarily due to time constraints and my limited

understanding of the various topics. Many parts of this book reflect my own personal interests, as well as my inclination for completeness rather than succinctness.

There are many, many people I wish to thank, most importantly my PhD advisor Marc Moeneclaey at Ghent University and my postdoc advisor Moe Win at MIT. They have very different personalities, but both are truly excellent in their own fields, both are genuinely nice people, and I feel privileged to have had the opportunity to work with them. I am also greatly indebted to the Belgian American Educational Foundation, the organization that sponsored my US fellowship, and gave me the financial freedom to work on my own project at MIT. A great deal of my knowledge on factor graphs is a direct consequence of the technical discussions I have had with some very smart people. Obviously, Justin Dauwels springs to mind, my own *encyclopaedia factorgraphica*. Over the years he has always been more than willing to answer my unending list of questions. Also critical in shaping the contents of this book was Frederik Simoens, one of Marc’s PhD students. My interactions with Frederik the past four years, from discussions on Kalman filters in the middle of the night in Stockholm, to getting wasted in dubious Boston nightclubs, have been nothing short of amazing. Another essential person in this list is Cédric Herzet, currently at UC Berkeley, a guy with an uncanny ability for lateral thinking. Some of the sweeter insights in this book are due to him. And then there is the long list of people who were at some point involved in my life this last crazy year, as a proof-reader, as a recipient of my many questions, or as a means to get away from it all. At MIT and surrounding areas, I thank Pedro Pinto, Faisal Kashif, Jaime Lien, Watcharapan Suwansantisuk, Damien Jourdan, Erik Sudderth, Wesley Gifford, Marco Chiani, Andrea Giorgetti, Gil Zussman, Alex Ihler, Hyundong Shin, Sejoon Lim, Sid Jaggi, Yuan Shen, Atilla Eryilmaz, Andrew Fletcher, Wee Peng Tay, Tony Quek, Eugene Baik, and the MIT Euroclub (in particular Bjorn-Mr. salsa-Maes). Back in Belgium, I would like to thank Xavier Jaspar, Valéry Ramon, Frederik Vanhaverbeke, Nele Noels, and Heidi Steendam. I am also grateful to have had the pleasure to work with Anna Littlewood and Phil Meyler at Cambridge University Press.

Finally, I wish to thank my family for their understanding, love, and patience, for helping me when times got tough (and boy, did they ever!), and for letting me pursue my dreams far from home. I miss you more than you know.

Henk Wymeersch

Cambridge, Massachusetts, October 2006

A Word on Notation

Vectors are written in bold, matrices in capital bold. The i th element in vector $\mathbf{x}$ is $[\mathbf{x}]_i$ or x_i . The element on the i th row, j th column of the matrix $\mathbf{A}$ is $[\mathbf{A}]_{ij}$ or A_{ij} . To access elements l_1 through l_2 ($l_1 \leq l_2$) in $\mathbf{x}$, we write $[\mathbf{x}]_{l_1:l_2}$ or $\mathbf{x}_{l_1:l_2}$. Logarithms are to the base e unless specified otherwise.

Abbreviations

APD	a-posteriori distribution
BICM	bit-interleaved coded modulation
CDMA	code-division multiple access
DS	direct sequence
FDMA	frequency-division multiple access
GMD	Gaussian mixture density
HMM	hidden Markov model
iid	independent and identically distributed
LDPC	low-density parity check
MAP	maximum a-posteriori
MC	Monte Carlo
MCMC	Markov-chain Monte Carlo
MIMO	multi-input, multi-output (multi-antenna system)
MMSE	minimum mean-squared error
OFDM	orthogonal frequency-division multiplexing
OFDMA	orthogonal frequency-division multiple access
pdf	probability density function
pmf	probability mass function
P/S	parallel-to-serial conversion
S/P	serial-to-parallel conversion
SPA	sum-product algorithm
SSM	state-space model
TCM	trellis-coded modulation
TDMA	time-division multiple access

Notations

Common notations A

$\mathcal{X}$	domain of variable X, x
$\mathbf{x}$	a vector
$\ \mathbf{x}\ $	Frobenius norm of $\mathbf{x}$
$\mathbf{X}$	a vector random variable or a matrix
$\mathbf{X}^H$	conjugate transpose of $\mathbf{X}$
$\mathbf{X}^T$	transpose of $\mathbf{X}$
$\hat{\mathbf{x}}$	estimate of $\mathbf{x}$
$\mathbb{E}\{\cdot\}$	expectation operator
$ \mathcal{S} $	cardinality of the set $\mathcal{S}$
$\mathbb{B}$	the set of binary numbers $\{0, 1\}$
$\mathbb{I}\{\cdot\}$	indicator function
δ_k	discrete Dirac distribution
$\delta(t)$	continuous Dirac distribution
$\mathbb{E}(\cdot)$	equality function
$\mathcal{R}_L(p_X(\cdot))$	particle representation of distribution $p_X(\cdot)$ with L samples
$\text{diag}\{\mathbf{x}\}$	a diagonal matrix with the elements of $\mathbf{x}$ on the diagonal
$f(x) \propto g(x)$	$f(x) = \gamma g(x), \forall x$, for some scalar γ
$\mathbf{I}_N$	an $N \times N$ identity matrix
$\mu_{X \rightarrow f}(x)$	message from edge X to node f , evaluated in $x \in \mathcal{X}$
$\mu_{f \rightarrow X}(x)$	message from node f to edge X , evaluated in $x \in \mathcal{X}$
$a \ll b$	a is much smaller than b
$\arg \max_x f(x)$	the value of x that achieves the maximum of $f(\cdot)$
$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$
$[a, b)$	the half-open interval $\{x \in \mathbb{R} : a \leq x < b\}$

Common notations B

B, b	information word
C, c	codeword
A, a	coded symbols
Y, y	observation
n	noise vector
h(·)	channel function
H	channel matrix
N_b	number of information bits per codeword
N_c	number of coded bits per codeword
N_s	number of coded symbols per codeword
N_T	number of transmit antennas
N_R	number of receive antennas
N_u	number of users
N_{FFT}	number of subcarriers per OFDM symbol
$\mathcal{M}$	the model
Ω	signaling constellation

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