Whistler and Alfvén Mode Cyclotron Masers in Space

The subject of wave–particle interactions occurring in space plasmas has developed strongly, both observationally and theoretically, since the discovery of the Van Allen radiation belts of energetic charged particles trapped in the Earth’s magnetosphere over 40 years ago. These wave–particle interactions are recognized today as being a most important research topic in space plasma physics. This is the first book to provide a full and systematic description of the physical theory of whistler and Alfvén cyclotron masers acting in planetary magnetospheres, and in the Sun’s outer atmosphere.

The book introduces current research topics by examining significant problems in the subject. It gives sufficient detail on the topic for readers to go on to apply the methods presented to new problems, helping them with their own research.

This book is a valuable reference for researchers and graduate students working in space science, solar–terrestrial physics, plasma physics, and planetary sciences.

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Both authors are well known in the field, each having published more than 200 papers, some of them jointly. Since 1997, both of them have been key members of international teams involved in three collaborative INTAS Projects and two NATO Science Programmes.
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Whistler and Alfvén mode cyclotron masers in space
WHISTLER AND ALFVÉN MODE CYCLOTRON MASERS IN SPACE

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Preface

The purpose of this monograph is to formulate a quantitative and self-consistent theoretical approach to wave–particle interactions occurring in space plasmas, and present a logical development of the subject. In the Earth’s magnetosphere, Nature has given us a plasma laboratory that is accessible to observations made by radio, magnetic and electric instruments on the ground, and a great variety of instruments aboard rockets and Earth-orbiting satellites. Spacecraft are making similar observations in the more distant solar system.

To understand such observations as fully as possible, with colleagues around the world we have been challenged to produce a rigorous description of the energetic charged particle distribution function interacting with electromagnetic waves across a wide frequency spectrum. The space plasma is, as a rule, a non-equilibrium system with sources and sinks of energy and charged particles. As such, electromagnetic waves are generated via the process of the stimulated emission of radiation. Together with the electrodynamic properties of the space plasma, determined by variations of the magnetic field and plasma density, this constitutes a maser system. It exerts a strong influence on the state of the space plasma.

Cyclotron masers (CMs) are a shining example of such maser systems operating in the Universe. Whether in the Earth’s magnetosphere or Jupiter’s, in the solar corona or in the laboratory, CMs are exciting systems to marvel at, to wonder about and to investigate in detail. Such is the theme of this book. We analyse waves in a resonant cavity (here termed the eigenmodes), the excitation conditions and different wave generation regimes. In these wave–particle interactions, feedback processes, which are inherently nonlinear, have to be taken into account. Energetic electrons interact in CMs with electromagnetic waves and are precipitated into the atmosphere; electrons
can be accelerated by these waves to produce secondary radiation. Similar results hold for the interaction between energetic ions and hydromagnetic (Alfvén) waves.

During this book’s preparation, we have good reason to be most grateful for support received from the Institute of Applied Physics of the Russian Academy of Sciences in Nizhny Novgorod, Russia, the Russian Foundation for Basic Research, NATO and INTAS in Brussels, Belgium, the Royal Society of London, United Kingdom, the International Space University in Strasbourg, France, the International Space Science Institute in Bern, Switzerland, and the Institute of Atmospheric Physics, Prague, Czech Republic. We are most grateful to Lyudmila Semenova for her excellent typing of the entire book. Special thanks are also given to Andrei Demekhov, Victor’s colleague at Nizhny Novgorod, for his detailed reading of the text, and a considerable amount of work on the illustrations and the references. Every effort has been made to secure the necessary permissions to reproduce copyright material in this work, though in some cases it has proved impossible to trace or receive replies from copyright holders. If any omissions are brought to our notice, we shall include appropriate acknowledgements on reprinting or in subsequent editions. Finally we thank all the staff at Cambridge University Press who have been involved in our book.

For their devotion and patience, we especially thank our wives, Galja and Mary, respectively.

Post Script, added by Michael Rycroft and Andrei Demekhov

We are greatly saddened to report the death of our good colleague and dear friend Victor Yurievich Trakhtengerts on 4 December 2007; he had battled against cancer for more than three years. At that time this book had been completed, and was being finally checked and collated. Poignantly, it was submitted electronically to the Cambridge University Press a few days later. We consider that it is an appropriate testament to Victor’s many achievements in research on space plasma physics over his entire career. We hope that his keen physical insight and imagination, and his superb analytical skills, are as evident to the reader as they are to us. We trust that the publication of this book may stimulate others to continue to pursue research in this fascinating field.