

DIFFUSION  
MASS TRANSFER IN FLUID SYSTEMS  
THIRD EDITION

*Diffusion: Mass Transfer in Fluid Systems* brings unsurpassed, engaging clarity to a complex topic. Diffusion is a key part of the undergraduate chemical engineering curriculum and at the core of understanding chemical purification and reaction engineering. This spontaneous mixing process is central to our daily lives, important in phenomena as diverse as the dispersal of pollutants to digestion in the small intestine. For students, this new edition goes to the basics of mass transfer and diffusion, illustrating the theory with worked examples and stimulating discussion questions. For professional scientists and engineers, it explores emerging topics and explains where new challenges are expected. Retaining its trademark enthusiastic style, the book's broad coverage now extends to biology and medicine.

This accessible introduction to diffusion and separation processes gives chemical and biochemical engineering students what they need to understand these important concepts.

New to this Edition

- **Diffusion:** Enhanced treatment of topics such as Brownian motion, composite materials, and barrier membranes.
- **Mass transfer:** Fundamentals supplemented by material on when theories work and why they fail.
- **Absorption:** Extensions include sections on blood oxygenators, artificial kidneys, and respiratory systems.
- **Distillation:** Split into two focused chapters on staged distillation and on differential distillation with structured packing.
- **Advanced Topics:** Including electrolyte transport, spinodal decomposition, and diffusion through cavities.
- **New Problems:** Topics are broad, supported by password-protected solutions found at [www.cambridge.org/cussler](http://www.cambridge.org/cussler).

Professor Cussler teaches chemical engineering at the University of Minnesota. His research, which centers on membrane separations, has led to over 200 papers and 4 books. A member of the National Academy of Engineering, he has received the Colburn and Lewis awards from the American Institute of Chemical Engineers, the Separations Science Award from the American Chemical Society, the Merryfield Design Award from the American Society for Engineering Education, and honorary doctorates from the Universities of Lund and Nancy.

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MASS TRANSFER IN FLUID  
SYSTEMS

THIRD EDITION

E. L. CUSSLER

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Frontmatter

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For Jason, Liz, Sarah, and Varick  
who wonder what I do all day

Contents

<i>List of Symbols</i>	xiii
<i>Preface to Third Edition</i>	xix
<i>Preface to Second Edition</i>	xxi
<b>1 Models for Diffusion</b>	<b>1</b>
1.1 The Two Basic Models	2
1.2 Choosing Between the Two Models	3
1.3 Examples	7
1.4 Conclusions	9
<i>Questions for Discussion</i>	10
<b>PART I Fundamentals of Diffusion</b>	
<b>2 Diffusion in Dilute Solutions</b>	<b>13</b>
2.1 Pioneers in Diffusion	13
2.2 Steady Diffusion Across a Thin Film	17
2.3 Unsteady Diffusion in a Semi-infinite Slab	26
2.4 Three Other Examples	33
2.5 Convection and Dilute Diffusion	41
2.6 A Final Perspective	49
<i>Questions for Discussion</i>	50
<i>Problems</i>	51
<i>Further Reading</i>	55
<b>3 Diffusion in Concentrated Solutions</b>	<b>56</b>
3.1 Diffusion With Convection	56
3.2 Different Forms of the Diffusion Equation	59
3.3 Parallel Diffusion and Convection	67
3.4 Generalized Mass Balances	75
3.5 A Guide to Previous Work	84
3.6 Conclusions	90
<i>Questions for Discussion</i>	90
<i>Problems</i>	91
<i>Further Reading</i>	94
<b>4 Dispersion</b>	<b>95</b>
4.1 Dispersion From a Stack	95
4.2 Dispersion Coefficients	97

4.3	Dispersion in Turbulent Flow	101
4.4	Dispersion in Laminar Flow: Taylor Dispersion	104
4.5	Conclusions	110
	<i>Questions for Discussion</i>	110
	<i>Problems</i>	111
	<i>Further Reading</i>	113
<b>PART II Diffusion Coefficients</b>		
<b>5</b>	<b>Values of Diffusion Coefficients</b>	<b>117</b>
5.1	Diffusion Coefficients in Gases	117
5.2	Diffusion Coefficients in Liquids	126
5.3	Diffusion in Solids	134
5.4	Diffusion in Polymers	135
5.5	Brownian Motion	139
5.6	Measurement of Diffusion Coefficients	142
5.7	A Final Perspective	156
	<i>Questions for Discussion</i>	157
	<i>Problems</i>	157
	<i>Further Reading</i>	159
<b>6</b>	<b>Diffusion of Interacting Species</b>	<b>161</b>
6.1	Strong Electrolytes	161
6.2	Associating Solutes	172
6.3	Solute–Solvent Interactions	183
6.4	Solute–Boundary Interactions	190
6.5	A Final Perspective	205
	<i>Questions for Discussion</i>	206
	<i>Problems</i>	206
	<i>Further Reading</i>	209
<b>7</b>	<b>Multicomponent Diffusion</b>	<b>211</b>
7.1	Flux Equations for Multicomponent Diffusion	211
7.2	Irreversible Thermodynamics	214
7.3	Solving the Multicomponent Flux Equations	218
7.4	Ternary Diffusion Coefficients	224
7.5	Tracer Diffusion	225
7.6	Conclusions	231
	<i>Questions for Discussion</i>	232
	<i>Problems</i>	232
	<i>Further Reading</i>	234
<b>PART III Mass Transfer</b>		
<b>8</b>	<b>Fundamentals of Mass Transfer</b>	<b>237</b>
8.1	A Definition of a Mass Transfer Coefficient	237
8.2	Other Definitions of Mass Transfer Coefficients	243

## Contents

ix

8.3	Correlations of Mass Transfer Coefficients	249
8.4	Dimensional Analysis: The Route to Correlations	257
8.5	Mass Transfer Across Interfaces	261
8.6	Conclusions	269
	<i>Questions for Discussion</i>	270
	<i>Problems</i>	270
	<i>Further Reading</i>	273
<b>9</b>	<b>Theories of Mass Transfer</b>	<b>274</b>
9.1	The Film Theory	275
9.2	Penetration and Surface-Renewal Theories	277
9.3	Why Theories Fail	281
9.4	Theories for Solid–Fluid Interfaces	284
9.5	Theories for Concentrated Solutions	294
9.6	Conclusions	298
	<i>Questions for Discussion</i>	300
	<i>Problems</i>	300
	<i>Further Reading</i>	303
<b>10</b>	<b>Absorption</b>	<b>304</b>
10.1	The Basic Problem	305
10.2	Absorption Equipment	307
10.3	Absorption of a Dilute Vapor	314
10.4	Absorption of a Concentrated Vapor	321
10.5	Conclusions	326
	<i>Questions for Discussion</i>	326
	<i>Problems</i>	327
	<i>Further Reading</i>	331
<b>11</b>	<b>Mass Transfer in Biology and Medicine</b>	<b>332</b>
11.1	Mass Transfer Coefficients	333
11.2	Artificial Lungs and Artificial Kidneys	339
11.3	Pharmacokinetics	347
11.4	Conclusions	350
	<i>Questions for Discussion</i>	351
	<i>Problems</i>	351
	<i>Further Reading</i>	352
<b>12</b>	<b>Differential Distillation</b>	<b>353</b>
12.1	Overview of Distillation	353
12.2	Very Pure Products	356
12.3	The Column's Feed and its Location	362
12.4	Concentrated Differential Distillation	366
12.5	Conclusions	371
	<i>Questions for Discussion</i>	371
	<i>Problems</i>	372
	<i>Further Reading</i>	374

<b>13 Staged Distillation</b>	<b>375</b>
13.1 Staged Distillation Equipment	376
13.2 Staged Distillation of Nearly Pure Products	379
13.3 Concentrated Staged Distillation	385
13.4 Stage Efficiencies	393
13.5 Conclusions	400
<i>Questions for Discussion</i>	400
<i>Problems</i>	401
<i>Further Reading</i>	403
<b>14 Extraction</b>	<b>404</b>
14.1 The Basic Problem	404
14.2 Extraction Equipment	407
14.3 Differential Extraction	409
14.4 Staged Extraction	413
14.5 Leaching	416
14.6 Conclusions	420
<i>Questions for Discussion</i>	420
<i>Problems</i>	421
<i>Further Reading</i>	423
<b>15 Adsorption</b>	<b>424</b>
15.1 Where Adsorption is Important	425
15.2 Adsorbents and Adsorption Isotherms	427
15.3 Breakthrough Curves	431
15.4 Mass Transfer Effects	439
15.5 Other Characteristics of Adsorption	443
15.6 Conclusions	450
<i>Questions for Discussion</i>	450
<i>Problems</i>	450
<i>Further Reading</i>	452
<b>PART IV Diffusion Coupled With Other Processes</b>	
<b>16 General Questions and Heterogeneous Chemical Reactions</b>	<b>455</b>
16.1 Is the Reaction Heterogeneous or Homogeneous?	456
16.2 What is a Diffusion-Controlled Reaction?	457
16.3 Diffusion and First-Order Heterogeneous Reactions	459
16.4 Finding the Mechanism of Irreversible Heterogeneous Reactions	465
16.5 Heterogeneous Reactions of Unusual Stoichiometries	469
16.6 Conclusions	473
<i>Questions for Discussion</i>	473
<i>Problems</i>	474
<i>Further Reading</i>	477



<b>17 Homogeneous Chemical Reactions</b>	<b>478</b>
17.1 Mass Transfer with First-Order Chemical Reactions	479
17.2 Mass Transfer with Second-Order Chemical Reactions	488
17.3 Industrial Gas Treating	492
17.4 Diffusion-Controlled Fast Reactions	500
17.5 Dispersion-Controlled Fast Reactions	504
17.6 Conclusions	507
<i>Questions for Discussion</i>	508
<i>Problems</i>	508
<i>Further Reading</i>	512
<b>18 Membranes</b>	<b>513</b>
18.1 Physical Factors in Membranes	514
18.2 Gas Separations	520
18.3 Reverse Osmosis and Ultrafiltration	526
18.4 Pervaporation	534
18.5 Facilitated Diffusion	539
18.5 Conclusions	545
<i>Questions for Discussion</i>	545
<i>Problems</i>	546
<i>Further Reading</i>	548
<b>19 Controlled Release and Related Phenomena</b>	<b>549</b>
19.1 Controlled Release by Solute Diffusion	551
19.2 Controlled Release by Solvent Diffusion	555
19.3 Barriers	558
19.4 Diffusion and Phase Equilibrium	562
19.5 Conclusions	565
<i>Questions for Discussion</i>	565
<i>Problems</i>	566
<i>Further Reading</i>	566
<b>20 Heat Transfer</b>	<b>568</b>
20.1 Fundamentals of Heat Conduction	568
20.2 General Energy Balances	575
20.3 Heat Transfer Coefficients	579
20.4 Rate Constants for Heat Transfer	585
20.5 Conclusions	591
<i>Questions for Discussion</i>	591
<i>Problems</i>	591
<i>Further Reading</i>	593
<b>21 Simultaneous Heat and Mass Transfer</b>	<b>594</b>
21.1 Mathematical Analogies Among Mass, Heat, and Momentum Transfer	594
21.2 Physical Equalities Among Mass, Heat, and Momentum Transfer	600

xii	<i>Contents</i>
21.3 Drying	604
21.4 Design of Cooling Towers	609
21.5 Thermal Diffusion and Effusion	615
21.6 Conclusions	621
<i>Questions for Discussion</i>	621
<i>Problems</i>	622
<i>Further Reading</i>	624
<i>Index</i>	626

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## List of Symbols

$a$	surface area per volume
$a$	major axis of ellipsoid (Section 5.2)
$a, a_i$	constant
$A$	area
$A$	absorption factor (Chapters 13 and 14)
$b$	constant
$b$	minor axis of ellipsoid (Section 5.2)
$B$	bottoms (Chapters 10, 12 and 13)
$B, b$	boundary positions (Section 7.3)
$c$	total molar concentration
$c_1$	concentration of species 1, in either moles per volume or mass per volume
$c_{\text{CMC}}$	critical micelle concentration (Section 6.2)
$c_{\text{T}}$	total concentration (Chapter 6)
$\bar{c}_1$	concentration of species 1 averaged over time (Sections 4.3 and 17.4)
$c'_1$	concentration fluctuation of species 1 (Sections 4.3, 17.3, and 17.4)
$\underline{c}$	vector of concentrations (Section 7.3)
$c_{1i}$	concentration of species 1 at an interface $i$
$C$	capacity factor (Section 13.1)
$\tilde{C}_p, \hat{C}_p$	molar and specific heat capacities respectively, at constant pressure
$\tilde{C}_v, \hat{C}_v$	molar and specific heat capacities respectively at constant volume
$d$	diameter or other characteristic length
$D$	binary diffusion coefficient
$D$	distillate (Chapters 12 and 13)
$D_{\text{eff}}$	effective diffusion coefficient, for example, in a porous solid
$D_i$	binary diffusion coefficient of species $i$
$D_0$	binary diffusion coefficient corrected for activity effects
$D_{ij}$	multicomponent diffusion coefficient (Chapter 7)
$D_{\text{Kn}}$	Knudsen diffusion coefficient of a gas in a small pore
$D_{\text{m}}$	micelle diffusion coefficient (Section 6.2)
$D^*$	intradiffusion coefficient (Section 7.5)
$E$	dispersion coefficient
$E$	extraction factor (Chapter 14)
$E(t)$	residence-time distribution (Section 9.2)
$f$	friction coefficient for a diffusing solute (Section 5.2)
$f$	friction factor for fluid flow (Chapter 21)
$F$	packing factor (Section 10.2)
$F$	feed (Chapters 12 and 13)
$F$	Faraday's constant (Section 6.1)
$F(D)$	solution to a binary diffusion problem (Section 7.3)
$g$	acceleration due to gravity

$G$	molar flux of gas
$G''$	mass flux of gas (Sections 10.2 and 13.1)
$G'$	molar flux of gas in stripping section (Chapters 12 and 13)
$h$	reduced plate height (Section 15.5)
$h, h_i$	heat transfer coefficients (Chapters 20 and 21)
$H$	partition coefficient
$\tilde{H}, \hat{H}$	molar and specific enthalpies (Chapters 20–21 and Chapter 7, respectively)
$\bar{H}_i$	partial specific enthalpy (Chapter 7)
HTU	height of transfer unit
$i$	current density (Section 6.1)
$j_v$	volume flux across a membrane (Section 18.3)
$j_T$	total electrolyte flux (Section 6.1)
$j_i$	diffusion flux of solute $i$ relative to the volume average velocity
$\mathbf{j}_i^m$	diffusion flux of solute $i$ relative to the mass average velocity
$j_i^*$	diffusion flux relative to the molar average velocity
$\mathbf{j}_1^{(2)}$	diffusion flux of solute (1) relative to velocity of solvent (2)
$\mathbf{j}_i^a$	diffusion flux of solute $i$ relative to reference velocity $a$
$J_s$	entropy flux (Section 7.2)
$J_T$	total solute flux in different chemical forms (Section 6.2)
$k$	mass transfer coefficient based on a concentration driving force
$k_p$	mass transfer coefficient based on a partial pressure driving force (Table 8.2-2)
$k_x, k_y$	mass transfer coefficients based on mole fraction driving forces in liquid and gas, respectively (Table 8.2-2)
$k_B$	Boltzmann's constant
$k_T$	thermal conductivity (Chapters 20–21)
$k^0$	mass transfer coefficient at low transfer rate (Section 9.5)
$k^0$	mass transfer coefficient without chemical reaction (Chapter 17)
$k'$	capacity factor (Sections 4.4 and 15.1)
$K$	equilibrium constant for chemical reaction
$K_G, K_L$	overall mass transfer coefficients based on concentration driving force in gas or liquid, respectively
$K_p$	overall mass transfer coefficient based on partial pressure difference in gas
$K_x, K_y$	overall mass transfer coefficient based on mole fraction driving force in liquid or gas, respectively
$Kn$	Knudsen number (Section 6.4)
$l$	length, e.g., of a membrane
$L$	length, e.g., of a pipe
$L$	molar flux of liquid
$L''$	mass flux of liquid (Sections 10.2 and 13.1)
$L'$	molar flux of liquid in stripping section (Sections 12.3 and 13.3)
$L_{ij}$	Onsager phenomenological coefficient (Section 7.2)
$L_p$	solvent permeability (Section 18.3)
$m$	partition coefficient relating mole fractions in gas and liquid
$M$	mass
$M$	total solute (Sections 4.2 and 5.5)
$\tilde{M}_i$	molecular weight of species $i$

List of Symbols

xv

$n$	micelle aggregation number or hydration number (Section 6.2)
$\mathbf{n}_i$	flux of species $i$ relative to fixed coordinates
$N$	number of ideal stages
$\tilde{N}$	Avogadro's number
$N_i$	flux of species $i$ at an interface
$N_i$	number of moles of species $i$
NTU	number of transfer units
$p$	pressure
$P$	power
$P$	membrane permeability (Chapter 18)
$P_{ij}$	weighting factor (Section 7.3)
$q$	scattering vector (Section 5.6)
$q$	feed quality (Sections 12.3 and 13.3)
$q$	solute concentration per volume adsorbent (Chapter 15)
$\mathbf{q}$	energy flux (Chapters 7, 20, and 21)
$r$	radius
$r, r_i$	rate of chemical reaction
$R$	gas constant
$R_D$	reflux ratio (Chapters 12 and 13)
$R_0$	characteristic radius
$s$	distance from pipe wall (Section 9.4)
$\hat{S}$	specific entropy (Chapter 7)
$\tilde{S}_i$	partial specific entropy of species $i$
$t$	time
$\mathbf{t}$	modal matrix (Section 7.3)
$t_i$	transference number of ion $i$ (Section 6.1)
$t_{1/2}$	reaction half-life
$T$	temperature
$u_i$	ionic mobility (Section 6.1)
$U$	overall heat transfer coefficient
$\hat{U}$	specific internal energy
$v_r, v_\theta$	velocities in the $r$ and $\theta$ directions
$v_x, v_y$	velocities in the $x$ and $y$ directions
$\mathbf{v}$	mass average velocity
$\mathbf{v}^a$	velocity relative to reference frame $a$
$\mathbf{v}^o$	volume average velocity
$\mathbf{v}'$	velocity fluctuation (Sections 4.3 and 17.4)
$\mathbf{v}^*$	molar average velocity
$\mathbf{v}_i$	velocity of species $i$
$V$	volume
$\bar{V}_i$	partial molar or specific volume of species $i$
$V_{ij}$	fraction of molecular volume (Section 5.1)
$W$	width
$W$	work (Section 20.2)
$W_s$	shaft work (Section 20.2)
$x$	mole fraction in liquid of more volatile species (Chapters 12 and 13)

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$x_B, x_D, x_F$	mole fractions of more volatile species in bottoms, distillate and feed, respectively (Chapters 12 and 13)
$x_i$	mole fraction of species $i$ , especially in a liquid or solid phase
$\mathbf{X}_i$	generalized force causing diffusion (Section 7.2)
$y$	mole fraction in vapor of more volatile species (Chapters 12 and 13)
$y_i$	mole fraction of species $i$ in a gas
$z$	position
$ z $	magnitude of charge (Section 6.1)
$z_i$	charge on species $i$
$\alpha$	thermal diffusivity (Chapters 20 and 21)
$\alpha$	thermal diffusion factor (Section 21.5)
$\alpha$	flake aspect ratio (Sections 6.4 and 9.5)
$\alpha_{ij}$	conversion factor (Section 7.1)
$\beta$	diaphragm cell calibration constant (Sections 2.2 and 5.5)
$\beta$	pervaporation selectivity (Section 18.4)
$\gamma$	interfacial influence (Section 6.3)
$\gamma$	surface tension (Section 6.4)
$\gamma_i$	activity coefficient of species $i$
$\delta$	thickness of thin layer, especially a boundary layer
$\delta(z)$	Dirac function of $z$
$\delta_{ij}$	Kronecker delta
$\varepsilon$	void fraction
$\varepsilon$	enhancement factor (Section 17.1)
$\varepsilon_{ij}$	interaction energy between colliding molecules (Sections 5.1 and 20.4)
$\zeta$	combined variable
$\eta$	Murphree efficiency (Section 13.4)
$\eta$	effectiveness factor (Section 17.1)
$\theta$	dimensionless concentration
$\theta$	fraction of unused adsorption bed (Section 15.3)
$\theta$	fraction of surface elements (Section 9.2)
$\kappa_i, \kappa_{-i}$	forward and reverse reaction rate constants respectively of reaction $i$
$\lambda$	length ratio (Section 6.4)
$\lambda$	heat of vaporization (Sections 12.3 and 13.3)
$\lambda_i$	equivalent ionic conductance of species $i$ (Section 6.1)
$\Lambda$	equivalent conductance
$\mu$	viscosity
$\mu_i$	chemical potential of species $i$
$\mu_i$	partial specific Gibbs free energy of species $i$ , i.e., the chemical potential divided by the molecular weight (Section 7.2)
$\nu$	kinematic viscosity
$\nu$	stoichiometric coefficient (Sections 16.5 and 17.2)
$\xi$	dimensionless position
$\xi$	correlation length (Section 6.3)
$\Pi$	osmotic pressure (Section 18.3)
$\rho$	total density, i.e., total mass concentration
$\rho_i$	mass concentration of species $i$
$\sigma$	rate of entropy production (Section 7.2)

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$\sigma$	standard deviation (Sections 5.5 and 15.4)
$\sigma, \sigma'$	reflection coefficients (Section 18.3)
$\sigma$	Soret coefficient (Section 21)
$\sigma$	diagonal matrix of eigenvalues (Chapter 7)
$\sigma_i$	eigenvalue (Section 7.3)
$\sigma_{ij}$	collision diameter
$\tau$	characteristic time
$\tau$	tortuosity (Section 6.4)
$\tau$	residence time for surface element (Section 9.2)
$\tau$	shear stress (Chapter 21)
$\tau_0$	shear stress at wall (Section 9.4)
$\phi$	Thiele modulus (Section 17.1)
$\phi_i$	volume fraction of species $i$
$\psi$	electrostatic potential
$\Psi$	combined concentration (Section 7.3)
$\omega$	jump frequency (Section 5.3)
$\omega$	regular solution parameter (Section 6.3)
$\omega$	coefficient of solute permeability (Section 18.3)
$\omega_i$	mass fraction of species $i$
$\Omega$	collision integral in Chapman–Enskog theory (Section 5.1)