

RADIATION IN THE ATMOSPHERE

A Course in Theoretical Meteorology

This book presents the theory and applications of radiative transfer in the atmosphere. It is written for graduate students and researchers in the fields of meteorology and related sciences.

The book begins with important basic definitions of the radiative transfer theory. It presents the hydrodynamic derivation of the radiative transfer equation and the principles of invariance. The authors examine in detail various quasi-exact solutions of the radiative transfer equation, such as the matrix operator method, the discrete ordinate method, and the Monte Carlo method. A thorough treatment of the radiative perturbation theory is given. The book also presents various two-stream methods for the approximate solution of the radiative transfer equation. The interaction of radiation with matter is discussed as well as the transmission in individual spectral lines and in bands of lines. It formulates the theory of gaseous absorption and analyzes the normal vibrations of linear and non-linear molecules. The book presents the Schrödinger equation and describes the computation of transition probabilities, before examining the mathematical formulation of spectral line intensities. A rigorous treatment of Mie scattering is given, including Rayleigh scattering as a special case, and the important efficiency factors for extinction, scattering and absorption are derived. Polarization effects are introduced with the help of Stokes parameters. The fundamentals of remote sensing applications of radiative transfer are presented.

Problems of varying degrees of difficulty are included at the end of each chapter, so readers can further their understanding of the materials covered in the book.

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This book is dedicated to the memory of
Professor Fritz Möller (1906–1983) and
Professor J. Vern Hales (1917–1997).
We have profited directly and indirectly
from their lectures and research.

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Preface

Radiation in the Atmosphere is the third volume in the series *A Course in Theoretical Meteorology*. The first two volumes entitled *Dynamics of the Atmosphere* and *Thermodynamics of the Atmosphere* were first published in the years 2003 and 2004.

The present textbook is written for graduate students and researchers in the field of meteorology and related sciences. Radiative transfer theory has reached a high point of development and is still a vastly expanding subject. Kourganoff (1952) in the postscript of his well-known book on radiative transfer speaks of the three olympians named completeness, up-to-date-ness and clarity. We have not made any attempt to be complete, but we have tried to be reasonably up-to-date, if this is possible at all with the many articles on radiative transfer appearing in various monthly journals. Moreover, we have tried very hard to present a coherent and consistent development of radiative transfer theory as it applies to the atmosphere. We have given principle allegiance to the olympian clarity and sincerely hope that we have succeeded.

In the selection of topics we have resisted temptation to include various additional themes which traditionally belong to the fields of physical meteorology and physical climatology. Had we included these topics, our book, indeed, would be very bulky, and furthermore, we would not have been able to cover these subjects in the required depth. Neither have we made any attempt to include radiative transfer theory as it pertains to the ocean, a subject well treated by Thomas and Stamnes (1999) in their book *Radiative Transfer in the Atmosphere and Ocean*.

As in the previous books of the series, we were guided by the principle to make the book as self-contained as possible. As far as space would permit, all but the most trivial mathematical steps have been included in the presentation of the theory to encourage students to follow the various developments. Nevertheless, here and there students may find it difficult to follow our reasoning. In this case, we encourage them not to get stuck with a particular detail but to continue with the

subject. Additional details given later may clarify any questions. Moreover, on a second reading everything will become much clearer.

We will now give a brief description of the various chapters and topics treated in this book. Chapter 1 gives the general introduction to the book. Various important definitions such as the radiance and the net flux density are given to describe the radiation field. The interaction of radiation with matter is briefly discussed by introducing the concepts of absorption and scattering. To get an overall view of the mean global radiation budget of the system Earth–atmosphere, it is shown that the incoming and outgoing energy at the top of the atmosphere are balanced.

In Chapter 2 the hydrodynamic derivation of the radiative transfer equation (RTE) is worked out; this is in fact the budget equation for photons. The radiatively induced temperature change is formulated with the help of the first law of thermodynamics. Some basic formulas from spherical harmonics, which are needed to evaluate certain transfer integrals, are presented. Various special cases are discussed.

Chapter 3 presents the principle of invariance which, loosely speaking, is a collection of common sense statements about the exact mathematical structure of the radiation field. At first glance the mathematical formalism looks much worse than it really is. A systematic study of the mathematical and physical principles of invariance is quite rewarding.

Quasi-exact solutions of the RTE, such as the matrix operator method together with the doubling algorithm are presented in Chapter 4. Various other prominent solutions such as the successive order of scattering and the Monte Carlo methods are discussed in some detail.

Chapter 5 presents the radiative perturbation theory. The concept of the adjoint formulation of the RTE is introduced, and it is shown that in the adjoint formulation certain radiative effects can be evaluated with much higher numerical efficiency than with the so-called forward mode methods.

For many practical purposes in connection with numerical weather prediction it is sufficient to obtain fast approximate solutions of the RTE. These are known as two-stream methods and are treated in Chapter 6. Partial cloudiness is introduced in the solution scheme on the basis of two differing assumptions. The method allows fairly general situations to be handled.

In Chapter 7, the theory of individual spectral lines and band models is treated in some detail. In those cases in which scattering effects can be ignored, formulas are worked out to describe the mean absorption of homogeneous atmospheric layers. A technique is introduced which makes it possible to replace the transmission through an inhomogeneous atmosphere by a nearly equivalent homogeneous layer.

The theory of gaseous absorption is formulated in Chapter 8. The analysis of normal vibrations of linear and nonlinear molecules is introduced. The Schrödinger equation is presented and the computation of transition probabilities is described,

which finally leads to the mathematical formulation of spectral line intensities. Simple but instructive analytic solutions of Schrödinger's equation are obtained leading, for example, to the description of the vibration–rotation spectrum of diatomic molecules.

Not only atmospheric gaseous absorbers influence the radiation field but also aerosol particles and cloud droplets. Chapter 9 gives a rigorous treatment of Mie scattering which includes Rayleigh scattering as a special case. The important efficiency factors for extinction, scattering and absorption are derived. The mathematical analysis requires the mathematical skill which the graduate student has acquired in various mathematics and physics courses. The effects of nonspherical particles are not treated in this book.

So far polarization has not been included in the RTE, which is usually satisfactory for energy considerations but may not be sufficient for optical applications. To give a complete description of the radiation field the polarization effects are introduced in Chapter 10 with the help of the Stokes parameters. This finally leads to the most general vector formulation of the RTE in terms of the phase matrix while the phase function is sufficient if polarization may be ignored.

Chapter 11 introduces remote sensing applications of radiative transfer. After the general description of some basic ideas, the RTE is presented in a form which is suitable to recover the atmospheric temperature profile by special inversion techniques. The chapter closes with a description of the way in which the atmospheric ozone profile can be retrieved using radiative perturbation theory.

The book closes with Chapter 12 in which a simple and brief account of the influence of clouds on climate is given. The student will be exposed to concepts such as cloud forcing and cloud radiative feedback.

Problems of various degrees of difficulty are included at the end of each chapter. Some of the included problems are almost trivial. They serve the purpose of making students familiar with new concepts and terminologies. Other problems are more demanding. Where necessary answers to problems are given at the end of the book.

One of the problems that any author of a physical science textbook is confronted with, is the selection of proper symbols. Inspection of the book shows that many times the same symbol is used to label several quite different physical entities. It would be ideal to represent each physical quantity by a unique symbol which is not used again in some other context. Consider, for example, the letter k . For the Boltzmann constant we could have written k_B , for Hooke's constant k_H , for the wave number k_w , and k_s for the climate sensitivity constant. It would have been possible, in addition to using the Greek alphabet, to also employ the letters of another alphabet, e.g. Hebrew, to label physical quantities in order to obtain uniqueness in notation. Since usually confusion is unlikely, we have tried to use standard notation even if the same symbol is used more than once. For example, the

climate sensitivity parameter k appears in Chapter 12, Hooke's constant in Chapter 8 and Boltzmann's constant in Chapter 1.

The book concludes with a list of frequently used symbols and a list of constants.

We would like to give recognition to the excellent textbooks *Radiative Transfer* by the late S. Chandrasekhar (1960), to *Atmospheric Radiation* by R. M. Goody (1964) and the updated version of this book by Goody and Yung (1989). These books have been an invaluable guidance to us in research and teaching.

We would like to give special recognition to Dr W. G. Panhans for his splendid cooperation in organizing and conducting our exercise classes. Recognition is due to Dr Jochen Landgraf for discussions related to the perturbation theory and to ozone retrieval. Moreover, we will be indebted to Sebastian Otto for carrying out the transfer calculations presented in Section 7.5. We also wish to express our gratitude to many colleagues and graduate students for helpful comments while preparing the text. Last but not least we wish to thank our families for their patience and encouragement during the preparation of this book.

It seems to be one of the unfortunate facts of life that no book as technical as this one can be written free of error. However, each author takes comfort in the thought that any errors appearing in this book are due to one of the other two. To remove such errors, we will be grateful for anyone pointing these out to us.