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978-0-521-87044-3 - An Introduction to Continuum Mechanics: With Applications

J. N. Reddy

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## AN INTRODUCTION TO CONTINUUM MECHANICS

This textbook on continuum mechanics reflects the modern view that scientists and engineers should be trained to think and work in multi-disciplinary environments. A course on continuum mechanics introduces the basic principles of mechanics and prepares students for advanced courses in traditional and emerging fields such as biomechanics and nanomechanics. This text introduces the main concepts of continuum mechanics simply with rich supporting examples but does not compromise mathematically in providing the invariant form as well as component form of the basic equations and their applications to problems in elasticity, fluid mechanics, and heat transfer. The book is ideal for advanced undergraduate and beginning graduate students. The book features: derivations of the basic equations of mechanics in invariant (vector and tensor) form and specializations of the governing equations to various coordinate systems; numerous illustrative examples; chapter-end summaries; and exercise problems to test and extend the understanding of concepts presented.

J. N. Reddy is a University Distinguished Professor and the holder of the Oscar S. Wyatt Endowed Chair in the Department of Mechanical Engineering at Texas A&M University, College Station, Texas. Dr. Reddy is internationally known for his contributions to theoretical and applied mechanics and computational mechanics. He is the author of over 350 journal papers and 15 books, including *Introduction to the Finite Element Method*, Third Edition; *Energy Principles and Variational Methods in Applied Mechanics*, Second Edition; *Theory and Analysis of Elastic Plates and Shells*, Second Edition; *Mechanics of Laminated Plates and Shells: Theory and Analysis*, Second Edition; and *An Introduction to Nonlinear Finite Element Analysis*. Professor Reddy is the recipient of numerous awards, including the Walter L. Huber Civil Engineering Research Prize of the American Society of Civil Engineers (ASCE), the Worcester Reed Warner Medal and the Charles Russ Richards Memorial Award of the American Society of Mechanical Engineers (ASME), the 1997 Archie Higdon Distinguished Educator Award from the American Society of Engineering Education (ASEE), the 1998 Nathan M. Newmark Medal from the ASCE, the 2000 Excellence in the Field of Composites from the American Society of Composites (ASC), the 2003 Bush Excellence Award for Faculty in International Research from Texas A&M University,

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and the 2003 Computational Solid Mechanics Award from the U.S. Association of Computational Mechanics (USACM).

Professor Reddy is a Fellow of the American Institute of Aeronautics and Astronautics (AIAA), the ASME, the ASCE, the American Academy of Mechanics (AAM), the ASC, the USACM, the International Association of Computational Mechanics (IACM), and the Aeronautical Society of India (ASI). Professor Reddy is the Editor-in-Chief of *Mechanics of Advanced Materials and Structures*, *International Journal of Computational Methods in Engineering Science and Mechanics*, and *International Journal of Structural Stability and Dynamics*; he also serves on the editorial boards of over two dozen other journals, including the *International Journal for Numerical Methods in Engineering*, *Computer Methods in Applied Mechanics and Engineering*, and *International Journal of Non-Linear Mechanics*.

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**J. N. Reddy**

Texas A&M University



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‘Tis the good reader that makes the good book; in every book he finds passages which seem confidences or asides hidden from all else and unmistakably meant for his ear; the profit of books is according to the sensibility of the reader; the profoundest thought or passion sleeps as in a mine, until it is discovered by an equal mind and heart.

Ralph Waldo Emerson

You cannot teach a man anything, you can only help him find it within himself.

Galileo Galilei

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## Preface

If I have been able to see further, it was only because I stood on the shoulders of giants.

Isaac Newton

Many of the mathematical models of natural phenomena are based on fundamental scientific laws of physics or otherwise are extracted from centuries of research on the behavior of physical systems under the action of natural forces. Today this subject is referred to simply as *mechanics* – a phrase that encompasses broad fields of science concerned with the behavior of fluids, solids, and complex materials. Mechanics is vitally important to virtually every area of technology and remains an intellectually rich subject taught in all major universities. It is also the focus of research in departments of aerospace, chemical, civil, and mechanical engineering, in engineering science and mechanics, and in applied mathematics and physics. The past several decades have witnessed a great deal of research in continuum mechanics and its application to a variety of problems. As most modern technologies are no longer discipline-specific but involve multidisciplinary approaches, scientists and engineers should be trained to think and work in such environments. Therefore, it is necessary to introduce the subject of mechanics to senior undergraduate and beginning graduate students so that they have a strong background in the basic principles common to all major engineering fields. A first course on *continuum mechanics* or *elasticity* is the one that provides the basic principles of mechanics and prepares engineers and scientists for advanced courses in traditional as well as emerging fields such as biomechanics and nanomechanics.

There are many books on mechanics of continua. These books fall into two major categories: those that present the subject as highly mathematical and abstract and those that are too elementary to be of use for those who will pursue further work in fluid dynamics, elasticity, plates and shells, viscoelasticity, plasticity, and interdisciplinary areas such as geomechanics, biomechanics, mechanobiology, and nanoscience. As is the case with all other books written (solely) by the author, the objective is to facilitate an easy understanding of the topics covered. While the author is fully aware that he is not an authority on the subject of this book, he feels that he understands the concepts well and feels confident that he can explain them to others. It is hoped that the book, which is simple in presenting the main concepts, will be mathematically rigorous enough in providing the invariant form as well as component form of the governing equations for analysis of practical problems of engineering. In particular, the book contains

formulations and applications to specific problems from heat transfer, fluid mechanics, and solid mechanics.

The motivation and encouragement that led to the writing of this book came from the experience of teaching a course on continuum mechanics at Virginia Polytechnic Institute and State University and Texas A&M University. A course on continuum mechanics takes different forms – abstract to very applied – when taught by different people. The primary objective of the course taught by the author is two-fold: (1) formulation of equations that describe the motion and thermomechanical response of materials and (2) solution of these equations for specific problems from elasticity, fluid flows, and heat transfer. This book is a formal presentation of the author's notes developed for such a course over past two-and-a-half decades.

After a brief discussion of the concept of a continuum in Chapter 1, a review of vectors and tensors is presented in Chapter 2. Since the language of mechanics is mathematics, it is necessary for all readers to familiarize themselves with the notation and operations of vectors and tensors. The subject of kinematics is discussed in Chapter 3. Various measures of strain are introduced here. In this chapter the deformation gradient, Cauchy–Green deformation, Green–Lagrange strain, Cauchy and Euler strain, rate of deformation, and vorticity tensors are introduced, and the polar decomposition theorem is discussed. In Chapter 4, various measures of stress – Cauchy stress and Piola–Kirchhoff stress measures – are introduced, and stress equilibrium equations are presented.

Chapter 5 is dedicated to the derivation of the field equations of continuum mechanics, which forms the heart of the book. The field equations are derived using the principles of conservation of mass, momenta, and energy. Constitutive relations that connect the kinematic variables (e.g., density, temperature, deformation) to the kinetic variables (e.g., internal energy, heat flux, and stresses) are discussed in Chapter 6 for elastic materials, viscous and viscoelastic fluids, and heat transfer.

Chapters 7 and 8 are devoted to the application of both the field equations derived in Chapter 5 and the constitutive models of Chapter 6 to problems of linearized elasticity, and fluid mechanics and heat transfer, respectively. Simple boundary-value problems, mostly linear, are formulated and their solutions are discussed. The material presented in these chapters illustrates how physical problems are analytically formulated with the aid of continuum equations. Chapter 9 deals with linear viscoelastic constitutive models and their application to simple problems of solid mechanics. Since a continuum mechanics course is mostly offered by solid mechanics programs, the coverage in this book is slightly more favorable, in terms of the amount and type of material covered, to solid and structural mechanics.

The book is written keeping the undergraduate seniors and first-year graduate students of engineering in mind. Therefore, it is most suitable as a textbook for adoption for a first course on continuum mechanics or elasticity. The book also serves as an excellent precursor to courses on viscoelasticity, plasticity, nonlinear elasticity, and nonlinear continuum mechanics.

The book contains so many mathematical equations that it is hardly possible not to have typographical and other kinds of errors. I wish to thank in advance those readers who are willing to draw the author's attention to typos and errors, using the following e-mail address: [jnreddy@tamu.edu](mailto:jnreddy@tamu.edu).

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