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978-0-521-86813-6 - The Mathematical Foundations of Mixing: The Linked Twist Map as a Paradigm in Applications Micro to Macro, Fluids to Solids

Rob Sturman, Julio M. Ottino and Stephen Wiggins

Table of Contents

[More information](#)


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## Contents

---

<i>Preface</i>	xiii
<i>Acknowledgments</i>	xx
<b>1 Mixing: physical issues</b>	<b>1</b>
1.1 Length and time scales	3
1.2 Stretching and folding, chaotic mixing	6
1.3 Reorientation	11
1.4 Diffusion and scaling	12
1.5 Examples	13
1.5.1 The Aref blinking vortex flow	14
1.5.2 Samelson's tidal vortex advection model	14
1.5.3 Chaotic stirring in tidal systems	15
1.5.4 Cavity flows	17
1.5.5 An electro-osmotic driven micromixer blinking flow	18
1.5.6 Egg beater flows	19
1.5.7 A blinking flow model of mixing of granular materials	22
1.5.8 Mixing in DNA microarrays	26
1.6 Mixing at the microscale	28
 <b>2 Linked twist maps: definition, construction and the relevance to mixing</b>	 <b>31</b>
2.1 Introduction	31
2.2 Linked twist maps on the torus	32
2.2.1 Geometry of mixing for toral LTMs	35
2.3 Linked twist maps on the plane	42
2.3.1 Geometry of mixing for LTMs on the plane	46
2.4 Constructing a LTM from a blinking flow	48

Cambridge University Press

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Rob Sturman, Julio M. Ottino and Stephen Wiggins

Table of Contents

[More information](#)

viii	<i>Contents</i>	
	2.5 Constructing a LTM from a duct flow	49
	2.6 More examples of mixers that can be analysed in the LTM framework	53
<b>3</b>	<b>The ergodic hierarchy</b>	<b>59</b>
	3.1 Introduction	59
	3.2 Mathematical ideas for describing and quantifying the flow domain, and a ‘blob’ of dye in the flow	60
	3.2.1 Mathematical structure of spaces	61
	3.2.2 Describing sets of points	64
	3.2.3 Compactness and connectedness	66
	3.2.4 Measuring the ‘size’ of sets	67
	3.3 Mathematical ideas for describing the movement of blobs in the flow domain	70
	3.4 Dynamical systems terminology and concepts	73
	3.4.1 Terminology for general fluid kinematics	73
	3.4.2 Specific types of orbits	74
	3.4.3 Behaviour near a specific orbit	75
	3.4.4 Sets of fluid particles that give rise to ‘flow structures’	76
	3.5 Fundamental results for measure-preserving dynamical systems	78
	3.6 Ergodicity	80
	3.6.1 A typical scheme for proving ergodicity	84
	3.7 Mixing	86
	3.8 The $K$ -property	93
	3.9 The Bernoulli property	94
	3.9.1 The space of (bi-infinite) symbol sequences, $\Sigma^N$	95
	3.9.2 The shift map	99
	3.9.3 What it means for a map to have the Bernoulli property	101
	3.10 Summary	104
<b>4</b>	<b>Existence of a horseshoe for the linked twist map</b>	<b>105</b>
	4.1 Introduction	105
	4.2 The Smale horseshoe in dynamical systems	106
	4.2.1 The standard horseshoe	106
	4.2.2 Symbolic dynamics	109
	4.2.3 Generalized horseshoes	111
	4.2.4 The Conley–Moser conditions	112

Cambridge University Press

978-0-521-86813-6 - The Mathematical Foundations of Mixing: The Linked Twist Map as a Paradigm in Applications Micro to Macro, Fluids to Solids

Rob Sturman, Julio M. Ottino and Stephen Wiggins

Table of Contents

[More information](#)

<i>Contents</i>		ix
4.3	Horseshoes in fluids	113
4.4	Linked twist mappings on the plane	115
4.4.1	A twist map on the plane	115
4.4.2	Linking a pair of twist maps	116
4.5	Existence of a horseshoe in the linked twist map	117
4.5.1	Construction of the invariant set $\Lambda_{j,k}$	117
4.5.2	The subshift of finite type	123
4.5.3	The existence of the conjugacy	124
4.5.4	Hyperbolicity of $\Lambda_{j,k}$	124
4.6	Summary	125
<b>5</b>	<b>Hyperbolicity</b>	<b>126</b>
5.1	Introduction	126
5.2	Hyperbolicity definitions	128
5.2.1	Uniform hyperbolicity	129
5.2.2	Nonuniform hyperbolicity	137
5.2.3	Partial nonuniform hyperbolicity	139
5.2.4	Other hyperbolicity definitions	139
5.3	Pesin theory	140
5.3.1	Lyapunov exponents	140
5.3.2	Lyapunov exponents and hyperbolicity	143
5.3.3	Stable and unstable manifolds	144
5.3.4	Ergodic decomposition	145
5.3.5	Ergodicity	146
5.3.6	Bernoulli components	148
5.4	Smooth maps with singularities	149
5.4.1	Katok–Strelecyn conditions	150
5.4.2	Ergodicity and the Bernoulli property	151
5.5	Methods for determining hyperbolicity	154
5.5.1	Invariant cones	154
5.6	Summary	158
<b>6</b>	<b>The ergodic partition for toral linked twist maps</b>	<b>159</b>
6.1	Introduction	159
6.2	Toral linked twist maps	160
6.2.1	Twist maps on the torus	161
6.2.2	Linking the twist maps	164
6.2.3	First return maps	166
6.2.4	Co-rotating toral linked twist maps	168
6.2.5	Counter-rotating toral linked twist maps	168

Cambridge University Press

978-0-521-86813-6 - The Mathematical Foundations of Mixing: The Linked Twist Map as a Paradigm in Applications Micro to Macro, Fluids to Solids

Rob Sturman, Julio M. Ottino and Stephen Wiggins

Table of Contents

[More information](#)

x

*Contents*

6.2.6	Smooth twists	168
6.2.7	Non-smooth twists	169
6.2.8	Linear twists	169
6.2.9	More general twists	170
6.3	The ergodic partition for smooth toral linked twist maps	171
6.3.1	Co-rotating smooth toral linked twist maps	171
6.3.2	Counter-rotating smooth toral linked twist maps	180
6.4	The ergodic partition for toral linked twist maps with singularities	180
6.4.1	Co-rotating toral linked twist maps with singularities	182
6.4.2	Counter-rotating toral linked twist maps with singularities	185
6.5	Summary	193
<b>7</b>	<b>Ergodicity and the Bernoulli property for toral linked twist maps</b>	194
7.1	Introduction	194
7.2	Properties of line segments	195
7.2.1	Definition, iteration and orientation of line segments	196
7.2.2	Growth of line segments	200
7.2.3	$v$ -segments and $h$ -segments	201
7.3	Ergodicity for the Arnold Cat Map	204
7.4	Ergodicity for co-rotating toral linked twist maps	205
7.5	Ergodicity for counter-rotating toral linked twist maps	207
7.6	The Bernoulli property for toral linked twist maps	213
7.7	Summary	216
<b>8</b>	<b>Linked twist maps on the plane</b>	217
8.1	Introduction	217
8.2	Planar linked twist maps	218
8.2.1	The annuli	218
8.2.2	The twist maps	219
8.2.3	Linked twist maps	221
8.3	The ergodic partition	222
8.3.1	Counter-rotating planar linked twist maps	225
8.3.2	Co-rotating planar linked twist maps	233
8.4	Ergodicity and the Bernoulli property for planar linked twist maps	238
8.5	Summary	239

Cambridge University Press

978-0-521-86813-6 - The Mathematical Foundations of Mixing: The Linked Twist Map as a Paradigm in Applications Micro to Macro, Fluids to Solids

Rob Sturman, Julio M. Ottino and Stephen Wiggins

Table of Contents

[More information](#)

<i>Contents</i>		xi
<b>9</b>	<b>Further directions and open problems</b>	240
9.1	Introduction	240
9.2	Optimizing mixing regions for linked twist maps	241
9.2.1	Toral linked twist maps	242
9.2.2	Planar linked twist maps	245
9.3	Breakdown of transversality: effect and mechanisms	251
9.3.1	Separatrices	255
9.3.2	Planar linked twist maps with a single intersection component	256
9.3.3	More than two annuli	257
9.4	Monotonicity of the twist functions	260
9.4.1	Lack of monotonicity in toral linked twist maps	262
9.4.2	Non-slip boundary conditions with breakdown of monotonicity	263
9.4.3	Non-slip boundary conditions	267
9.5	Final remarks	267
	<i>References</i>	271
	<i>Index</i>	279