CALCULUS

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Third Edition

Michael Spivak

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Dedicated to the Memory of Y. P.

PREFACE

I hold every man a debtor
to his profession,
from the which as men of course
doe seeke to receive countenance and profit,
so ought they of duty to endeavour
themselves by way of amends,
to be a help and
ornament therunto.

FRANCIS BACON

PREFACE TO THE FIRST EDITION

Every aspect of this book was influenced by the desire to present calculus not merely as a prelude to but as the first real encounter with mathematics. Since the foundations of analysis provided the arena in which modern modes of mathematical thinking developed, calculus ought to be the place in which to expect, rather than avoid, the strengthening of insight with logic. In addition to developing the students' intuition about the beautiful concepts of analysis, it is surely equally important to persuade them that precision and rigor are neither deterrents to intuition, nor ends in themselves, but the natural medium in which to formulate and think about mathematical questions.

This goal implies a view of mathematics which, in a sense, the entire book attempts to defend. No matter how well particular topics may be developed, the goals of this book will be realized only if it succeeds as a whole. For this reason, it would be of little value merely to list the topics covered, or to mention pedagogical practices and other innovations. Even the cursory glance customarily bestowed on new calculus texts will probably tell more than any such extended advertisement, and teachers with strong feelings about particular aspects of calculus will know just where to look to see if this book fulfills their requirements.

A few features do require explicit comment, however. Of the twenty-nine chapters in the book, two (starred) chapters are optional, and the three chapters comprising Part V have been included only for the benefit of those students who might want to examine on their own a construction of the real numbers. Moreover, the appendices to Chapters 3 and 11 also contain optional material.

The order of the remaining chapters is intentionally quite inflexible, since the purpose of the book is to present calculus as the evolution of one idea, not as a collection of "topics." Since the most exciting concepts of calculus do not appear until Part III, it should be pointed out that Parts I and II will probably require less time than their length suggests—although the entire book covers a one-year course, the chapters are not meant to be covered at any uniform rate. A rather natural dividing point does occur between Parts II and III, so it is possible to reach differentiation and integration even more quickly by treating Part II very briefly, perhaps returning later for a more detailed treatment. This arrangement corresponds to the traditional organization of most calculus courses, but I feel that it will only diminish the value of the book for students who have seen a small amount of calculus previously, and for bright students with a reasonable background.

The problems have been designed with this particular audience in mind. They range from straightforward, but not overly simple, exercises which develop basic techniques and test understanding of concepts, to problems of considerable difficulty and, I hope, of comparable interest. There are about 625 problems in all. Those which emphasize manipulations usually contain many examples, numbered with small Roman numerals, while small letters are used to label interrelated parts in other problems. Some indication of relative difficulty is provided by a system of starring and double starring, but there are so many criteria for judging difficulty, and so many hints have been provided, especially for harder problems, that this guide is not completely reliable. Many problems are so difficult, especially if the hints are not consulted, that the best of students will probably have to attempt only those which especially interest them; from the less difficult problems it should be easy to select a portion which will keep a good class busy, but not frustrated. The answer section contains solutions to about half the examples from an assortment of problems that should provided a good test of technical competence. A separate answer book contains the solutions of the other parts of these problems, and of all the other problems as well. Finally, there is a Suggested Reading list, to which the problems often refer, and a glossary of symbols.

I am grateful for the opportunity to mention the many people to whom I owe my thanks. Jane Bjorkgren performed prodigious feats of typing that compensated for my fitful production of the manuscript. Richard Serkey helped collect the material which provides historical sidelights in the problems, and Richard Weiss supplied the answers appearing in the back of the book. I am especially grateful to my friends Michael Freeman, Jay Goldman, Anthony Phillips, and Robert Wells for the care with which they read, and the relentlessness with which they criticized, a preliminary version of the book. Needles to say, they are not responsible for the deficiencies which remain, especially since I sometimes rejected suggestions which would have made the book appear suitable for a larger group of students. I must express my admiration for the editors and staff of W. A. Benjamin, Inc., who were always eager to increase the appeal of the book, while recognizing the audience for which it was intended.

The inadequacies which preliminary editions always involve were gallantly endured by a rugged group of freshmen in the honors mathematics course at Brandeis University during the academic year 1965–1966. About half of this course was devoted to algebra and topology, while the other half covered calculus, with the preliminary edition as the text. It is almost obligatory in such circumstances to report that the preliminary version was a gratifying success. This is always safe—after all, the class is unlikely to rise up in a body and protest publicly—but the students themselves, it seems to me, deserve the right to assign credit for the thoroughness with which they absorbed an impressive amount of mathematics. I am content to hope that some other students will be able to use the book to such good purpose, and with such enthusiasm.

MICHAEL SPIVAK
Waltham, Massachusetts
February 1967

PREFACE TO THE SECOND EDITION

I have often been told that the title of this book should really be something like "An Introduction to Analysis," because the book is usually used in courses where the students have already learned the mechanical aspects of calculus—such courses are standard in Europe, and they are becoming more common in the United States. After thirteen years it seems too late to change the title, but other changes, in addition to the correction of numerous misprints and mistakes, seemed called for. There are now separate Appendices for many topics that were previously slighted: polar coordinates, uniform continuity, parameterized curves, Riemann sums, and the use of integrals for evaluating lengths, volumes and surface areas. A few topics, like manipulations with power series, have been discussed more thoroughly in the text, and there are also more problems on these topics, while other topics, like Newton's method and the trapezoid rule and Simpson's rule, have been developed in the problems. There are in all about 160 new problems, many of which are intermediate in difficulty between the few routine problems at the beginning of each chapter and the more difficult ones that occur later.

Most of the new problems are the work of Ted Shifrin. Frederick Gordon pointed out several serious mistakes in the original problems, and supplied some non-trivial corrections, as well as the neat proof of Theorem 12-2, which took two Lemmas and two pages in the first edition. Joseph Lipman also told me of this proof, together with the similar trick for the proof of the last theorem in the Appendix to Chapter 11, which went unproved in the first edition. Roy O. Davies told me the trick for Problem 11-66, which previously was proved only in Problem 20-8 [21-8 in the third edition], and Marina Ratner suggested several interesting problems, especially ones on uniform continuity and infinite series. To all these people go my thanks, and the hope that in the process of fashioning the new edition their contributions weren't too badly botched.

MICHAEL SPIVAK

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PREFACE TO THE THIRD EDITION

The most significant change in this third edition is the inclusion of a new (starred) Chapter 17 on planetary motion, in which calculus is employed for a substantial physics problem.

In preparation for this, the old Appendix to Chapter 4 has been replaced by three Appendices: the first two cover vectors and conic sections, while polar coordinates are now deferred until the third Appendix, which also discusses the polar coordinate equations of the conic sections. Moreover, the Appendix to Chapter 12 has been extended to treat vector operations on vector-valued curves.

Another large change is merely a rearrangement of old material: "The Cosmopolitan Integral," previously a second Appendix to Chapter 13, is now an Appendix to the chapter on "Integration in Elementary Terms" (previously Chapter 18, now Chapter 19); moreover, those problems from that chapter which used the material from that Appendix now appear as problems in the newly placed Appendix.

A few other changes and renumbering of Problems result from corrections, and elimination of incorrect problems.

I was both startled and somewhat dismayed when I realized that after allowing 13 years to elapse between the first and second editions of the book, I have allowed another 14 years to elapse before this third edition. During this time I seem to have accumulated a not-so-short list of corrections, but no longer have the original communications, and therefore cannot properly thank the various individuals involved (who by now have probably lost interest anyway). I have had time to make only a few changes to the Suggested Reading, which after all these years probably requires a complete revision; this will have to wait until the next edition, which I hope to make in a more timely fashion.

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