

An Introduction to the Theory of Stellar Structure and Evolution

Second Edition

Using fundamental physics, the theory of stellar structure and evolution can predict how stars are born, how their complex internal structure changes, what nuclear fuel they burn, and their ultimate fate. This textbook is a stimulating introduction for students of astronomy, physics and applied mathematics, taking a course on the physics of stars. It uniquely emphasizes the basic physical principles governing stellar structure and evolution.

This second edition contains two new chapters on mass loss from stars and interacting binary stars, and new exercises. Clear and methodical, it explains the processes in simple terms, while maintaining mathematical rigour. Starting from general principles, this textbook leads students step-by-step to a global, comprehensive understanding of the subject. Fifty exercises and full solutions allow students to test their understanding. No prior knowledge of astronomy is required, and only a basic background in undergraduate physics and mathematics is necessary.

Dina Prialnik is a Professor of Planetary Physics at Tel Aviv University. Her research interests lie in stellar evolution; the structure and evolution of cataclysmic variables; comet nuclei and other small solar system bodies; and the evolution of planets.

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Tel Aviv University



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To my son

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Preface to the second edition

It is now a decade since the publication of the first edition of this book. Despite the large number of research papers devoted to the subject during this period of time, the basic principles and their applications that are addressed in the book remain valid and hence the original text has been mostly left unchanged. And yet a major development did occur soon after the book first appeared in print: the ‘solar neutrino problem’ that had puzzled physicists and astrophysicists for almost four decades finally found its solution, which indeed necessitated new physics. However, the new physics belongs to the theory of elementary particles, which must now account for neutrino masses, rather than to the theory of stars. Also worth mentioning is a major recent discovery that finally provides support to the theory proposed about four decades ago regarding the end of very massive stars in powerful supernova explosions triggered by pair-production instability: SN2006gy, the first observed candidate for such a mechanism. Thus the section on solar neutrinos is now complete and that on supernovae expanded.

Stellar evolution calculations have made great progress in recent years, following the rapid development of computational means: increasingly faster CPUs and greater memory volumes. Nevertheless, I have made use of new results only when they provide better illustration for points raised in text. For the most part, old results are still valid and this long-term validity is worth emphasizing; the theory of stellar structure and evolution, with all its complexity, is a well-established physical theory.

The text was expanded to include two new chapters on topics that were not addressed in the first edition: mass loss and interacting binary stars. Both are complicated subjects, some aspects of which are still not well understood, similarly to star formation. Although this may justify their exclusion from a basic textbook on stellar structure and evolution theory, an exposition of the theory would not be complete without some reference to them. Each one deserves a full textbook by itself, and in fact books have been devoted to each in the last decade, not to mention older texts dealing with these subjects. In the new chapters I have

touched upon them briefly enough to adapt the treatment to the general level and scope of this book, but also in sufficient detail to arouse interest and enable a basic understanding of where the problems lie.

I have also added an appendix that explains and develops more rigorously the concept of degeneracy pressure in an attempt to dispel some confusion related to the applicability of complete degeneracy, which was the only form developed in the early edition: is the omission of temperature an assumption or a justified result? Another, minor, addition is a concise discussion of the mixing-length treatment of convection. Finally, I have included a few more exercises, which are mostly of the same nature and serve the same purpose as the older ones: to elucidate points made in the text or provide additional information.

While I am still grateful to those who have helped, supported and encouraged me during the writing of the original version of this book, it is with new pleasure and gratitude that I thank those who have commented on it since, who have used the book in their classes and have helped to improve it. Among them are Nuria Calvet, Aparna Venkatesan, Allan Walstad, Werner Däppen, Nicolay Samus, Bill Herbst, Phil Armitage, Silvia Rossi and Barry Davids, and my long-time friends Mike Shara, Mario Livio and Oded Regev. Special thanks are due to Robert Smith for pointing out a number of inaccuracies and for making important suggestions.

Preface to the first edition

For over ten years I have been teaching an introductory course in astrophysics for undergraduate students in their second or third year of physics or planetary sciences studies. In each of these classes, I have witnessed the growing interest and enthusiasm building up from the beginning of the course toward its end.

It is not surprising that astrophysics is considered interesting; the field is continually gaining in popularity and acclaim due to the development of very sophisticated telescopes and to the frequent space missions, which seem to bring the universe closer and make it more accessible. But students of physics have an additional reason of their own for this interest. The first years of undergraduate studies create the impression that physics is made up of several distinct disciplines, which appear to have little in common: mechanics, electromagnetism, thermodynamics and atomic physics, each dealing with a separate class of phenomena. Astrophysics – in its narrowest sense, as *the physics of stars* – presents a unique opportunity for teachers to demonstrate and for students to discover that complex structures and processes do occur in Nature, for the understanding of which all the different branches of physics must be invoked and combined. Therefore, a course devoted to the physics of stars should perhaps be compulsory, rather than elective, during the second or third year of physics undergraduate studies. The present book may serve as a guide or textbook for such a course.

Books on astrophysics fall mostly into two categories: on the one hand, extensive introductions to the field covering all its branches, from planets to galaxies and cosmology, quite often including an introduction to the main fields of physics as well; and on the other hand, specialized books, often including up to date results of ongoing studies. The former are aimed at readers who have not yet received any real training in physics, the latter, at graduate students who are specializing in astronomy or astrophysics. The present book is aimed at students who fall between these extremes: undergraduates who have acquired a basic mathematical background and have been introduced to the basic laws of physics

during the first two or three semesters of studies, but have no prior knowledge of astronomy.

The purpose of this book is to satisfy the eagerness to comprehend the realm of stars, by focusing on fundamental principles. The students are made to understand, rather than become familiar with, the different types of stars and their evolutionary trends. As far as possible, I have refrained from burdening the reader with astronomical concepts and details, in an attempt to make the text suitable for students of physics who do not necessarily intend to pursue astrophysics any further. Thus, odd as it may seem, there is no mention of concepts that are so familiar to astronomers, such as magnitude, colour index, spectral class and so forth. Equally odd may appear the use of SI units, which is still alien to astrophysics, but has become common, in fact mandatory, in physics studies. I have complied with this demand, despite my conviction that, perhaps surprisingly, astrophysicists still think in terms of cgs units. (One hardly comes across stellar opacities expressed in square metres per kilogram, or densities in kilograms per metre cubed.) As is customary in textbooks, exercises are scattered throughout the book and solutions are provided in an appendix.

The theory of stellar evolution is developed in a methodical manner. The student is led step by step from the formulation of the problem to its solution on a path that appears very natural, even obvious at times. I have tried to avoid the widely adopted alternative of following the progress of a star's evolution, enumerating the different phases with their inherent physical aspects. I find the *logical*, rather than the *chronological*, method the best way of presenting this theory, the way any other established theory is usually presented. When each chapter of a scientific book relies on the preceding one and leads to the next, there is hope of arousing in the reader sufficient curiosity for reading on. The fascinating history of the theory of stellar structure and evolution is sometimes alluded to in 'Notes' and quotations.

The first chapter introduces the subject of stellar evolution, as it arises from observations: the problem is defined and the basic assumptions (axioms) are laid down. The following six chapters are essentially theoretical: the second formulates the problem mathematically by introducing the equations of stellar evolution; the third summarizes briefly the basic physical laws involved in the study of stellar structure, serving for reference later on. Chapters 4, 5 and 6 – dealing with nucleosynthesis in stars, simple stellar models and stability – build up to Chapter 7, which is the heart of this book. Combining the material of Chapters 3–6, it presents a general, almost schematic picture of the evolution of stars in all its aspects. From my experience, this picture remains imprinted in the students' minds long after the details have faded away. Chapter 8 is, in a way, a recapitulation of the previous chapter from a different angle: the story of stellar evolution is retold, filling in many details, as it emerges from numerical computations. Emphasis is now put on comparison with observations, thereby closing the circuit opened

in Chapter 1. The next chapter deals with special objects: supernovae and their remnants, pulsars, black holes (very briefly) and other radiation sources. Finally, Chapter 10 touches on the global picture of the stellar evolution cycle, from the galactic point of view.

I have tried to give proper credit where it was due, but occasionally I may have failed or erred. I apologize for any such failure or error, my only defence being that it was not intentional. I have refrained from referring to original papers in the text, in order not to interfere with fluency. A selection of references (by no means complete) is given in the bibliography.

Enthusiasm toward a subject of study is instigated not only by the subject itself, but quite often by the teacher. In this respect I was lucky to have been introduced to astrophysics by Giora Shaviv and I hope to have carried on some of his passion to my own students. Computing and numerical modeling, on which the subject matter of this book relies, are not merely a skill but a true art of unique beauty and elegance. For having introduced me to this art long ago and for having been a constant source of encouragement and advice during the writing of this book, I am grateful to my husband (and former teacher) Attay Kovetz. I would like to express my gratitude and appreciation to Leon Mestel for a very careful and thorough reading of the original manuscript. This book has tremendously benefited from his countless observations, comments and suggestions. Special thanks are due to Michal Semo and her team at the Desktop Publishing unit of Tel Aviv University for their skilful and painstaking graphics work, not to mention their endless patience and cheerfulness. Above all, I am grateful to my son Ely for gracefully bearing with a busy and preoccupied mother during the rather demanding years of adolescence.

CHOOSE SOMETHING LIKE A STAR

by Robert Frost

*O Star, (the fairest one in sight),
We grant your loftiness the right
To some obscurity of cloud –
It would not do to say of night,
Since dark is what brings out your light.
Some mystery becomes the proud.
But to be wholly taciturn
In your reserve is not allowed.
Say something to us we can learn
By heart and when alone repeat.
Say something! And it says 'I burn'.
But say with what degree of heat.
Talk Fahrenheit, talk Centigrade.
Use language we can comprehend.
Tell us what elements you blend.
It gives us strangely little aid,
But does tell something in the end.*

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