1

Observational background and basic assumptions

1.1 What is a star?

A *star* can be defined as a body that satisfies two conditions: (a) it is bound by self-gravity; (b) it radiates energy supplied by an internal source. From the first condition it follows that the shape of such a body must be spherical, for gravity is a spherically symmetric force field. Or, it might be spheroidal, if axisymmetric forces are also present. The source of radiation is usually nuclear energy released by fusion reactions that take place in stellar interiors, and sometimes gravitational potential energy released in contraction or collapse. By this definition, a *planet*, for example, is not a star, in spite of its stellar appearance, because it shines (mostly) by reflection of solar radiation. Nor can a *comet* be considered a star, although in early Chinese and Japanese records comets belonged with the 'guest *stars*' – those stars that appeared suddenly in the sky where none had previously been observed. Comets, like planets, shine by reflection of solar radiation and, moreover, their masses are too small for self-gravity to be of importance.

A direct implication of the definition is that stars must evolve: as they release energy produced internally, changes necessarily occur in their structure or composition, or both. This is precisely the meaning of *evolution*. From the above definition we may also infer that the *death* of a star can occur in two ways: violation of the first condition – self-gravity – meaning breakup of the star and scattering of its material into interstellar space, or violation of the second condition – internally supplied radiation of energy – that could result from exhaustion of the nuclear fuel. In the latter case, the star fades slowly away, while it gradually cools off, radiating the energy accumulated during earlier phases of evolution. Eventually, it will become extinct, disappearing from the field of view of even the most powerful telescopes. This is what we call a *dead star*. We shall see that most stars end their *lives* by a combination of these two processes: partial breakup (or shedding of matter) and extinction. As to the *birth* of a star, this is a complex process, which presents many problems that are still under intensive investigation. We shall deal with this phase only briefly, mainly by pointing out the circumstances under which it is expected to occur.

We shall therefore start pursuing the evolution of a star from the earliest time when both conditions of the definition have been fulfilled, and we shall stop when at least one condition has ceased to be satisfied, completely and irreversibly. Finally, we shall consider the life cycle of stellar populations and the effect of stellar evolution on the evolution of galaxies within which stars reside. Galaxies are large systems of stars (up to 10^{11} or so), which also contain interstellar clouds of gas and dust. Many of the stars in a galaxy are aggregated in clusters, the largest among them containing more than 10^5 stars. The object of reference in stellar physics is, naturally, the Sun, and in galactic physics, the Galaxy to which it belongs, also known as the Milky Way galaxy.

1.2 What can we learn from observations?

Astrophysics (the physics of stars) does not lend itself to experimental study, as do the other fields of physical science. We cannot devise and conduct experiments in order to test and validate theories or hypotheses. Validation of a theory is achieved by accumulating observational evidence that supports it and its predictions or inferences. The evidence is derived from events that have occurred in the past and are completely beyond our control. The task is rather similar to that of a detective. As a rule of thumb, a theory is accepted as valid (or at least highly probable) if it withstands two radically different and independent observational tests, and of course, so long as no contradictory evidence has been found.

The information we can gather from an individual star is quite restricted. The primary characteristic that can be measured is the *apparent brightness*, which is the amount of radiation from the star falling per unit time on unit area of a collector (usually, a telescope). This radiation flux, which we shall denote $I_{\rm obs}$ is not, however, an intrinsic property of the observed star, for it depends on the distance of the star from the observer. The stellar property is the *luminosity* L, defined as the amount of energy radiated per unit time – the power of the stellar engine. Since L is also the amount of energy crossing, per unit time, a spherical surface area at the distance d of the observer from the star, the measured apparent brightness is

$$I_{\rm obs} = \frac{L}{4\pi d^2}, \tag{1.1}$$

and L may be inferred from $I_{\rm obs}$ if d is known. The luminosity of a star is usually expressed relative to that of the Sun, the *solar luminosity* $L_\odot = 3.85 \times 10^{26}$ J s^{-1}. Stellar luminosities range between less than $10^{-5} L_\odot$ and over $10^5 L_\odot$.

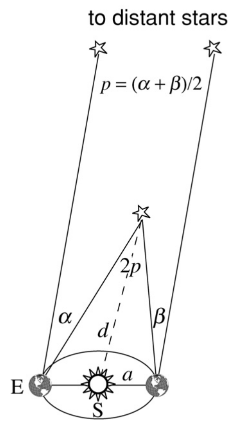

Figure 1.1 Sketch of the parallax method for measuring distances to stars.

Note: The only *direct* method of determining distances to stars (and other celestial bodies) is based on the old concept of *parallax* – the angle between the lines of sight of a star from two different positions of the observer. The lines of sight and the line connecting the observer's positions form a triangle, with the star at the apex, as shown in Figure 1.1. The larger the distance to the object, the wider the baseline required for obtaining a discernible parallax: for objects within the solar system distant points on Earth suffice; for stars, a much larger baseline is needed. This is provided by the Earth's orbit around the Sun, yielding a maximal baseline of $\sim 3 \times 10^{11}$ m, twice the Earth-Sun distance $a(= 1\ AU)$. Thus, the stellar parallax is obtained by determining a star's position relative to very distant, fixed stars, at an interval of half a year. Even so, the triangle obtained is very nearly isosceles, with almost right base angles, while the parallax p, defined as half the apex angle, is less than $1''$ (the largest known stellar parallax is that of *Proxima Centauri* – the star closest to our Sun, $p = 0''.76$). Consequently, to a good approximation, $d \approx a/p$. Based on this method, distances of up to about 500 light-years may be directly measured. (One light-year, 9.46×10^{15} m, is the distance travelled in one year at the speed of light.) A common astronomical unit for measuring distances, called *parsec*, is based on the parallax method: as its name indicates, it is the distance corresponding to a parallax of $1''$, amounting to about 3 light-years. Recently, the number of stars for which we have accurate distances has grown a hundredfold as a result of the activity of the satellite specially designed for this task, *Hipparcos* (High Precision Parallax Collecting Satellite), named after the greatest astronomer of antiquity, Hipparchus of Nicea (second century BC), who measured the celestial positions and brightnesses of almost a thousand stars and produced the first star catalogue. The satellite *Hipparcos*, which operated during 1989–93, gathered data on more than a million nearby stars. But on the astronomical scale, distances that can be directly measured are quite small and hence *indirect* methods have to be devised, some of which are based on the theory of stellar structure and evolution, as we shall see in Chapter 9.

The surface temperature of a star may be obtained from the general shape of its spectrum, the *continuum*, which is very similar to that of a blackbody. The *effective temperature* of a star $T_{\rm eff}$ is thus defined as the temperature of a blackbody that would emit the same radiation flux. It provides a good approximation to the temperature of the star's outermost layer, called the *photosphere*, where the bulk of the emitted radiation originates. If R is the stellar radius, the surface flux is $L/4\pi R^2$, and hence:

$$\sigma T_{\rm eff}^4 = \frac{L}{4\pi R^2}, \tag{1.2}$$

where σ is the Stefan-Boltzmann constant. Thus

$$L = 4\pi R^2 \sigma T_{\rm eff}^4. \tag{1.3}$$

The surface temperatures of stars range between a few thousand to a few hundred thousand degrees Kelvin (K), the wavelength of maximum radiation $\lambda_{\rm max}$ shifting, according to Wien's law

$$\lambda_{\rm max} T = \text{constant}, \tag{1.4}$$

from infra-red to soft X-rays. The effective temperature of the Sun is 5780 K. We should bear in mind, however, that conclusions regarding internal temperatures cannot be drawn from surface temperatures without a theory.

The chemical composition, too, can be inferred from the spectrum. Each chemical element has its characteristic set of spectral lines. These lines can be observed in the light received from stars, superimposed upon the continuous spectrum, either as emission lines, when the intensity is enhanced, or as absorption lines, when it is diminished. The elements that make up the photosphere of a star, which emits the observed radiation, may thus be identified in the stellar spectrum. But since the photosphere is very thin, the deduced composition is not representative of the bulk, opaque interior of the star. Most of the chemical elements were found to be present in the solar spectrum. In fact the existence of the element *helium* was first suggested by spectral lines from the Sun (in the 1860s); its name is derived from 'helios', the Greek word for Sun.

Under certain conditions, the mass of a star that is a member of a binary system can be calculated, based on spectral line shifts, as we shall show in Chapter 11. Very seldom, in eclipsing binary systems, may the radius of a star be directly derived; it can, however, be estimated from the independently derived luminosity (when possible) and effective temperature using Equation (1.3). Stellar masses and radii are measured in units of the *solar mass*, $M_\odot = 1.99 \times 10^{30}$ kg, and the *solar radius*, $R_\odot = 6.96 \times 10^8$ m. The mass range is quite narrow – between $\sim 0.1 M_\odot$ and a few tens $M_\odot$; stellar radii vary typically between less than $0.01 R_\odot$ to more than $1000 R_\odot$. Much more compact stars exist, though, with radii of a few tens of kilometres.

Besides being sparse, the information one can gather is confined to a very brief moment in a star's life, even if observations are carried on for hours or years, or, hypothetically, hundreds of years. To illustrate this point, let us compare the life span of a star to that of a human being: uninterrupted observation of a star since, say, the discovery of the telescope some 400 years ago, would be tantamount to watching a person for about 3 minutes! Obviously, it would be impossible to learn anything (directly) about the evolution of the star from such a fleeting observation. The body of data available to the astrophysicist consists of accumulated momentary information on a very large number of stars, at different evolutionary stages. From these data, the astrophysicist is required to form a scenario describing the evolution of a single star.

Imagine, for comparison, an explorer who has never seen human beings, trying to figure out the nature and evolutionary course of these creatures, based solely on a large sample of photographs of many different humans chosen at random. The explorer will find that humans differ in many properties, such as height, colour of skin, etc., and will note, for example, that the height of the majority varies within a narrow range around a mean of, say, 1.75 m, and only the height of a small minority is significantly below this mean. These findings may be interpreted in two ways: (a) humans are intrinsically different, the tall ones being more numerous than the short ones; (b) humans are similar to one another, but their properties change in the course of their lives, their height either increasing or decreasing with age (one would not be able to tell which). In the latter case, based on the hypothesis that humans evolve, it may also be inferred that individual human beings are tall for a longer part of their lives than they are short. It might even be possible to calculate the rate of change of the human height from the relative number of individuals in different height ranges.

In a similar manner, if we find that a certain property is common to a great number of stars, we may infer – on the basis of the evolution hypothesis – that such a property prevails in stars for long periods of time. By the same token, rarely observed phenomena might not be rare events, but simply short-lived ones. At the same time, the possibility of actually rare phenomena cannot be entirely ruled out. This is a sample of the problems one would have to face if the understanding of stars and their evolution were to rest entirely on observation.

As the information available for any given star is so limited, the theory of stellar evolution is not meant to describe in detail the structure and expected evolutionary course of any individual star (with the exception of the Sun). Its purpose is rather to construct a general model that explains the large variety of stellar types, as well as the relations between different stellar properties revealed by observations (such as the correlation between luminosity and surface temperature, or between luminosity and mass, which we shall shortly encounter).

1.3 Basic assumptions

Guided by the observational evidence, we may add several fundamental assumptions (or axioms) to the general definition of a star, on which to base the theory of stellar structure and evolution.

Isolation

Regarding its structure and evolution, a single star may be considered isolated in empty space, although it is invariably a member of a large group – a galaxy – or even a denser group within a galaxy – a stellar cluster. (We exclude from the present discussion binary stars – a pair of stars that form a bound system that we shall address in Chapter 11.) Consequently, the initial conditions will exclusively determine the course of a star's evolution. Thus the evolutionary process of a star (metaphorically termed *life*) differs from that of live creatures, the latter being influenced to a large extent by interaction with their environment. To better grasp the isolation of stars, consider the star closest to our Sun (*Proxima Centauri*), which is at a distance of 4.3 light-years. This distance is larger than the solar diameter by a factor of 3×10^7. Such a situation would be similar to nearest neighbours on Earth being separated by a distance 3×10^7 times their height, which roughly amounts to 50 000 km. This is four times the Earth diameter or one seventh of the distance to the Moon. We *would* call this isolation! Both the gravitational field and the radiation flux, which vary in proportion to $1/d^2$, are diminished by a factor of at least $1/(3 \times 10^7)^2 \sim 10^{-15}$ from one star to another.

Uniform initial composition

A star is born with a given mass and a given, presumably homogeneous, composition. The latter depends on the time of formation and on the location within the galaxy where the star is formed. The composition of stars has been a question of intense debate for a long time. It turned out, finally, that most of the material of a newly formed star, about 70% of its mass, consists of hydrogen. The second most important element is helium, amounting to 25–30% of the mass, and there are traces of heavier elements, of which the most abundant are oxygen, carbon and nitrogen (in that order), known collectively as the CNO group. In the Sun, for example, for every 10 000 hydrogen atoms, there are about 1000 helium atoms, 8 oxygen atoms, almost 4 carbon atoms, one atom of nitrogen, one of neon and less than one atom of each of the other species. The composition of stellar material is usually described by the *mass fractions* of different elements, the mass of each element per unit mass of material. It is common to denote the mass fraction of hydrogen by X, that of helium by Y, and the total mass fraction of all the other elements by Z, so that $X + Y + Z = 1$.

Exercise 1.1: Calculate the mass fractions of hydrogen, helium, carbon, oxygen, nitrogen and neon in the Sun.

Thus, since both hydrogen and helium, the predominant stellar components, are found in the gas phase unless the temperature is extremely low or the density (pressure) extremely high, we may quite safely deduce that stars are made of gas. We shall return to this point later on, when we gain more insight into stellar interiors.

With very few exceptions, the abundances of the chemical elements, as derived from stellar spectra, are remarkably similar. Moreover, they are very similar to those prevailing in the interstellar medium. As stars are born in interstellar clouds, and the composition of their surface layers is expected to be the least affected by evolutionary processes, it may be concluded that there is little difference in the *initial* composition of stars. The largest differences occur for the abundances of the heavy elements, which vary among different stars between less than 0.001 to a few per cent of the entire stellar mass. But differences in the initial abundances of these elements are of secondary importance to stellar evolution. For simplicity, we shall ignore differences in the initial composition of stars. In numerical examples we shall generally adopt the solar composition. The fate of a star will then be solely dependent upon its initial mass M.

Historical Note: The first to show that the Sun's atmosphere is dominated by hydrogen was Cecilia Payne in her doctoral dissertation completed in 1925. Not only did she show that the most abundant elements were hydrogen and helium, but she also suggested that the relative abundances of the heavier elements were roughly constant throughout the galaxy, thus indicating the homogeneity of the universe. These findings followed from Saha's equation (see Section 3.6), then new, according to which, the strength of spectral lines depends on physical conditions as well as on elemental abundances. These conclusions, very much opposed to the common wisdom of the time, were largely ignored. It was only a few years later, when, corroborated by further evidence, the prevalence of hydrogen and helium in the Sun's atmosphere was convincingly argued by Henry Norris Russell, whose fame will become apparent shortly.

A doctoral degree awarded to a woman was extremely unusual in those days. In her autobiography, Cecilia Payne-Gaposchkin writes 'One serious obstacle existed: there was no advanced degree in astronomy, and I should have to be accepted as a candidate by the Department of Physics. The redoubtable Chairman of that department was Theodore Lyman, and Shapley [Harlow Shapley, her mentor] reported to me that he refused to accept a woman candidate.' In the end she became the first person to earn a doctorate in astronomy from Harvard University.

Spherical symmetry

Departure from spherical symmetry may be caused by rotation or by the star's own magnetic field (since by assuming isolation, we have excluded all possible external force fields). In the overwhelming majority of cases, the energy associated with these factors is much smaller than the gravitational binding energy. We know, for example, that the period of revolution of the Sun around its axis is about 27 days, so that its angular velocity is $\omega \simeq 2.5 \times 10^{-6}\ \mathrm{s}^{-1}$. The spin velocity of more distant stars can be deduced from the broadening of spectral lines caused by the Doppler effect. The kinetic energy of rotation relative to the gravitational binding energy is of the order:

$$\frac{M\omega^2 R^2}{GM^2/R} = \frac{\omega^2 R^3}{GM} \sim 2 \times 10^{-5},$$

where G is the constant of gravitation. (This is also the ratio of the centrifugal acceleration to the gravitational acceleration at the equator.)

The magnetic fields of stars similar to the Sun range from a few thousandths to a few tenths of a tesla. The larger ones may be directly deduced from split spectral lines caused by the Zeeman effect, whose separation can be measured. The energy density associated with a magnetic field B is $B^2/2\mu_0$, while the gravitational energy density is of the order of GM^2/R^4; for the Sun, even taking $B = 0.1$ T (typical of sunspots, but larger than the average magnetic field), we have

$$\frac{B^2/\mu_0}{GM^2/R^4} = \frac{B^2 R^4}{\mu_0 GM^2} \sim 10^{-11}.$$

Compact stars tend to have higher magnetic fields, but their small radii (large binding energies) compensate for them. Hence, magnetic effects on the structure of a star can usually be ignored.

Neglecting deviations from spherical symmetry, the physical properties within a star change only with the radial distance r from the centre and are uniform over a spherical surface of radius r. The spatial variable r may be replaced by the mass m enclosed in a sphere of radius r, as shown in Figure 1.2. The transformation between these variables is given in terms of the density ρ:

$$m(r) = \int_0^r 4\pi r^2 \rho(r) dr$$

or, in differential form,

$$dm = \rho\, 4\pi r^2 dr. \tag{1.5}$$

The advantage of using m instead of r in calculations of the changing stellar structure is that its range of variation is bounded, $0 \le m \le M$, whereas the radius may change by several orders of magnitude in the course of evolution of a star.

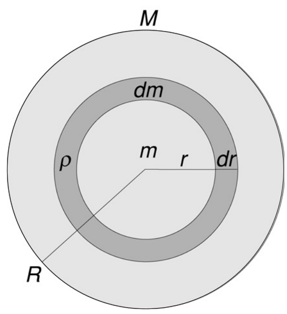

Figure 1.2 The relationship between space variables r and m in spherical symmetry.

Exercise 1.2: In a star of mass M, the density decreases from the centre to the surface as a function of radial distance r, according to

$$\rho = \rho_c \left[1 - \left(\frac{r}{R} \right)^2 \right],$$

where ρ_c is a given constant and R is the star's radius. (a) Find $m(r)$. (b) Derive the relation between M and R. (c) Show that the average density of the star (total mass divided by total volume) is $0.4\rho_c$.

1.4 The H–R diagram: a tool for testing stellar evolution

As we have seen, the two most fundamental properties of a star that can be inferred from observation are the luminosity L and the effective temperature $T_{\rm eff}$. It is only natural that a possible correlation between them be sought. This was initiated independently by two astronomers at about the same time: Ejnar Hertzsprung in 1911 and Henry Norris Russell in 1913. Hence the diagram whose axes are the (decreasing) surface temperature (or related properties) and the luminosity (or related properties) bears their names, being known as *the H–R diagram*. Each observed star is represented by a point in such a diagram, an example of which is given in Figure 1.3. The results depend to some extent on the criterion used for choosing the sample of stars, for example, stars within a limited volume in the solar neighbourhood, or members of a given star cluster, or stars of apparent brightness greater than a prescribed limit, etc. The question we are interested in is whether something can be learned from this diagram regarding the evolution of stars.

It is immediately obvious from the examination of *any* H–R diagram that only certain combinations of L and $T_{\rm eff}$ values are possible (a priori there is nothing to impose such a constraint): most points are found to lie along a thin strip that

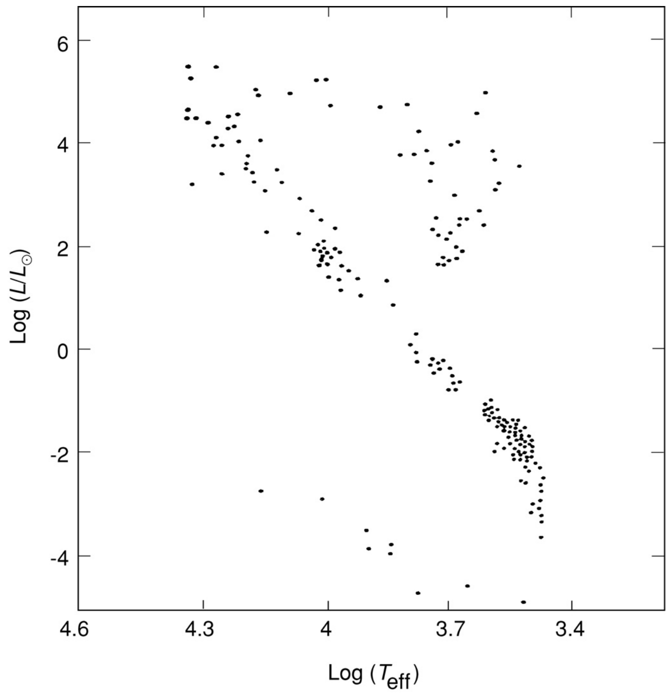

Figure 1.3 The H–R diagram of stars in the neighbourhood of the Sun.

runs diagonally through the ($\log T_{\mathrm{eff}}$, $\log L$) plane. This strip is called *the main sequence* and the corresponding stars are known as *main-sequence stars*.

Another populated area of the diagram is found to the right and above the main sequence: it represents stars that are brighter than main-sequence stars of same T_{eff}, or of lower T_{eff} for the same L, meaning that their spectrum is shifted toward longer wavelengths and their colour is reddish. A higher L and lower T_{eff} implies, according to Equation (1.3), a large radius. Such stars are therefore called *red giants*. Their radii may attain several hundred solar radii and even more. If the Sun were to become a red giant, it would engulf the Earth and reach beyond Mars.

Another region of the ($\log T_{\mathrm{eff}}$, $\log L$) plane that is relatively rich in points is located at the lower left corner: low luminosities and high effective temperatures. Stars that fall in this region have a small radius and a bluish-white colour; accordingly, they are named *white dwarfs*. White dwarf radii are of the order of the Earth's, although their masses are close to the Sun's. The typical densities of such stars are therefore tremendous; one cubic centimetre of white dwarf material would weigh more than a ton on Earth.

There are points outside these three main regions and there are conspicuously empty spots within densely populated areas of the diagram, but we shall ignore them for the moment and concentrate on the three main ones. What, if anything, can we learn from them? We recall that, in view of our basic assumptions, stars