

## GEOLOGICAL FLUID DYNAMICS

### Sub-surface Flow and Reactions

Owen Phillips's textbook, *Flow and Reactions in Permeable Rocks*, published in 1991, became a classic in the field of geological fluid dynamics. This book is its long-awaited successor. In the intervening years, significant advances have been made in our understanding of subterranean flow, especially through the vast amount of research into underground storage of nuclear waste and aquifer pollution. This new book integrates and extends these modern ideas and techniques and applies them to the physics and chemistry of sub-surface flows in water-saturated, sandy, and rocky media. It describes essential scientific concepts and tools for hydrologists and public health ecologists concerned with present-day flow and transport, and also for geologists who interpret present-day patterns of mineralization in terms of fluid flow in the distant past. The book is ideal for graduate students and professionals in hydrology, water resources, and aqueous geochemistry.

OWEN M. PHILLIPS is a Fellow of the Royal Society and a member of the US National Academy of Engineering. He has held academic posts at Cambridge University and the Johns Hopkins University. He was awarded the Sverdrup Gold Medal of the American Meteorological Society for his contributions to oceanography, and a fellowship in the American Geophysical Union for his contributions to geological fluid dynamics. His *Last Chance Energy Book*, published in 1979, anticipated the first global energy crisis of the 1980s, while his recent research has been on sub-surface aquifer flows, the dispersal of contaminants and flow-controlled reactions in rocks. He has two other publications with Cambridge University Press – *The Dynamics of the Upper Ocean* (1966), which was awarded the Adams Prize from Cambridge University, and *Flow and Reactions in Permeable Rocks* (1991).

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O. M. Phillips

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Sub-surface Flow and Reactions

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For Merle

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## *Preface*

This book is concerned with the dynamics of subterranean flows in the natural environment, with the transport and dispersal of contaminants that they may carry and the chemistry of the interactions between the matrix and the fluid that percolates through it in spatially random conduits or aquifer pores. It is intended for anyone with a quantitative interest in the world around them, and particularly for professionals and graduate students in hydrology, geology and environmental science and engineering. Many of the basic concepts originated in the late nineteenth or mid twentieth centuries, but they were usually developed in a very idealized and simplified form because few field measurements of actual sub-surface seepage or flow patterns had been attempted. Only recently have extensive and detailed hydrological field measurements been undertaken and their findings contain surprises that are re-defining the way we view this part of the natural world and begin to understand how it works.

In writing this book, I have relied heavily on the guidance and advice of many colleagues, friends and students who listened patiently, corrected gently and pointed in new directions. In particular, I must thank Lawrence Hardy, John Ferry, Jim Wood and Gordon Wolman, all colleagues, Robert Shedlock of the US Geological Survey and Emory Cleaves of the Maryland Geological Survey and my Cambridge colleagues, Andrew Woods and Herbert Huppert who always had something new to show me. I am particularly grateful to my wife for her tireless reading of the manuscript and her suggestions for improvement.

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