Geophysical Continua presents a systematic treatment of deformation in the Earth from seismic to geologic time scales, and demonstrates the linkages between different aspects of the Earth's interior that are often treated separately.

A unified treatment of solids and fluids is developed to include thermodynamics and electrodynamics, in order to cover the full range of tools needed to understand the interior of the globe. A close link is made between microscopic and macroscopic properties manifested through elastic, viscoelastic and fluid rheologies, and their influence on deformation. Following a treatment of geological deformation, a global perspective is taken on lithospheric and mantle properties, seismology, mantle convection, the core and Earth's dynamo. The emphasis throughout the book is on relating geophysical observations to interpretations of earth processes. Physical principles and mathematical descriptions are developed that can be applied to a broad spectrum of geodynamic problems.

Incorporating illustrative examples and an introduction to modern computational techniques, this textbook is designed for graduate-level courses in geophysics and geodynamics. It is also a useful reference for practising Earth Scientists. Supporting resources for this book, including exercises and full-colour versions of figures, are available at www.cambridge.org/9780521865531.

BRIAN KENNETT is Director and Distinguished Professor of Seismology at the Research School of Earth Sciences in The Australian National University. Professor Kennett's research interests are directed towards understanding the structure of the Earth through seismological observations. He is the recipient of the 2006 Murchison Medal of the Geological Society of London, and the 2007 Gutenberg Medal of the European Geosciences Union, and he is a Fellow of the Royal Society of London. Professor Kennett is the author of three other books for Cambridge University Press: *Seismic Wave Propagation in Stratified Media* (1983), *The Seismic Wavefield: Introduction and Theoretical Development* (2001), and *The Seismic Wavefield: Interpretation of Seismograms on Regional and Global Scales* (2002).

HANS-PETER BUNGE is Professor and Chair of Geophysics at the Department of Earth and Environmental Sciences, University of Munich, and is Head of the Munich Geo-Center. Prior to his Munich appointment, he spent 5 years on the faculty at Princeton University. Professor Bunge's research interests lie in the application of high performance computing to problems of Earth and planetary evolution, including core, mantle and lithospheric dynamics. A member of the Bavarian Academy of Sciences, Bunge is also President of the Geodynamics Division of the European Geosciences Union (EGU).

Geophysical Continua

Deformation in the Earth's Interior

B.L.N. KENNETT
Research School of Earth Sciences, The Australian National University

H.-P. BUNGE
Department of Geosciences, Ludwig Maximilians University, Munich

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9780521865531

First published 2008

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-86553-1 Hardback

Contents

1 Introduction *page* 1
1.1 Continuum properties 2
1.1.1 Deformation and strain 3
1.1.2 The stress-field 3
1.1.3 Constitutive relations 3
1.2 Earth processes 6
1.3 Elements of Earth structure 8
1.3.1 Mantle 12
1.3.2 Core 13
1.4 State of the Earth 14

PART I: CONTINUUM MECHANICS IN GEOPHYSICS 19
2 Description of Deformation 21
2.1 Geometry of deformation 21
2.1.1 Deformation of a vector element 23
2.1.2 Successive deformations 24
2.1.3 Deformation of an element of volume 24
2.1.4 Deformation of an element of area 25
2.1.5 Homogeneous deformation 25
2.2 Strain 27
2.2.1 Stretch 27
2.2.2 Principal fibres and principal stretches 28
2.2.3 The decomposition theorem 29
2.2.4 Pure rotation 30
2.2.5 Tensor measures of strain 32
2.3 Plane deformation 34
2.4 Motion 36
2.5 The continuity equation 38
2.A Appendix: Properties of the deformation gradient determinant 39

3 The Stress-Field Concept 41
3.1 Traction and stress 41
3.2 Local equations of linear motion 44
3.2.1 Symmetry of the stress tensor 44
3.2.2 Stress jumps (continuity conditions) 46
3.3 Principal basis for stress 48
3.4 Virtual work rate principle 51
3.5 Stress from a Lagrangian viewpoint 53
4 Constitutive Relations 54
4.1 Constitutive relation requirements 54
4.1.1 Simple materials 55
4.1.2 Material symmetry 56
4.1.3 Functional dependence 57
4.2 Energy balance 57
4.3 Elastic materials 60
4.4 Isotropic elastic material 61
4.4.1 Effect of rotation 61
4.4.2 Coaxiality of the Cauchy stress tensor and the Eulerian triad 62
4.4.3 Principal stresses 62
4.4.4 Some isotropic work functions 63
4.5 Fluids 64
4.6 Viscoelasticity 67
4.7 Plasticity and flow 69
5 Linearised Elasticity and Viscoelasticity 71
5.1 Linearisation of deformation 71
5.2 The elastic constitutive relation 72
5.2.1 Isotropic response 73
5.2.2 Nature of moduli 73
5.2.3 Interrelations between moduli 74
5.2.4 An example of linearisation 74
5.2.5 Elastic constants 75
5.2.6 The uniqueness Theorem 76
5.3 Integral representations 79
5.3.1 The reciprocal Theorem 80
5.3.2 The representation Theorem 81
5.4 Elastic Waves 83
5.4.1 Isotropic media 83
5.4.2 Green's tensor for isotropic media 85
5.4.3 Interfaces 86
5.5 Linear viscoelasticity 88
5.6 Viscoelastic behaviour 91
5.7 Damping of harmonic oscillations 92

6 Continua under Pressure 95
6.1 Effect of radial stratification 95
6.1.1 Hydrostatic pressure 96
6.1.2 Thermodynamic relations 97
6.2 Finite strain deformation 100
6.3 Expansion of Helmholtz free energy and equations of state 102
6.4 Incremental stress and strain 105
6.4.1 Perturbations in stress 106
6.4.2 Perturbations in boundary conditions 107
6.5 Elasticity under pressure 107
7 Fluid Flow 110
7.1 The Navier–Stokes equation 110
7.1.1 Heat flow 111
7.1.2 The Prandtl number 112
7.2 Non-dimensional quantities 113
7.2.1 The Reynolds number 115
7.2.2 Stokes Flow 115
7.2.3 Compressibility 115
7.2.4 The Péclet number 117
7.3 Rectilinear shear flow 117
7.4 Plane two-dimensional flow 118
7.5 Thermal convection 121
7.5.1 The Rayleigh and Nusselt numbers 121
7.5.2 The Boussinesq approximation 121
7.5.3 Onset of convection 122
7.5.4 Styles of convection 125
7.6 The effects of rotation 126
7.6.1 Rapid rotation 127
7.6.2 The Rossby and Ekman numbers 128
7.6.3 Geostrophic flow 128
7.6.4 The Taylor–Proudman theorem 129
7.6.5 Ekman layers 129
8 Continuum Equations and Boundary Conditions 131
8.1 Conservation equations 131
8.1.1 Conservation of mass 132
8.1.2 Conservation of momentum 132
8.1.3 Conservation of energy 133
8.2 Interface conditions 134
8.3 Continuum electrodynamics 135
8.3.1 Maxwell's equations 135
8.3.2 Electromagnetic constitutive equations 136
8.3.3 Electromagnetic continuity conditions 137

8.3.4 Energy equation for the electromagnetic field 138
8.3.5 Electromagnetic disturbances 139
8.3.6 Magnetic fluid dynamics 141
8.4 Diffusion and heat flow 145
8.4.1 Equilibrium heat flow 146
8.4.2 Time-varying problems 148

PART II: EARTH DEFORMATION 151
9 From the Atomic Scale to the Continuum 153
9.1 Transport properties and material defects 153
9.1.1 Grains and crystal defects 153
9.1.2 General transport properties 156
9.1.3 Atomic diffusion 157
9.2 Lattice vibrations 158
9.3 Creep and rheology 162
9.3.1 Crystal elasticity 162
9.3.2 Deformation behaviour 163
9.4 Material properties at high temperatures and pressures 166
9.4.1 Shock-wave techniques 166
9.4.2 Pressure concentration by reduction of area 168
9.5 Computational methods 171
9.5.1 Electronic structure calculations 171
9.5.2 Atomistic simulations 174
9.5.3 Simulation of crystal structures 175
9.5.4 Finite temperature 176
9.5.5 Influence of defects 178
10 Geological Deformation 180
10.1 Microfabrics 181
10.1.1 Crystal defects 181
10.1.2 Development of microstructure 182
10.1.3 Formation of crystallographically preferred orientations 184
10.2 Macroscopic structures 186
10.2.1 Multiple phases of deformation 187
10.2.2 Folding and boudinage 189
10.2.3 Fractures and faulting 193
10.2.4 Development of thrust complexes 204
11 Seismology and Earth Structure 207
11.1 Seismic Waves 207
11.1.1 Reflection and refraction 208
11.1.2 Attenuation effects 209
11.2 Seismic sources 212
11.3 Building the response of the Earth to a source 216

11.3.1 Displacements as a normal mode sum 218
11.3.2 Free oscillations of the Earth 220
11.4 Probing the Earth 231
11.4.1 Seismic phases 231
11.4.2 Normal mode frequencies 239
11.4.3 Comparison with observations 243
11.4.4 Imaging three-dimensional structure 247
11.5 Earthquakes and faulting 253
12 Lithospheric Deformation 257
12.1 Definitions of the lithosphere 257
12.2 Thermal and mechanical structure 258
12.2.1 Thermal conduction in the oceanic lithosphere 258
12.2.2 Mechanical deformation 261
12.2.3 Estimates of the elastic thickness of the lithosphere 265
12.2.4 Strength envelopes and failure criteria 266
12.3 Plate boundaries and force systems 271
12.3.1 Nature of plate boundaries 271
12.3.2 Plate boundary forces 272
12.4 Measures of stress and strain 274
12.4.1 Stress measurements 274
12.4.2 Strain measurements 276
12.5 Glacial rebound 280
12.6 Extension and convergence 283
12.6.1 Extension 283
12.6.2 Convergence 288
13 The Influence of Rheology: Asthenosphere to the Deep Mantle 294
13.1 Lithosphere and asthenosphere 294
13.1.1 Seismic imaging 295
13.1.2 Seismic attenuation 297
13.1.3 Seismic anisotropy 299
13.1.4 Asthenospheric flow 302
13.1.5 The influence of a low-viscosity zone 302
13.2 Subduction zones and their surroundings 308
13.2.1 Configuration of subduction zones 309
13.2.2 Flow around the slab 311
13.2.3 Temperatures in and around the subducting slab 314
13.2.4 Subduction and orogeny 316
13.3 The influence of phase transitions 319
13.4 The deeper mantle 323
13.4.1 Viscosity variations in the mantle and the geoid 323
13.4.2 The lower boundary layer 328

14 Mantle Convection 330
14.1 Convective forces 330
14.1.1 Boundary layer theory 330
14.1.2 Basic equations 332
14.1.3 Boundary conditions 333
14.1.4 Non-dimensional treatment 334
14.1.5 Computational convection 335
14.2 Convective planform 339
14.3 Thermal structure and heat budget 342
14.3.1 Thermal boundary layers and the geotherm 342
14.3.2 Plates 346
14.3.3 Hot spots and plumes 348
14.4 Circulation of the mantle 353
14.4.1 Present-day and past plate motion models 354
14.4.2 Implications of plate motion models for mantle circulation 357
14.4.3 Mantle circulation models 361
14.5 Mantle rheology 371
14.5.1 Temperature dependence 372
14.5.2 Strain dependence 373
14.6 Coupled lithosphere–mantle convection models 375
14.7 Thermochemical convection 377
15 The Core and the Earth's Dynamo 379
15.1 The magnetic field at the surface and at the top of the core 380
15.2 Convection and dynamo action 384
15.2.1 Basic equations 384
15.2.2 Boundary conditions 387
15.2.3 Interaction of the flow with the magnetic field 388
15.2.4 Deviations from the reference state 389
15.2.5 Non-dimensional treatment 390
15.3 Numerical dynamos 392
15.4 Evolution of the Earth's core 397
15.4.1 Energy balance 397
15.4.2 Thermal and compositional effects 399
15.4.3 Inner core growth in a well-mixed core 400
Appendix: Table of Notation 407
Bibliography 413
Index 423

Preface

Geophysical Continua is designed to present a systematic treatment of deformation in the Earth from seismic to geologic time scales. In this way we demonstrate the linkages between different aspects of the Earth's interior that are commonly treated separately. We provide a coherent presentation of non-linear continuum mechanics with a uniform notation, and then specialise to the needs of particular topics such as elastic, viscoelastic and fluid behaviour. We include the concepts of continuum thermodynamics and link to the properties of material under pressure in the deep interior of the Earth, and also provide the continuum electrodynamics needed for conducting fluids such as the Earth's core.

Following an introduction to continuum methods and the structure of the Earth, Part I of the book takes the development of continuum techniques to the level where they can be applied to the diverse aspects of Earth structure and dynamics in Part II. At many levels there is a close relation between microscopic properties and macroscopic consequences such as effective rheology, and so Part II opens with a discussion of the relation of phenomena at the atomic scale to continuum properties. We follow this with a treatment of geological deformation at the grain and outcrop scale. In the subsequent chapters we emphasise the physical principles that allow understanding of Earth processes, taking a global perspective towards lithospheric and mantle properties, seismology, mantle convection, the core and Earth's dynamo. We make links to experimental results and seismological observations to provide insight into geodynamic interpretations.

The material in the book has evolved over a considerable time period and has benefited from interactions with many students in Cambridge, Canberra, Princeton and Munich. Particular thanks go to the participants in the Geodynamics Seminar in Munich in 2005, which helped to refine Part I and the discussion of the lithosphere in Part II.

In a work of this complexity covering many topics with their own specific notation it is difficult to avoid reusing symbols. Nevertheless we have have tried to sustain a unified notation throughout the whole book and to minimise multiple use.

We have had stimulating discussions with Jason Morgan, John Suppe and Geoff Davies over a wide range of topics. Gerd Steinle-Neumann provided very helpful

input on mineral properties and *ab initio* calculations, and Stephen Cox provided valuable insight into the relation of continuum mechanics and structural geology.

Special thanks go to the Alexander von Humboldt Foundation for the Research Award to Brian Kennett that led to the collaboration on this volume.

Acknowledgements

We are grateful to the many people who have gone to trouble to provide us with figures, in particular: A. Barnhoorn, G. Batt, J. Besse, C. Bina, S. Cox, J. Dawson, E. Debayle, U. Faul, A. Fichtner, S. Fishwick, J. Fitz Gerald, E. Garnero, A. Gorbatov, B. Goleby, O. Heibach, M. Heintz, G. Houseman, R. Holme, G. Iaffeldano, M. Ishii, A. Jackson, I. Jackson, J. Jackson, M. Jessell, J. Kung, P. Lorinczi, S. Micklethwaite, M. Miller, D. Mueller, A. Piazzoni, K. Priestley, M. Sandiford, W. Spakman, B. Steinberger, J. Suppe, F. Takahashi, and K. Yoshizawa.