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Introduction

The development of quantitative methods for the study of the Earth rests firmly on the application of physical techniques to the properties of materials without recourse to the details of atomic level structure. This has formed the basis of seismological methods for investigating the internal structure of the Earth, and for modelling of mantle convection through fluid flow. The deformation behaviour of materials is inextricably tied to microscopic properties such as the elasticity of individual crystals and processes such as the movement of dislocations. In the continuum representation such microscopic behaviour is encapsulated in the description of the rheology of the material through the connection between stress and strain (or strain rate).

Different classes of behaviour are needed to describe the diverse aspects of the Earth both in depth and as a function of time. For example, in the context of the rapid passage of a seismic wave the lithosphere may behave elastically, but under the sustained load of a major ice sheet will deform and interact with the deeper parts of the Earth. When the ice sheet melts at the end of an ice age, the lithosphere recovers and the pattern of post-glacial uplift can be followed through raised beaches, as in Scandinavia.

The Earth's core is a fluid and its motions create the internal magnetic field of the Earth through a complex dynamo interaction between fluid flow and electromagnetic interactions. The changes in the magnetic field at the surface on time scales of a few tens of years are an indirect manifestation of the activity in the core. By contrast, the time scales for large-scale flow in the silicate mantle are literally geological, and have helped to frame the configuration of the planet as we know it.

We can link together the many different facets of Earth behaviour through the development of a common base of continuum mechanics before branching into the features needed to provide a detailed description of specific classes of behaviour. We start therefore by setting the scene for the continuum representation. We then review the structure of the Earth and the different types of mechanical behaviour that occur in different regions, and examine some of the ways in which information at the microscopic level is exploited to infer the properties of the Earth through both experimental and computational studies.

1.1 Continuum properties

A familiar example of the concept of a continuum comes from the behaviour of fluids, but we can use the same approach to describe solids, glasses and other more general substances that have short-term elastic and long-term fluid responses. The behaviour of such continua can then be established by using the conservation laws for linear and angular momentum and energy, coupled to explicit descriptions of the relationship between the stress, describing the force system within the material, and the strain, which summarises the deformation.

We adopt the viewpoint of continuum mechanics and thus ignore all the fine detail of atomic level structure and assume that, for sufficiently large samples,:

- the highly discontinuous structure of real materials can be replaced by a smoothed hypothetical continuum; and
- every portion of this continuum, however small, exhibits the macroscopic physical properties of the bulk material.

In any branch of continuum mechanics, the field variables (such as density, displacement, velocity) are conceptual constructs. They are taken to be defined at all points of the imagined continuum and their values are calculated via axiomatic rules of procedure.

The continuum model breaks down over distances comparable to interatomic spacing (in solids about 10^{-10} m). Nonetheless the *average* of a field variable over a small but *finite* region is meaningful. Such an average can, in principle, be compared directly to its nominal counterpart found by experiment – which will itself represent an average of a kind taken over a region containing many atoms, because of the physical size of any measuring probe.

For solids the continuum model is valid in this sense down to a scale of order 10^{-8} m, which is the side of a cube containing a million or so atoms.

Further, when field variables change slowly with position at a microscopic level $\sim 10^{-6}$ m, their averages over such volumes (10^{-20} m^3 say) differ insignificantly from their centroidal values. In this case pointwise values can be compared directly to observations.

Within the continuum we take the behaviour to be determined by
a) conservation of mass;
b) linear momentum balance: the rate of change of total linear momentum is equal to the sum of the external forces; and
c) angular momentum balance.
The continuum hypothesis enables us to apply these laws on a local as well as a global scale.

1.1.1 Deformation and strain

If we take a solid cube and subject it to some deformation, the most obvious change in external characteristics will be a modification of its shape. The specification of this deformation is thus a geometrical problem, that may be carried out from two different viewpoints:
a) with respect to the undeformed state (Lagrangian), or
b) with respect to the deformed state (Eulerian).
Locally, the mapping from the deformed to the undeformed state can be assumed to be linear and described by a differential relation, which is a combination of pure stretch (a rescaling of each coordinate) and a pure rotation.

The mechanical effects of the deformation are confined to the stretch and it is convenient to characterise this by a *strain* measure. For example, for a wire under load the strain ϵ would be the relative extension, i.e.,

$$\epsilon = \frac{\text{change in length}}{\text{initial length}}, \tag{1.1.1}$$

The generalisation of this idea requires us to introduce a *strain tensor* at each point of the continuum to allow for the three-dimensional nature of deformation.

1.1.2 The stress field

Within a deformed continuum a force system acts. If we were able to cut the continuum in the neighbourhood of a point, we would find a force acting on a cut surface which would depend on the inclination of the surface and is not necessarily perpendicular to the surface (Figure 1.1).

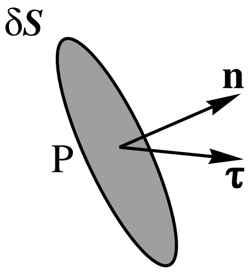

Figure 1.1. The force vector $\boldsymbol{\tau}$ acting on an internal surface specified by the vector normal $\mathbf{n}$ will normally not align with $\mathbf{n}$.

This force system can be described by introducing a *stress tensor* $\boldsymbol{\sigma}$ at each point, whose components describe the loading characteristics, and from which the force vector $\boldsymbol{\tau}$ can be found for a surface with arbitary normal $\mathbf{n}$.

For a loaded wire, the stress σ would just be the force per unit area.

1.1.3 Constitutive relations

The specification of the stress and strain states of a body is insufficient to describe its full behaviour, we need in addition to link these two fields. This is achieved by introducing a *constitutive relation*, which prescribes the response of the continuum to arbitrary loading and thus defines the connection between the stress and strain tensors for the particular material.

At best, a mathematical expression provides an approximation to the actual behaviour of the material. But, as we shall see, we can simulate the behaviour of a wide class of media by using different mathematical forms.

We shall assume that the forces acting at a point depend on the *local* geometry of deformation and its history, and possibly also on the history of the local temperature. This concept is termed the *principle of local action*, and is designed to exclude 'action at a distance' for stress and strain.

Solids

Solids are a familiar part of the Earth through the behaviour of the outer layers, which exhibit a range of behaviours depending on time scale and loading.

We can illustrate the range of behaviour with the simple case of extension of a wire under loading. The tensile stress σ and tensile strain ϵ are then typically related as shown in Figure 1.2.

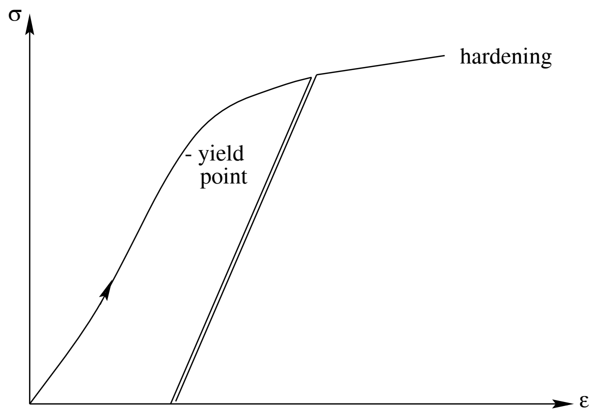

Figure 1.2. Behaviour of a wire under load

Elasticity

If the wire returns to its original configuration when the load is removed, the behaviour is said to be elastic:
(i) linear elasticity $\sigma = E\epsilon$ – usually valid for small strains;
(ii) non-linear elasticity $\sigma = f(\epsilon)$ – important for rubber-like materials, but not significant for the Earth.

Plasticity

Once the yield point is exceeded, permanent deformation occurs and there is no unique stress–strain curve, but a unique $d\sigma - d\epsilon$ relation. As a result of microscopic processes the yield stress rises with increasing strain, a phenomenon known as work hardening. Plastic flow is important for the movement of ice, e.g., in glacier flow.

Viscoelasticity (rate-dependent behaviour)

Materials may creep and show slow long-term deformation, e.g., plastics and metals at elevated temperatures. Such behaviour also seems to be appropriate to the Earth, e.g., the slow uplift of Fennoscandia in response to the removal of the loading of the glacial ice sheets.

Elementary models of viscoelastic behaviour can be built up from two basic building blocks: the *elastic spring* for which

$$\sigma = m\,\epsilon, \tag{1.1.2}$$

and the *viscous dashpot* for which

$$\sigma = \eta\,\dot{\epsilon}. \tag{1.1.3}$$

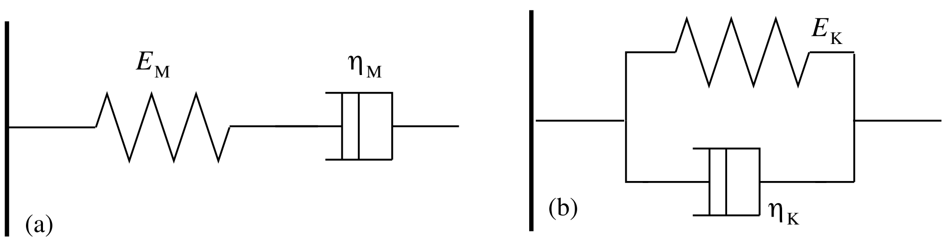

Figure 1.3. Mechanical models for linear viscoelastic behaviour combining a spring and viscous dashpot: (a) Maxwell model, (b) Kelvin–Voigt model.

The stress-strain relations depend on how these elements are combined.

(i) Maxwell model
The spring and dashpot are placed in series (Figure 1.3a) so that

$$\dot{\sigma} \;=\; E_M(\dot{\epsilon} + \epsilon/\tau_M); \tag{1.1.4}$$

this allows for instantaneous elasticity and represents a crude description of a fluid. The constitutive relation can be integrated using, e.g., Laplace transform methods and we find

$$\sigma(t) \;=\; E_M\left(\epsilon(t) + \int^t dt'\,\epsilon(t')\exp[-(t-t')/\tau_M]\right), \tag{1.1.5}$$

so the stress state depends on the history of strain.

(ii) Kelvin–Voigt model
The spring and dashpot are placed in parallel (Figure 1.3b) and so

$$\sigma \;=\; E_K(\dot{\epsilon} + \epsilon/\tau_K), \tag{1.1.6}$$

which displays long-term elasticity. For the initial condition $\epsilon = 0$ at $t = 0$ and constant stress σ_0, the evolution of strain in the Kelvin–Voigt model is

$$\epsilon = \frac{\sigma_0}{2E_K}\left[1 - \exp\left(-\frac{t}{\tau_K}\right)\right], \tag{1.1.7}$$

so that the viscous damping is not relevant on long time scales.

More complex models can be generated, but all have the same characteristic that the stress depends on the time history of deformation.

Fluids

The simplest constitutive equation encountered in continuum mechanics is that for an ideal fluid, where the pressure field p is isotropic and depends on density and temperature

$$\sigma = -p(\rho, T), \tag{1.1.8}$$

where ρ is the density, and T is the absolute temperature. If the fluid is incompressible ρ is a constant.

The next level of complication is to allow the pressure to depend on the flow of the fluid. The simplest such form includes a linear dependence on strain rate $\dot{\epsilon}$ – a Newtonian viscous fluid:

$$\sigma = -p(\rho, T) + \eta\dot{\epsilon}. \tag{1.1.9}$$

Further complexity can be introduced by allowing a non-linear dependence of stress on strain rate, as may be required for the flow of glacier ice.

1.2 Earth processes

The Earth displays a broad spectrum of continuum properties varying with both depth and time. A dominant influence is the effect of pressure with increasing depth, so that properties of materials change as phase transitions in minerals accommodate closer packed structures. Along with the pressure the temperature increases, so we need to deal with the properties of materials at conditions that are not simple to reproduce under laboratory conditions.

The nature of the deformation processes within the Earth depends strongly on the frequency of excitation. At high frequencies appropriate to the passage of seismic waves the dominant contribution is elastic, with some seismic attenuation that can be represented with a small linear viscoelastic component. However, as the frequency decreases and the period lengthens viscous flow effects become more prominent, so that elastic contributions can be ignored in the study of mantle convection. This transition in behaviour is illustrated in Figure 1.4, and is indicative of a very complex rheology for the interior of the Earth as different facets of material behaviour become important in different frequency bands. The observed behaviour reflects competing influences at the microscopic level, and

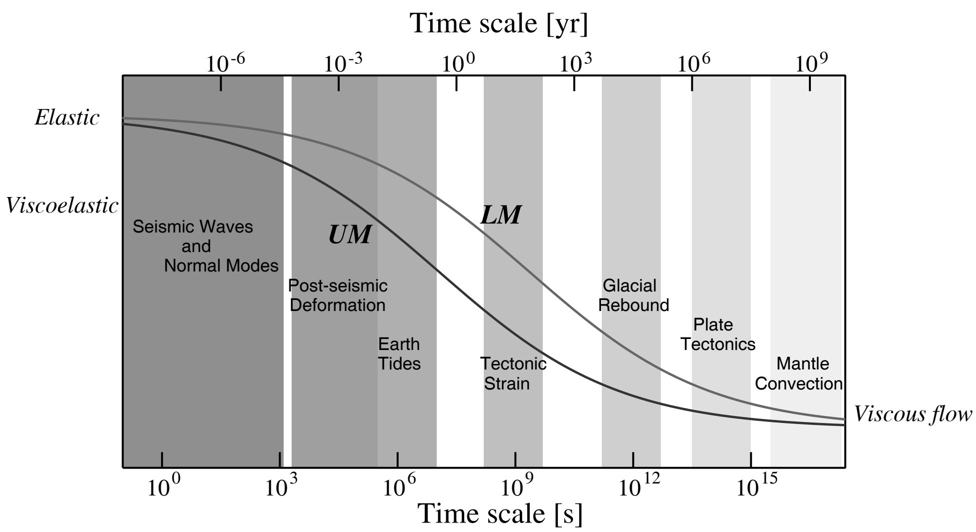

Figure 1.4. Spectrum of Earth deformation processes indicating the transition from viscoelastic to fully viscous behaviour as the frequency decreases. The upper curve refers to the lower mantle (*LM*), and the lower curve to the upper mantle (*UM*), indicating the differences in viscosity and deformation history.

varies significantly with depth as indicated by the two indicative curves (*UM*, *LM*) in Figure 1.4 for the transition from near elastic behaviour to fully viscous flow behaviour, representing the states for the upper and lower mantle.

Further, the various classes of deformation occur over a very wide range of spatial scales (Figure 1.5). As a result, a variety of different techniques is needed to examine the behaviour from seismological to geodetic through to geological observations. There is increasing overlap in seismic and space geodetic methods for studying the processes associated with earthquake sources that has led to new insights for fault behaviour. Some phenomena, such as the continuing recovery of the Earth from glacial loading, can be studied using multiple techniques that provide direct constraints on rheological properties.

Our aim is to integrate understanding of continuum properties and processes with the nature of the Earth itself, and to show how the broad range of terrestial phenomena can be understood within a common framework. We therefore now turn our attention to the structure of the Earth and the classes of geodynamic and deformation processes that shape the planet we live on.

In Part I that follows, we embark on a more detailed examination of the development of continuum methods, in a uniform treatment encompassing solid, fluid and intermediate behaviour. Then in Part II we address specific Earth issues building on the continuum framework

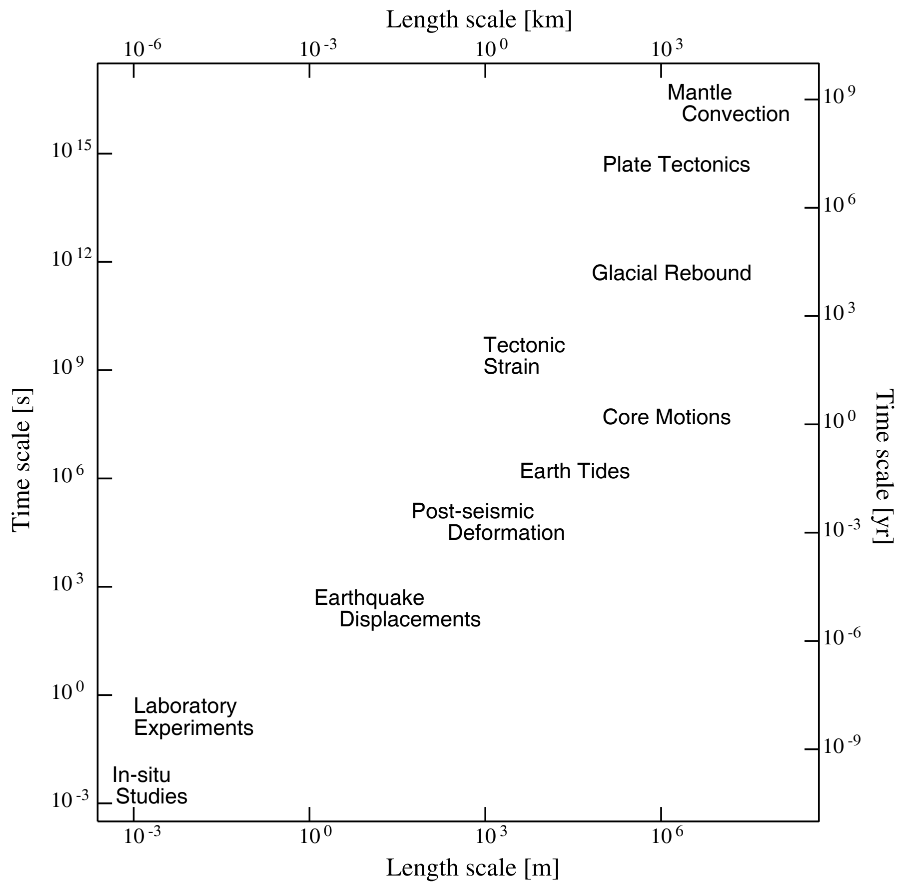

Figure 1.5. Temporal and spatial spectrum of Earth deformation processes.

1.3 Elements of Earth structure

Much of our knowledge of the interior of the Earth comes from the analysis of seismological data, notably the times of passage of seismic body waves at high frequencies ($\approx$ 1 Hz) and the behaviour of the free oscillations of the Earth at lower frequencies (0.03 – 3 mHz). Such studies provide information both on the dominant radial variations in physical properties, and on the three-dimensional variations in the solid parts of the Earth. Important additional constraints are provided by the mass and moments of inertia of the Earth, which can be deduced from satellite observations. The moments of inertia are too low for the Earth to have uniform density, there has to be a concentration of mass towards the centre that can be identified with the seismologically defined *core*.

The resulting picture of the dominant structure of the Earth is presented in Figure 1.6. The figure of the Earth is close to an oblate spheroid with a flattening of 0.003356. The radius to the pole is 6357 km and the equatorial radius is 6378 km,

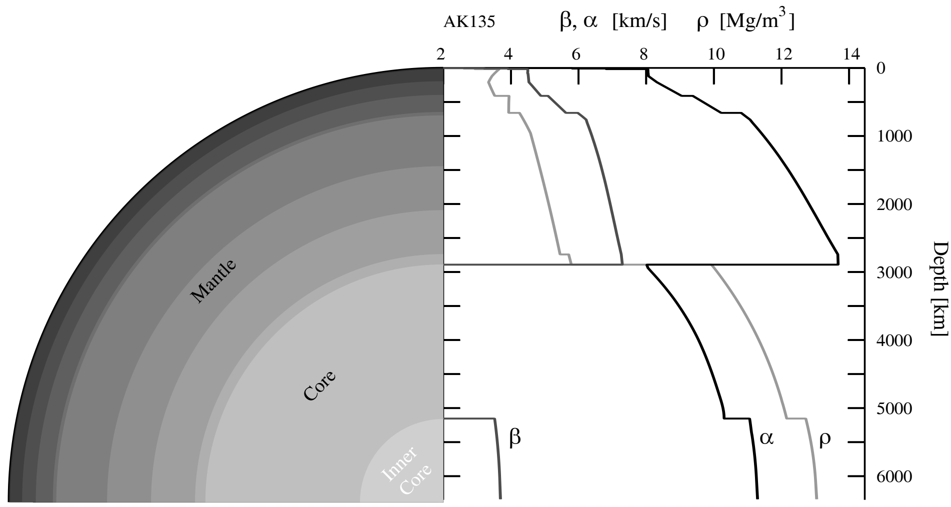

Figure 1.6. The major divisions of the radial structure of the Earth linked to the radial reference Earth model AK135, seismic wave speeds α (P), β (S): Kennett et al. (1995); density ρ: Montagner & Kennett (1996). The gradations in tone in the Earth's mantle indicate the presence of discontinuities at 410 and 660 km depth, and the presence of the D″ near the core–mantle boundary.

but for most purposes a spherical model of the Earth with a mean radius of 6371 km is adequate. Thus reference models for internal structure in which the physical properties depend on radius can be used. Three-dimensional variations can then be described by deviations from a suitable reference model.

Beneath the thin crustal shell lies the silicate mantle which extends to a depth of 2890 km. The mantle is separated from the metallic core by a major change of material properties that has a profound effect on global seismic wave propagation. The outer core behaves as a fluid at seismic frequencies and does not allow the passage of shear waves, while the inner core appears to be solid.

The existence of a discontinuity at the base of the crust was found by Mohorovičić in the analysis of the Kupatal earthquake of 1909 from only a limited number of records from permanent seismic stations. Knowledge of crustal structure from seismic methods has developed substantially in past decades through the use of controlled sources, e.g., explosions. Indeed most of the information on the oceanic crust comes from such work. The continental crust varies in thickness from around 20 km in rift zones to 70 km under the Tibetan Plateau. Typical values are close to 35 km. The oceanic crust is thinner, with a basalt pile about 7 km thick whose structure changes somewhat with the age of the oceanic crust.

Earthquakes and man-made sources generate two types of seismic waves that propagate through the Earth. The earliest arriving (P) wave has longitudinal motion; the second (S) wave has particle motion perpendicular to the path. In the Earth the direct P and S waves are accompanied by multiple reflections and conversions, particulary from the free surface. These additional seismic phases follow the main arrivals, so that seismograms have a quite complex character with many distinct arrivals. Behind the *S* wave a large-amplitude train of waves builds up from surface waves trapped between the Earth's surface and the increase in seismic wavespeed with depth. These surface waves have dominantly *S* character and are most prominent for shallow earthquakes. The variation in the properties of surface waves with frequency provides valuable constraints on the structure of the outer parts of the Earth.

The times of arrival of seismic phases on their different paths through the globe constrain the variations in *P* and *S* wavespeed, and can be used to produce models of the variation with radius. A very large volume of arrival time data from stations around the world has been accumulated by the International Seismological Centre and is available in digital form. This data set has been used to develop high-quality travel-time tables, that can in turn be used to improve the locations of events. With reprocessing of the arrival times to improve locations and the identification of the picks for later seismic phases, a set of observations of the relation between travel time and epicentral distance have been produced for a wide range of phases. The reference model AK135 of Kennett et al. (1995) for both *P* and *S* wave speeds, illustrated in figure 1.6, gives a good fit to the travel times of mantle and core phases. The reprocessed data set and the AK135 reference model have formed the basis of much recent work on high-resolution travel-time tomography to determine three-dimensional variations in seismic wavespeed.

The need for a core at depth with greatly reduced seismic wave speeds was recognised at the end of the nineteenth century by Oldham in his analysis of the great Assam earthquake of 1890, because of a zone without distinct *P* arrivals (a 'shadow zone' in *PKP*). By 1914 Gutenburg had obtained an estimate for the radius of the core which is quite close to the current value. The presence of the inner core was inferred by Inge Lehmann in 1932 from careful analysis of arrivals within the shadow zone (*PKiKP*), which had to be reflected from some substructure within the core.

The mantle shows considerable variation in seismic properties with depth, with strong gradients in seismic wavespeed in the top 800 km. The presence of distinct structure in the upper mantle was recognised by Jeffreys in the 1930's from the change in the slope of the travel time as a function of distance from events near 20°. Detailed analysis at seismic arrays in the late 1960s provided evidence for significant discontinuities in the upper mantle. Subsequent studies have demonstrated the global presence of discontinuities near 410 and 660 km depth, but also significant variations in seismic structure within the upper mantle (for a review see Nolet et al., 1994).

The use of the times of arrival of seismic phases enables the construction of models for *P* and *S* wavespeed, but more information is needed to provide a full model for Earth structure. The density distribution in the Earth has to be