Introduction

This book is written with the graduate student in mind. I had in mind to write a text that would introduce my students to the basic ideas and concepts behind many-body physics. At the same time, I felt very strongly that I would like to share my excitement about this field, for without feeling the thrill of entering uncharted territory I do not think one has the motivation to learn and to make the passage from learning to research.

Traditionally, as physicists we ask "what are the microscopic laws of nature?", often proceeding with the brash certainty that, once revealed, these laws will have such profound beauty and symmetry that the properties of the universe at large will be self-evident. This basic philosophy can be traced from the earliest atomistic philosophy of Democritus to the most modern quests to unify quantum mechanics and gravity.

The dreams and aspirations of many-body physics interwine the reductionist approach with a complementary philosophy: that of *emergent phenomena*. In this view, fundamentally new kinds of phenomena emerge within complex assemblies of particles which cannot be anticipated from an *a priori* knowledge of the microscopic laws of nature. Many-body physics aspires to synthesize, from the microscopic laws, new principles that govern the macroscopic realm, asking:

> *What emergent principles and laws develop as we make the journey from the microscopic to the macroscopic?*

This is a comparatively modern and far less familiar scientific philosophy. Charles Darwin was perhaps the first to seek an understanding of emergent laws of nature. Following in his footsteps, Ludwig Boltzmann and James Clerk Maxwell were among the first physicists to appreciate the need to understand how emergent principles are linked to microscopic physics. From Boltzmann's biography [1], we learn that he was strongly influenced and inspired by Charles Darwin. In more modern times, a strong advocate of this philosophy has been Philip W. Anderson, who first introduced the phrase "emergent phenomenon" into physics. In an influential article entitled "More is different," written in 1967 [2], he captured the philosophy of emergence, writing:

> The behavior of large and complex aggregations of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other.

In an ideal world, I would hope that from this short course your knowledge of many-body techniques will grow hand-in-hand with an appreciation of the motivating philosophy.

In many ways, this dual track is essential, for often one needs both inspiration and overview to steer one lightly through the formalism, without getting bogged down in mathematical quagmires.

I have tried in the course of the book to mention aspects of the history of the field. We often forget that the act of discovering the laws of nature is a very human and very passionate one. Indeed, the act of creativity in physics research is very similar to the artistic process. Sometimes, scientific and artistic revolutions even go hand-in-hand, for the desire for change and revolution often crosses between art and the sciences [3]. I think it is important for students to gain a feeling of this passion behind the science, and for this reason I have often included a few words about the people and the history behind the ideas that appear in this text. There are, unfortunately, very few texts that tell the history of many-body physics. Abraham Pais' book *Inward Bound* [4] has some important chapters on the early stages of many-body physics. A few additional references are included at the end of this chapter [5–7].

There are several texts that can be used as reference books in parallel with this book, of which a few deserve special mention. The student reading this book will need to consult standard references on condensed matter and statistical mechanics. Among these let me recommend *Statistical Physics, Part 2* by Lifshitz and Pitaevksii [8]. For a conceptual underpining of the field, I recommend Anderson's classic *Basic Notions of Condensed Matter Physics* [9]. For an up-to-date perspective on solid state physics from a many-body physics perspective, may I refer you to *Advanced Solid State Physics* by Philip Phillips [10]. Among the classic references to many-body physics let me also mention *Methods of Quantum Field Theory* by Abrikosov, Gor'kov and Dzyaloshinskii ("AGD") [11]. This is the text that drove the quantum many-body revolution of the 1960s and 1970s, yet it is still very relevant today, if rather terse. Other many-body texts which introduce the reader to the Green's function approach to many-body physics include *Many-Particle Physics* by G. Mahan [12], notable for the large number of problems he provides, *Green's Functions for Solid State Physicists* by Doniach and Sondheimer [13] and a very light introduction to the subject, *A Guide to Feynman Diagrams in the Many-Body Problem* by Richard Mattuck [14]. Among the more recent treatments, let me note Alexei Tsvelik's *Quantum Field Theory in Condensed Matter Physics* [15], which provides a wonderful introduction to bosonization and conformal field theory, and *Condensed Matter Field Theory* by Alexander Altland and Ben Simons [17], a perfect companion volume to my own work. As a reference to the early developments of many-body physics, I recommend *The Many-Body Problem* by David Pines [16], which contains a compilation of the classic early papers in the field. Lastly, let me direct the reader to numerous excellent online reference sources; in addition to the online physics archive http://arXiv.org, let me mention the lecture notes on solid state physics and many-body theory by Chetan Nayak [18].

Here is a brief summary of the book:

- Scales and complexity: the gulf of time (T), length scale (L), particle number (N), and complexity that separates the microscopic from the macroscopic (Chapter 1)
- Second quantization: the passage from the wavefunction to the field operator, and an introduction to the excitation concept (Chapters 2–4)

- The Green's function: the fundamental correlator of quantum fields (Chapter 5)
- Landau Fermi-liquid theory: an introduction to Landau's phenomenological theory of interacting fermions and his concept of the quasiparticle (Chapter 6)
- Feynman diagrams: an essential tool for visualizing and calculating many-body processes (Chapter 7)
- Finite temperature and imaginary time: by replacing $it \longrightarrow \tau$, $e^{-iHt} \longrightarrow e^{-H\tau}$, quantum field theory is extended to finite temperature, where we find an intimate link between fluctuations and dissipation (Chapters 8–9)
- Electron transport theory: using many-body physics to calculate the resistance of a metal; second-quantized treatment of weakly disordered metals: the Drude metal and the derivation of Ohm's law (Chapter 10)
- The concepts of broken symmetry and generalized rigidity (Chapter 11)
- Path integrals: using the coherent state to link the partition function and the S-matrix by an integral over all possible time-evolved paths of the many-body system ($Z = \int_{PATH} e^{-S/\hbar}$); using path integrals to study itinerant magnetism (Chapters 12–13)
- Superconductivity and BCS theory, including anisotropic pairing (Chapters 14–15)
- Introduction to the physics of local moments and the Kondo effect (Chapter 16)
- Introduction to the physics of heavy electrons and mixed valence using the large-N and slave boson approaches (Chapters 17–18).

Finally, some notes on conventions. This book uses standard SI notation, which means abandoning some of the notational elegance of cgs units, but brings the book into line with international standards.

Following early Russian texts on physics and many-body physics, and Mahan's *Many Particle Physics* [12], I use the convention that the charge on the electron is

$$e = -1.602\ldots \times 10^{-19}\,\text{C}.$$

In other words, $e = -|e|$ denotes the magnitude *and* the sign of the electron charge. This convention minimizes the number of minus signs required. The magnitude of the electron charge is denoted by $|e|$ in formulae such as the electron cyclotron frequency $\omega_c = \frac{|e|B}{m}$. With this notation, the Hamiltonian of an electron in a magnetic field is given by

$$H = \frac{(\mathbf{p} - e\mathbf{A})^2}{2m} + e\phi,$$

where $\mathbf{A}$ is the vector potential and ϕ the electric potential. We choose the notation ϕ for electric potential in order to avoid confusion with the frequent use of V for potential and also for hybridization.

Following a tradition started in the Landau and Lifshitz series, this book uses the notation

$$F = E - TS - \mu N$$

for the Landau free energy – the grand canonical version of the traditional Helmholtz free energy ($E - TS$), which for simplicity will be referred to as the free energy.

One of the more difficult choices in the book concerns the notation for the density of states of a Fermi gas. To deal with the different conventions used in Fermi-liquid theory, in superconductivity and in local-moment physics, I have adopted the notation

$$\mathcal{N}(0) \equiv 2N(0)$$

to denote the total density of states at the Fermi energy, where $N(0)$ is the density of states per spin. The alternative notation $N(0) \equiv \rho$ is used in Chapters 16, 17, and 18, in keeping with traditional notation for the Kondo effect.

References

[1] E. Broda and L. Gray, *Ludwig Boltzmann: Man, Physicist, Philosopher*, Woodbridge, 1983.
[2] P. W. Anderson, More is different, *Science*, vol. 177, p. 393, 1972.
[3] R. March, *Physics for Poets*, McGraw-Hill, 1992.
[4] A. Pais, *Inward Bound: Of Matter and Forces in the Physical World*, Oxford University Press, 1986.
[5] L. Hoddeson, G. Baym, and M. Eckert, The development of the quantum mechanical electron theory of metals: 1928–1933, *Rev. Mod. Phys.*, vol. 59, p. 287, 1987.
[6] M. Riordan and L. Hoddeson, *Crystal Fire*, W.W. Norton, 1997.
[7] L. Hoddeson and V. Daitch, *True Genius: The Life and Science of John Bardeen*, National Academy Press, 2002.
[8] E.M.L. Lifshitz and L. P. Pitaevksii, *Statistical Mechanics, Part 2: Theory of the Condensed State*, Landau and Lifshitz Course on Theoretical Physics, vol. 9, trans. J.B. Sykes and M.J. Kearsley, Pergamon Press, 1980.
[9] P. W. Anderson, *Basic Notions of Condensed Matter Physics*, Benjamin Cummings, 1984.
[10] P. Phillips, *Advanced Solid State Physics*, Cambridge University Press, 2nd edn., 2012.
[11] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics*, Dover, 1977.
[12] G. D. Mahan, *Many-Particle Physics*, Plenum, 3rd edn., 2000.
[13] S. Doniach and E. H. Sondheimer, *Green's Functions for Solid State Physicists*, Imperial College Press, 1998.
[14] R. Mattuck, *A Guide to Feynman Diagrams in the Many-Body Problem*, Dover, 2nd edn., 1992.
[15] A. Tsvelik, *Quantum Field Theory in Condensed Matter Physics*, Cambridge University Press, 2nd edn., 2003.
[16] D. Pines, *The Many-Body Problem*, Wiley Advanced Book Classics, 1997.
[17] A. Altland and B. Simons, *Condensed Matter Field Theory*, Cambridge University Press, 2006.
[18] C. Nayak, *Quantum Condensed Matter Physics*, http://stationq.cnsi.ucsb.edu/nayak/courses.html, 2004.

Cambridge University Press
978-0-521-86488-6 - Introduction to Many-Body Physics
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1 Scales and complexity

We do in fact know the microscopic physics that governs all metals, chemistry, materials and possibly life itself. In principle, all can be determined from the many-particle wavefunction

$$\Psi(\vec{x}_1, \vec{x}_2, \ldots \vec{x}_N, t), \tag{1.1}$$

which in turn is governed by the Schrödinger equation [1, 2], written out for identical particles as

$$\left\{ -\frac{\hbar^2}{2m} \sum_{j=1}^{N} \nabla_j^2 + \sum_{i<j} V(\vec{x}_i - \vec{x}_j) + \sum_j U(\vec{x}_j) \right\} \Psi = i\hbar \frac{\partial \Psi}{\partial t}. \tag{1.2}$$

There are of course a few details that we have omitted. For instance, if we're dealing with electrons, then

$$V(\vec{x}) = \frac{e^2}{4\pi\epsilon_0} \frac{1}{|\vec{x}|} \tag{1.3}$$

is the Coulomb interaction potential, where $e = -|e|$ is the charge on the electron. In an electromagnetic field we must also *gauge* the derivatives $\nabla \to \nabla - i(e/\hbar)\mathbf{A}$ and $U(\vec{x}) \to U(\vec{x}) + e\phi(\vec{x})$, where $\vec{A}$ is the vector potential and $\phi(\vec{x})$ is the electric potential. Furthermore, to be complete we must discuss spin and the antisymmetry of Ψ under particle exchange; moreoever, if we are to treat the vibrations of the lattice, we must include the nuclear locations in the wavefunction. But with these various provisos, we have every reason to believe that the many-body Schrödinger equation contains the essential microscopic physics that governs the macrosopic behavior of materials.

Unfortunately this knowledge is only the beginning. Why? Because, at the most pragmatic level, we are defeated by the sheer complexity of the problem. Even the task of solving the Schrödinger equation for modest multi-electron atoms proves insurmountable without bold approximations. The problem facing the condensed matter physicist, with systems involving 10^{23} atoms, is qualitatively more severe. The amount of storage required for numerical solution of Schrödinger equation grows exponentially with the number of particles, so with a macroscopic number of interacting particles this becomes far more than a technical problem: it becomes one of *principle*. Indeed, we believe that the gulf between the microscopic and the macroscopic is something qualitative and fundamental, so much so that new properties emerge in macroscopic systems that we cannot anticipate a priori by using brute-force analysis of the Schrödinger equation.

The *Hitchhiker's Guide to the Galaxy* [3] describes a super-computer called Deep Thought that, after millions of years spent calculating "the answer to the ultimate question of life and the universe", reveals it to be "42." Douglas Adams' cruel parody of reductionism holds a certain sway in physics today. Our "42," is Schrödinger's many-body equation: a set of relations whose complexity grows so rapidly that we can't trace its full consequences to macroscopic scales. All is fine, provided we wish to understand the workings of isolated atoms or molecules up to sizes of about a nanometer, but, between the nanometer and the micron, wonderful things start to occur that severely challenge our understanding. Physicists have coined the term "emergence" from evolutionary biology to describe these phenomena [4–7].

The pressure of a gas is an example of emergence: it's a cooperative property of large numbers of particles which cannot be anticipated from the behavior of one particle alone. Although Newton's laws of motion account for the pressure in a gas, 180 years elapsed before James Clerk Maxwell developed the statistical description of atoms needed to understand pressure.

Let us dwell a little more on this gulf of complexity that separates the microscopic from the macroscopic. We can try to describe this gulf using four main catagories of scale:

- T: time
- L: length
- N: number of particles
- $\mathcal{C}$: complexity.

1.1 *T*: Time scale

We can make an estimate of the characteristic quantum time scale by using the uncertainty principle, $\Delta\tau\Delta E \sim \hbar$, so that

$$\Delta\tau \sim \frac{\hbar}{[1\,\text{eV}]} \sim \frac{\hbar}{10^{-19}\,\text{J}} \sim 10^{-15}\text{s}. \tag{1.4}$$

Although we know the physics on this time scale, in our macroscopic world the characteristic time scale $\sim$ 1s, so that

$$\frac{\Delta\tau_{\text{macroscopic}}}{\Delta\tau_{\text{quantum}}} \sim 10^{15}. \tag{1.5}$$

To link quantum and macroscopic time scales, we must make a leap comparable with an extrapolation from the time scale of a heartbeat to the age of the universe (10 billion years $\sim 10^{17}$s).

1.2 *L*: length scale

An approximate measure for the characteristic length scale in the quantum world is the de Broglie wavelength of an electron in a hydrogen atom,

$$L_{\text{quantum}} \sim 10^{-10}\text{m}, \tag{1.6}$$

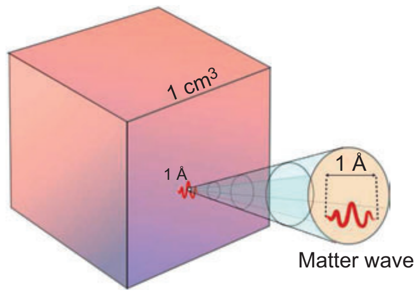

Fig. 1.1 The typical size of a de Broglie wave is 10^{-10} m, to be compared with a typical scale of 1 cm for a macroscopic crystal.

so

$$\frac{L_{\text{macroscopic}}}{L_{\text{quantum}}} \sim 10^8 \tag{1.7}$$

(see Figure 1.1). At the beginning of the twentieth century, a leading philosopher-physicist of the era, Ernst Mach, argued to Boltzmann that the atomic hypothesis was metaphysical, for, he argued, one could simply not envisage any device with the resolution to detect or image something so small. Today, this incredible gulf of scale is routinely spanned in the lab with scanning tunneling microscopes, able to view atoms at sub-angstrom resolution.

1.3 *N*: particle number

To visualize the number of particles in a single mole of a substance, it is worth reflecting that a crystal containing a mole of atoms occupies a volume of roughly 1 cm^3. From the quantum perspective, this is a cube with approximately 100 million atoms along each edge. Avagadro's number,

$$N_{\text{macroscopic}} = 6 \times 10^{23} \sim (100 \text{ million})^3, \tag{1.8}$$

is placed in perspective by reflecting that the number of atoms in a grain of sand is roughly comparable with the number of sand-grains in a 1-mile-long beach. But in quantum matter, the sand-grains, which are electrons, quantum mechanically interfere with one another, producing a state that is much more than the simple sum of its constituents.

1.4 *C*: complexity and emergence

Real materials are like "macroscopic atoms," where the quantum interference among the constituent particles gives rise to a range of complexity and diversity that constitutes the largest gulf of all. We can attempt to quantify the "complexity" axis by considering the

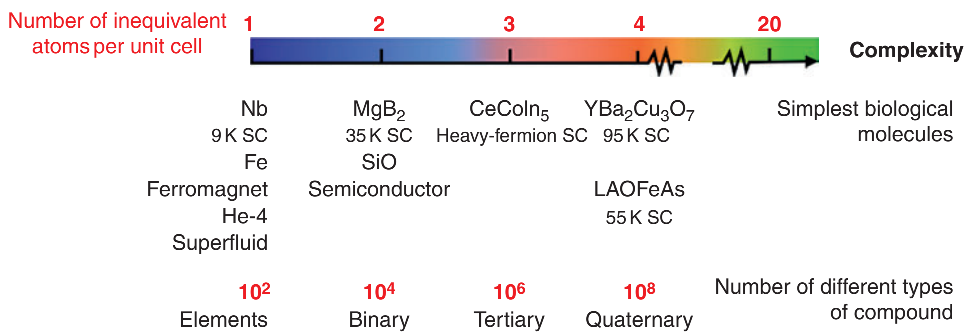

Fig. 1.2 Condensed matter of increasing complexity. As the number of inequivalent atoms per unit cell grows, the complexity of the material and the potential for new types of behavior grows. In the labels, "*X* K SC" denotes a superconductor with a transition temperature $T_c = X$; thus MgB_2 has a 35 K transition temperature.

number of atoms per unit cell of a crystal. Whereas there are roughly 100 stable elements, there are roughly 100^2 stable binary compounds. The number of stable tertiary compounds is conservatively estimated at more than 10^6, of which still only a tiny fraction have been explored experimentally. At each step the range of diversity increases (see Figure 1.2), and there is reason to believe that, at each level of complexity, new types of phenomena begin to emerge.

But it is really the confluence of the length and time scales, particle number and complexity that provides the canvas for "more is different." While classical matter develops new forms of behavior on large scales, the potential for quantum matter to develop emergent properties is far more startling. Let us take an example given by Anderson [8]. Consider similar atoms of niobium and gold: at the angstrom level, there is really nothing to distinguish them, yet as crystals on the macroscale one is a lustrous metal, the other a superconductor. Up to about 30 nm, there is little to distinguish copper and niobium, yet at longer length scales everything changes. Electrons can roam freely across gold crystals, forming the conducting fluid that gives it its lustrous metallic properties. But in niobium, beyond 30 nm the electrons pair up into "Cooper pairs". By the time we reach the micron scale, these pairs congregate by the billions into a pair condensate, transforming the crystal into an entirely new metallic state: a type II superconductor,[1] which conducts without resistance, excludes magnetic fields and has the ability to levitate magnets.

Niobium is an elemental superconductor, with a transition temperature $T_c = 9.2\,\mathrm{K}$ that is pretty typical of conventional low-temperature superconductors. When experimenters began to explore the properties of quaternary compounds in the 1980s, they came across the completely unexpected phenomenon of high-temperature superconductivity. Even today, three decades later, research has only begun to explore the vast universe of quaternary compounds, and the pace of discovery has not slackened. In the two years preceeding publication of this book, superconductivity has been tentatively observed at 108 K in a single-layer FeSe superconductor, and at 190 K in H_2S at high pressure. I'd like to think

[1] Niobium, together with vanadium and technitium, is one of only three elements to exhibit type II superconductivity, which allows magnetic fields to penetrate, forming a vortex lattice. See Section 11.5.4.

that before this book goes out of print, many more families of superconductor will have been discovered.

Superconductivity is only a beginning. First of all, it is only one of a large number of broken-symmetry states that can develop in "hard" quantum matter. But in assemblies of softer organic molecules, a tenth of a micron is already enough for the emergence of life. Self-sustaining microbes little more than 200 nm in size have recently been discovered. While we understand, more or less, the principles that govern the superconductor, we do not yet understand those that govern the emergence of life on roughly the same spatial scale [9].

References

[1] E. Schrödinger, Quantisierung als Eigenwertproblem I (Quantization as an eigenvalue problem), *Ann. Phys.*, vol. 79, p. 361, 1926.
[2] E. Schrödinger, Quantisierung als Eigenwertproblem IV (Quantization as an eigenvalue problem), *Ann. Phys.*, vol. 81, p. 109, 1926.
[3] D. Adams, *The Hitchhiker's Guide to the Galaxy*, Pan Macmillan, 1979.
[4] P. W. Anderson, More is different, *Science*, vol. 177, p. 393, 1972.
[5] R. B. Laughlin, D. Pines, J. Schmalian, B. P. Stojkovic, and P. Wolynes, The middle way, *Proc. Natl. Acad. Sci. U.S.A.*, vol. 97, 2000.
[6] R. B. Laughlin, *A Different Universe*, Basic Books, 2005.
[7] P. Coleman, The frontier at your fingertips, *Nature*, vol. 446, 2007.
[8] P. W. Anderson, *Basic Notions of Condensed Matter Physics*, Benjamin Cummings, 1984.
[9] J. C. Séamus Davis, *Music of the Quantum*, http://musicofthequantum.rutgers.edu, 2005.

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978-0-521-86488-6 - Introduction to Many-Body Physics
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2 Quantum fields

2.1 Overview

At the heart of quantum many-body theory lies the concept of the quantum field. Like a classical field $\phi(x)$, a quantum field is a continuous function of position, except that now this variable is an operator $\hat{\phi}(x)$. Like all other quantum variables, the quantum field is in general a strongly fluctuating degree of freedom that only becomes sharp in certain special eigenstates; its function is to add or subtract particles to the system. The appearance of particles or *quanta* of energy $E = \hbar\omega$ is perhaps the greatest single distinction between quantum and classical fields.

This astonishing feature of quantum fields was first recognized by Albert Einstein, who in 1905 and 1907 made the proposal that the fundamental excitations of continuous media – the electromagnetic field and crystalline matter in particular – are carried by quanta [1–4], with energy

$$E = \hbar\omega.$$

Einstein made this bold leap in two stages, first showing that Planck's theory of black-body radiation could be reinterpreted in terms of photons [1, 2], and one year later generalizing the idea to the vibrations inside matter [3] which, he reasoned, must also be made up of tiny wavepackets of sound that we now call *phonons*. From his phonon hypothesis Einstein was able to explain the strong temperature dependence of the specific heat in diamond – a complete mystery from a classical standpoint. Yet despite these early successes, it took a further two decades before the machinery of quantum mechanics gave Einstein's ideas a concrete mathematical formulation.

Quantum fields are intimately related to the idea of second quantization. First quantization permits us to make the jump from the classical world to the simplest quantum systems. The classical momentum and position variables are replaced by operators such as

$$\begin{aligned} E &\rightarrow i\hbar\partial_t \\ p &\rightarrow \hat{p} = -i\hbar\partial_x, \end{aligned} \tag{2.1}$$

while the Poisson bracket which relates canonical conjugate variables is now replaced by the quantum commutator [5, 6]:

$$[x, p] = i\hbar. \tag{2.2}$$

The commutator is the key to first quantization, and it is the non-commuting property that leads to quantum fluctuations and the Heisenberg uncertainty principle. (See Examples