QUANTUM FIELD THEORY

Quantum field theory is the basic mathematical framework that is used to describe elementary particles. It is a cornerstone of modern physics.

This textbook provides a complete and essential introduction to this subject. Assuming only an undergraduate knowledge of quantum mechanics and special relativity, it is ideal for graduate students beginning the study of elementary particles, and will also be of value to those in related fields such as condensed-matter physics.

The step-by-step presentation begins with basic concepts illustrated by simple examples, and proceeds through historically important results to thorough treatments of modern topics such as the renormalization group, spinor-helicity methods for quark and gluon scattering, magnetic monopoles, instantons, supersymmetry, and the unification of forces.

The book is written in a modular format, with each chapter as self-contained as possible, and with the necessary prerequisite material clearly identified. This structure results in great flexibility, and allows readers to reach topics of specific interest easily. The book is based on a year-long course given by the author and contains extensive problems, with password-protected solutions available to lecturers at www.cambridge.org/9780521864497.

Mark Srednicki is Professor of Physics at the University of California, Santa Barbara. He gained his undergraduate degree from Cornell University in 1977, and received a Ph.D. from Stanford University in 1980. Professor Srednicki has held postdoctoral positions at Princeton University and the European Organization for Nuclear Research (CERN).

"This accessible and conceptually structured introduction to quantum field theory will be of value not only to beginning students but also to practicing physicists interested in learning or reviewing specific topics. The book is organized in a modular fashion, which makes it easy to extract the basic information relevant to the reader's area(s) of interest. The material is presented in an intuitively clear and informal style. Foundational topics such as path integrals and Lorentz representations are included early in the exposition, as appropriate for a modern course; later material includes a detailed description of the Standard Model and other advanced topics such as instantons, supersymmetry, and unification, which are essential knowledge for working particle physicists, but which are not treated in most other field theory texts."

Washington Taylor, Massachusetts Institute of Technology

"Over the years I have used parts of Srednicki's book to teach field theory to physics graduate students not specializing in particle physics. This is a vast subject, with many outstanding textbooks. Among these, Srednicki's stands out for its pedagogy. The subject is built logically, rather than historically. The exposition walks the line between getting the idea across and not shying away from a serious calculation. Path integrals enter early, and renormalization theory is pursued from the very start, with the excellent choice of φ^3 in six dimensions as the training workhorse. By the end of the course the student should understand both beta functions and the Standard Model, and be able to carry through a calculation when a perturbative calculation is called for."

Predrag Cvitanović, Georgia Institute of Technology

"This book should become a favorite of quantum field theory students and instructors. The approach is systematic and comprehensive, but the friendly and encouraging voice of the author comes through loud and clear to make the subject feel accessible. Many interesting examples are worked out in pedagogical detail."

Ann Nelson, University of Washington

"I expect that this will be the textbook of choice for many quantum field theory courses. The presentation is straightforward and readable, with the author's easy-going 'voice' coming through in his writing. The organization into a large number of short chapters, with the prerequisites for each chapter clearly marked, makes the book flexible and easy to teach from or to read independently. A large and varied collection of special topics is available, depending on the interests of the instructor and the student."

Joseph Polchinski, University of California, Santa Barbara

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CAMBRIDGE
UNIVERSITY PRESS

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University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of
education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9780521864497

First published 2007
10th printing 2015

Printed in the United Kingdom by TJ International Ltd, Padstow, Cornwall

A catalog record for this publication is available from the British Library

ISBN-13 978-0-521-86449-7 hardback

To my parents
Casimir and Helen Srednicki
with gratitude

Cambridge University Press
978-0-521-86449-7 - Quantum Field Theory
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Preface for students

Quantum field theory is the basic mathematical language that is used to describe and analyze the physics of elementary particles. The goal of this book is to provide a concise, step-by-step introduction to this subject, one that covers all the key concepts that are needed to understand the Standard Model of elementary particles, and some of its proposed extensions.

In order to be prepared to undertake the study of quantum field theory, you should recognize and understand the following equations:

$$\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2 \,,$$

$$a^\dagger|n\rangle = \sqrt{n{+}1}\,|n{+}1\rangle \,,$$

$$J_\pm|j,m\rangle = \sqrt{j(j{+}1){-}m(m{\pm}1)}\,|j,m{\pm}1\rangle \,,$$

$$A(t) = e^{+iHt/\hbar}Ae^{-iHt/\hbar} \,,$$

$$H = p\dot{q} - L \,,$$

$$ct' = \gamma(ct - \beta x) \,,$$

$$E = (\mathbf{p}^2c^2 + m^2c^4)^{1/2} \,,$$

$$\mathbf{E} = -\dot{\mathbf{A}}/c - \nabla\varphi \,.$$

This list is not, of course, complete; but if you are familiar with these equations, you probably know enough about quantum mechanics, classical mechanics, special relativity, and electromagnetism to tackle the material in this book.

Quantum field theory has the reputation of being a subject that is hard to learn. The problem, I think, is not so much that its basic ingredients are unusually difficult to master (indeed, the conceptual shift needed to go from quantum mechanics to quantum field theory is not nearly as severe as the one needed to go from classical mechanics to quantum mechanics), but rather that there are a *lot* of these ingredients. Some are fundamental, but many are just technical aspects of an unfamiliar form of perturbation theory.

In this book, I have tried to make the subject as accessible to beginners as possible. There are three main aspects to my approach.

Logical development of the basic concepts. This is, of course, very different from the historical development of quantum field theory, which, like the historical development of most worthwhile subjects, was filled with inspired guesses and brilliant extrapolations of sometimes fuzzy ideas, as well as its fair share of mistakes, misconceptions, and dead ends. None of that is in this book. From this book, you will (I hope) get the impression that the whole subject is effortlessly clear and obvious, with one step following the next like sunshine after refreshing rain.

Illustration of the basic concepts with the simplest examples. In most fields of human endeavor, newcomers are not expected to do the most demanding tasks right away. It takes time, dedication, and lots of practice to work up to what the accomplished masters are doing. There is no reason to expect quantum field theory to be any different in this regard. Therefore, we will start off by analyzing quantum field theories that are not immediately applicable to the real world of electrons, photons, protons, etc., but that will allow us to gain familiarity with the tools we will need, and to practice using them. Then, when we do work up to "real physics," we will be fully ready for the task. To this end, the book is divided into three parts: Spin Zero, Spin One Half, and Spin One. The technical complexities associated with a particular type of particle increase with its spin. We will therefore first learn all we can about spinless particles before moving on to the more difficult (and more interesting) nonzero spins. Once we get to them, we will do a good variety of calculations in (and beyond) the Standard Model of elementary particles.

User friendliness. Each of the three parts is divided into numerous sections. Each section is intended to treat one idea or concept or calculation, and each is written to be as self-contained as possible. For example, when an equation from an earlier section is needed, I usually just repeat it, rather than ask you to leaf back and find it (a reader's task that I've always found annoying). Furthermore, each section is labeled with its immediate prerequisites, so you can tell exactly what you need to have learned in order to

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proceed. This allows you to construct chains to whatever material may interest you, and to get there as quickly as possible.

That said, I expect that most readers of this book will encounter it as the textbook in a course on quantum field theory. In that case, of course, your reading will be guided by your professor, who I hope will find the above features useful. If, however, you are reading this book on your own, I have two pieces of advice.

The first (and most important) is this: find someone else to read it with you. I promise that it will be far more fun and rewarding that way; talking about a subject to another human being will inevitably improve the depth of your understanding. And you will have someone to work with you on the problems. (As with all physics texts, the problems are a key ingredient. I will not belabor this point, because if you have gotten this far in physics, you already know it well.)

The second piece of advice echoes the novelist and Nobel laureate William Faulkner. An interviewer asked, "Mr. Faulkner, some of your readers claim they still cannot understand your work after reading it two or three times. What approach would you advise them to adopt?" Faulkner replied, "Read it a fourth time."

That's my advice here as well. After the fourth attempt, though, you should consider trying something else. This is, after all, not the only book that has ever been written on the subject. You may find that a different approach (or even the same approach explained in different words) breaks the logjam in your thinking. There are a number of excellent books that you could consult, some of which are listed in the Bibliography. I have also listed particular books that I think could be helpful on specific topics in Reference Notes at the end of some of the sections.

This textbook (like all finite textbooks) has a number of deficiencies. One of these is a rather low level of mathematical rigor. This is partly endemic to the subject; rigorous proofs in quantum field theory are relatively rare, and do not appear in the overwhelming majority of research papers. Even some of the most basic notions lack proof; for example, currently you can get a million dollars from the Clay Mathematics Institute simply for proving that nonabelian gauge theory actually exists and has a unique ground state. Given this general situation, and since this is an introductory book, the proofs that we do have are only outlined.

Another deficiency of this book is that there is no discussion of the application of quantum field theory to condensed matter physics, where it also plays an important role. This connection has been important in the historical development of the subject, and is especially useful if you already know a lot of advanced statistical mechanics. I do not want this to be a prerequisite, however, and so I have chosen to keep the focus on applications within elementary particle physics.

Yet another deficiency is that *there are no references to the original literature*. In this regard, I am following a standard trend: as the foundations of a branch of science retreat into history, textbooks become more and more synthetic and reductionist. For example, it is now rare to see a new textbook on quantum mechanics that refers to the original papers by the famous founders of the subject. For guides to the original literature on quantum field theory, there are a number of other books with extensive references that you can consult; these include *Peskin & Schroeder*, *Weinberg*, and *Siegel*. (Italicized names refer to works listed in the Bibliography.) Unless otherwise noted, experimental numbers are taken from the Review of Particle Properties, available online at http://pdg.lbl.gov. Experimental numbers quoted in this book have an uncertainty of roughly ± 1 in the last significant digit. The Review should be consulted for the most recent experimental results, and for more precise statements of their uncertainty.

To conclude, let me say that you are about to embark on a tour of one of humanity's greatest intellectual endeavors, and certainly the one that has produced the most precise and accurate description of the natural world as we find it. I hope you enjoy the ride.

Preface for instructors

On learning that a new text on quantum field theory has appeared, one is surely tempted to respond with Isidor Rabi's famous comment about the muon: "Who ordered *that*?" After all, many excellent textbooks on quantum field theory are already available. I, for example, would not want to be without my well-worn copies of *Quantum Field Theory* by Lowell S. Brown (Cambridge 1994), *Aspects of Symmetry* by Sidney Coleman (Cambridge 1985), *Introduction to Quantum Field Theory* by Michael E. Peskin and Daniel V. Schroeder (Westview 1995), *Field Theory: A Modern Primer* by Pierre Ramond (Addison-Wesley 1990), *Fields* by Warren Siegel (arXiv.org 2005), *The Quantum Theory of Fields*, Volumes I, II, and III, by Steven Weinberg (Cambridge 1995), and *Quantum Field Theory in a Nutshell* by my colleague Tony Zee (Princeton 2003), to name just a few of the more recent texts. Nevertheless, despite the excellence of these and other books, I have never followed any of them very closely in my twenty years of on-and-off teaching of a year-long course in relativistic quantum field theory.

As discussed in the Preface for Students, this book is based on the notion that quantum field theory is most readily learned by starting with the simplest examples and working through their details in a logical fashion. To this end, I have tried to set things up at the very beginning to anticipate the eventual need for renormalization, and not be cavalier about how the fields are normalized and the parameters defined. I believe that these precautions take a lot of the "hocus pocus" (to quote Feynman) out of the "dippy process" of renormalization. Indeed, with this approach, even the anharmonic oscillator is in need of renormalization; see problem 14.7.

A field theory with many pedagogical virtues is φ^3 theory in six dimensions, where its coupling constant is dimensionless. Perhaps because six dimensions used to seem too outré (though today's prospective string theorists do not even blink), the only introductory textbook I know of that treats this model is *Quantum Field Theory* by George Sterman (Cambridge 1993), though it is also discussed in some more advanced books, such as *Renormalization* by John Collins (Cambridge 1984) and *Foundations of Quantum Chromodynamics* by T. Muta (World Scientific 1998). (There is also a series of lectures by Ed Witten on quantum field theory for mathematicians, available online, that treat φ^3 theory.) The reason φ^3 theory in six dimensions is a nice example is that its Feynman diagrams have a simple structure, but still exhibit the generic phenomena of renormalizable quantum field theory at the one-loop level. (The same cannot be said for φ^4 theory in four dimensions, where momentum-dependent corrections to the propagator do not appear until the two-loop level.) Thus, in Part I of this text, φ^3 theory in six dimensions is the primary example. I use it to give introductory treatments of most aspects of relativistic quantum field theory for spin-zero particles, with a minimum of the technical complications that arise in more realistic theories (like QED) with higher-spin particles.

Although I eventually discuss the Wilson approach to renormalization and effective field theory (in section 29), and use effective field theory extensively for the physics of hadrons in Part III, I do not feel it is pedagogically useful to bring it in at the very beginning, as is sometimes advocated. The problem is that the key notion of the decoupling of physical processes at different length scales is an unfamiliar one for most students; there is nothing in typical courses on quantum mechanics or electromagnetism or classical mechanics to prepare students for this idea (which was deemed worthy of a Nobel Prize for Ken Wilson in 1982). It also does not provide for a simple calculational framework, since one must deal with the infinite number of terms in the effective lagrangian, and then explain why most of them do not matter after all. It is noteworthy that Wilson himself did not spend a lot of time computing properly normalized perturbative S-matrix elements, a skill that we certainly want our students to have; we want them to have it because a great deal of current research still depends on it. Indeed, the vaunted success of quantum field theory as a description of the real world is based almost entirely on our ability to carry out these perturbative calculations. Studying renormalization early on has other pedagogical advantages. With the Nobel Prizes to Gerard 't Hooft and Tini Veltman in 1999 and to David Gross, David Politzer, and Frank Wilczek in 2004, today's students are well aware of beta functions and running couplings, and would like to understand them. I find that they are generally much more excited about this (even in the context of toy models) than they are about learning to reproduce the nearly century-old tree-level calculations of QED. And φ^3 theory in six dimensions is asymptotically free, which ultimately provides for a nice segue to the "real physics" of QCD.

In general, I have tried to present topics so that the more interesting aspects (from a present-day point of view) come first. An example is anomalies; the traditional approach is to start with the $\pi^0 \to \gamma\gamma$ decay rate, but such a low-energy process seems like a dusty relic to most of today's students. I therefore begin by demonstrating that anomalies destroy the self-consistency of the great majority of chiral gauge theories, a fact that strikes me (and, in my experience, most students) as much more interesting and dramatic than an incorrect calculation of the π^0 decay rate. Then, when we do eventually get to this process (in section 90), it appears as a straightforward consequence of what we already learned about anomalies in sections 75–77.

Nevertheless, I want this book to be useful to those who disagree with my pedagogical choices, and so I have tried to structure it to allow for maximum flexibility. Each section treats a particular idea or concept or calculation, and is as self-contained as possible. Each section also lists its immediate prerequisites, so that it is easy to see how to rearrange the material to suit your personal preferences.

In some cases, alternative approaches are developed in the problems. For example, I have chosen to introduce path integrals relatively early (though not before canonical quantization and operator methods are applied to free-field theory), and use them to derive Dyson's expansion. For those who would prefer to delay the introduction of path integrals (but since you will have to cover them eventually, why not get it over with?), problem 9.5 outlines the operator-based derivation in the interaction picture.

Another point worth noting is that a textbook and lectures are ideally complementary. Many sections of this book contain rather tedious mathematical detail that I would not and do not write on the blackboard during a lecture. (Indeed, the earliest origins of this book are supplementary notes that I typed up and handed out.) For example, much of the development of Weyl spinors in sections 34–37 can be left to outside reading. I do encourage you not to eliminate this material entirely, however; pedagogically, the problem with skipping directly to four-component notation is explaining that (in four dimensions) the hermitian conjugate of a left-handed field is right handed, a deeply important fact that is the key to solving problems such as 36.5 and 83.1, which are in turn vital to understanding the structure of the Standard Model and its extensions. A related topic is computing scattering amplitudes for Majorana fields; this is essential for modern research

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on massive neutrinos and supersymmetric particles, though it could be left out of a time-limited course.

While I have sometimes included more mathematical detail than is ideal for a lecture, I have also tended to omit explanations based on "physical intuition". For example, in section 90, we compute the $\pi^- \to \ell^- \bar{\nu}_\ell$ decay amplitude (where ℓ is a charged lepton) and find that it is proportional to the lepton mass. There is a well-known heuristic explanation of this fact that goes something like this: "The pion has spin zero, and so the lepton and the antineutrino must emerge with opposite spin, and therefore the same helicity. An antineutrino is always right handed, and so the lepton must be as well. But only the left-handed lepton couples to the W^-, so the decay amplitude vanishes if the left- and right-handed leptons are not coupled by a mass term."

This is essentially correct, but the reasoning is a bit more subtle than it first appears. A student may ask, "Why can't there be orbital angular momentum? Then the lepton and the antineutrino could have the same spin." The answer is that orbital angular momentum must be perpendicular to the linear momentum, whereas helicity is (by definition) parallel to the linear momentum; so adding orbital angular momentum cannot change the helicity assignments. (This is explored in a simplified model in problem 48.4.) The larger point is that intuitive explanations can almost always be probed more deeply. This is fine in a classroom, where you are available to answer questions, but a textbook author has a hard time knowing where to stop. Too little detail renders the explanations opaque, and too much can be overwhelming; furthermore, the happy medium tends to differ from student to student. The calculation, on the other hand, is definitive (at least within the framework being explored, and modulo the possibility of mathematical error). As Roger Penrose once said, "The great thing about physical intuition is that it can be adjusted to fit the facts." So, in this book, I have tended to emphasize calculational detail at the expense of heuristic reasoning. Lectures should ideally invert this to some extent.

I should also mention that a section of the book is not intended to coincide exactly with a lecture. The material in some sections could easily be covered in less than an hour, and some would clearly take more. My approach in lecturing is to try to keep to a pace that allows the students to follow the analysis, and then try to come to a more-or-less natural stopping point when class time is up. This sometimes means ending in the middle of a long calculation, but I feel that this is better than trying to artificially speed things along to reach a predetermined destination.

It would take at least three semesters of lectures to cover this entire book, and so a year-long course must omit some. A sequence I might follow (I tend to change things around a bit every year) is 1–23, 26–28, 33–43, 45–48, 51, 52, 54–59, 62–64, 66–68, 24, 69, 70, 44, 53, 71–73, 75–77, 30, 32, 84, 87–89, 29, 82, 83, 90, and, if any time was left, a selection of whatever seemed of most interest to me and the students of the remaining material.

To conclude, I hope you find this book to be a useful tool in working towards our mutual goal of bringing humanity's understanding of the physics of elementary particles to a new audience.

Acknowledgments

Every book is a collaborative effort, even if there is only one author on the title page. Any skills I may have as a teacher were first gleaned as a student in the classes of those who taught me. My first and most important teachers were my parents, Casimir and Helen Srednicki, to whom this book is dedicated. In our small town in Ohio, my excellent public-school teachers included Thelma Kieffaber, Marie Casher, Carol Baird, Jim Chase, Joe Gerin, Hugh Laughlin, and Tom Murphy. In college at Cornell, Don Hartill, Bruce Kusse, Bob Siemann, John Kogut, and Saul Teukolsky taught particularly memorable courses. In graduate school at Stanford, Roberto Peccei gave me my first exposure to quantum field theory, in a superb course that required bicycling in by 8:30 AM (which seemed like a major sacrifice at the time). Everyone in that class very much hoped that Roberto would one day turn his extensive hand-written lecture notes (which he put on reserve in the library) into a book. He never did, but I'd like to think that perhaps a bit of his consummate skill has found its way into this text. I have also used a couple of his jokes.

My thesis advisor at Stanford, Lenny Susskind, taught me how to think about physics without getting bogged down in the details. This book includes a lot of detail that Lenny would no doubt have left out, but while writing it I have tried to keep his exemplary clarity of thought in mind as something to strive for.

During my time in graduate school, and subsequently in postdoctoral positions at Princeton and CERN, and finally as a faculty member at UC Santa Barbara, I was extremely fortunate to be able to interact with many excellent physicists, from whom I learned an enormous amount. These include Stuart Freedman, Eduardo Fradkin, Steve Shenker, Sidney Coleman, Savas Dimopoulos, Stuart Raby, Michael Dine, Willy Fischler, Curt Callan, David Gross, Malcolm Perry, Sam Treiman, Arthur Wightman, Ed Witten, Hans-Peter Nilles, Daniel Wyler, Dmitri Nanopoulos, John Ellis, Keith Olive, John Cardy, Jose Fulco, Ray Sawyer, Frank Wilczek, Jim Hartle, Gary Horowitz, Andy Strominger, and Tony Zee. I am especially grateful to my Santa Barbara colleagues David Berenstein, Steve Giddings, Don Marolf, Joe Polchinski, and Bob Sugar, who used various drafts of this book while teaching quantum field theory, and made various suggestions for improvement.

I am also grateful to physicists at other institutions who read parts of the manuscript and also made suggestions, including Zvi Bern, Oliver de Wolfe, Lance Dixon, Marcelo Gleiser, Steve Gottlieb, Arkady Tseytlin, and Arkady Vainshtein. I must single out for special thanks Professor Heidi Fearn of Cal State Fullerton, whose careful reading of Parts I and II allowed me to correct many unclear passages and outright errors that would otherwise have slipped by.

Students over the years have suffered through my varied attempts to arrive at a pedagogically acceptable scheme for teaching quantum field theory. I thank all of them for their indulgence. I am especially grateful to Sam Pinansky, Tae Min Hong, and Sho Yaida for their diligence in finding and reporting errors, and to Brian Wignal for help with formatting the manuscript. Also, a number of readers from around the world (as well as Santa Barbara) kindly reported errors in earlier versions; these include Mark Alford, Curtis Asplund, Omri Bahat-Treidel, Hee-Joong Chung, Claudio Coriano, Chris Duston, Daniel J. Feldman, Edson Fernando Ferrari, Gregory Giecold, Julian Heeck, Idse Heemskerk, Tae Min Hong, Ziyang Hu, Nathan Johnson-McDaniel, Nikhil Jayant Joshi, Yevgeny Kats, Sue Ann Koay, Hwasung Lee, Peter Lee, Shu-Ping Lee, Chris Lee Lin, Guilin Liu, Joyce Myers, Matan Mussel, Ahsan Nazer, Hiromichi Nishimura, Chris Pagnutti, Ari Pakman, Jess Riedel, Jorge Rocha, Mauricio Romo, Dusan Simic, Yushu Song, Daniel Vangheluwe, Miles Stoudenmire, Kevin Weil, Masaru Watanabe, Mark Weitzman, Brian Willett, Ting Yu, Ryan Zelen, Jianhui Zhou, and Fabio Zocchi. I thank them for their help, and apologize to anyone who I may have missed.

Throughout this project, the assistance and support of my wife Eloïse and daughter Julia were invaluable. Eloïse read through the manuscript and made suggestions that often clarified the language. Julia offered advice on the cover design (a highly stylized Feynman diagram). And they both kindly indulged the amount of time I spent working on this book that you now hold in your hands.