Introduction

This book is an introduction to some of the ways mathematics can be used to obtain useful, profitable and extremely attractive results in finance. It is now widely recognised that the financial world has become a profitable hunting ground for mathematicians. Indeed, without a confident grasp of basic mathematics, many of the most important financial products in the market will not be understood. It is the aim of this book to explain, in simple terms, some of the most important ideas of basic financial mathematics. A significant feature of the book is that Excel spreadsheets are used to assist the reader with the more tricky algebraic manipulations. If the reader is strong in algebra, the spreadsheets act as an aid with the calculations. If the reader is not so strong, these spreadsheets will show, in a numerical framework, what is 'going on' in the algebra. By seeing what the spreadsheet is doing, the reader grasps the purpose of the algebra. An introduction to those parts of Excel used in this book is given in the first chapter. However, this is not meant to be a tutorial in Excel; rather, it is a basic covering of those features of Excel the reader will need. It is important to emphasise that to move ahead with this subject, familiarity and confidence with Excel (or some other programming language) are essential. Some references are given in 'An introduction to Excel'.

One of the really attractive features of financial mathematics is that the subject is so new. The major breakthrough came in 1973 with the classic Black–Scholes result on the pricing of European call and put options. Almost everything in this subject has happened since that fairly recent date. This means that not only are the results new and fresh but so is the thinking that led to these results. We can still see – very clearly – the problems the originators were trying to overcome when they produced these wonderful ideas. Because we are so near the beginning of the subject, there is no long history to absorb before today's ideas can be seen. There is no vast theory to plough through before you can understand today's problems. In Chapter 8, we look at recent developments. There are fewer exercises attached to this chapter. In a sense, the problems that could be attached to Chapter 8 are close to what

mathematicians are working on today. In this subject, it is not a long bus ride to the frontier.

So who is the book for?

1. Level of mathematical ability:
   We have aimed the book at:
   - very good GCSE candidates, who are confident enough to try new things in mathematics
   - those who have at least some AS Mathematics experience: ideally in the C1 and C2 pure modules and in the S1 statistics module.
2. Courses, examinations involving the subject matter of the book:
   - Business Studies, Finance, Investment and Economics courses in institutions of higher education and universities.
   - The Securities and Investment Institute: Certificate in Investment Administration; Certificate in Investments; also, the Financial Derivatives Module.
   - The Faculty of Actuaries and Institute of Actuaries Finance and Investment, Specialist Technical B syllabus: Certificate in Derivatives.
   - The CFA Program (Chartered Financial Analyst): Analysis of Debt Investments (VII); Analysis of Derivatives (VIII).

What is the book about?

There are eight chapters.

Chapter 1 describes the building blocks of the subject. We describe interest rates, how they are calculated and how they may be used. Then interest rates are used to describe the present value (or the discounted value) of money. We define and explain the important idea of arbitrage, after which we illustrate risk neutral probabilities. Finally, we take a first look at a curve illustrating the development of interest rates: the zero curve.

Chapter 2 implements some of these ideas and describes in full detail how forward contracts operate and change in value as time passes.

Chapter 3 takes forward contracts into the market place and provides a full description of futures contracts. We describe the mechanics of futures contracts and illustrate how they might be traded. We show how futures contracts can be used for speculation and for hedging risk.

Chapter 4 looks at bonds: what they are and how they are priced. We look at quotations, day count conventions, bond yields and how to compare different bonds. We show how futures contracts and bonds can be used together and, finally, we make a second attempt to construct a zero curve.

Chapter 5 takes forward the idea of interest rates – literally – and considers interest rates over future time periods. We look at forward rate agreements and show how to achieve, over a future time period, an interest rate that is fixed today. We define and describe interest rate swaps and the swap rate and illustrate some applications of swaps. We describe and illustrate caps and floors. We show how to enter a futures contract on an interest rate and, finally, we construct and plot a realistic zero curve.

Chapter 6 takes a pictorial look at options. We describe many options (including call and put options) with their pay-offs and the strategies determining their use. We illustrate put–call parity. We show how the options can be used to hedge risk and for racheting up profits.

Chapter 7 shows how options can be priced. We describe in elegant detail a binomial tree method for pricing European and American options. We describe and illustrate the Black–Scholes formula.

Chapter 8 points to the future. In this chapter, we remove the idea that all debts will be honoured in full on the day they fall due and see what happens. We consider, from two different standpoints, the probability that a company will default on its obligations and look at an outstanding problem in this area. Finally, we describe four ways in which, for a price, risk can be considerably reduced, if not eliminated.

What further help is available?

There are internet sites. We give below some that the author has found helpful, but as can be imagined, in this rapidly developing field where communication is all important, there are many, many more.

Educational (helpful with definitions, explanations and background material):

www.defaultrisk.com	Mainly credit risk and credit derivatives
www.investopedia.com	
www.investorguide.com	
www.investorwords.com	

www.riskglossary.com	
en.wikipedia.org	

Information and data:

www.moneyextra.com	
www.moneyfacts.co.uk	
yahoo finance	An excellent and really useful site offering both information and large amounts of data.

Exchanges and markets:

www.bankofengland.co.uk	Bank of England
www.cbot.com	Chicago Board of Trade
www.cme.com	Chicago Mercantile Exchange
www.ftse.com	FTSE International
www.liffe.com	London International Financial Futures and Options Exchange
www.londonstockexchange.com	London Stock Exchange

Newspapers and news (also helpful with data and background information):

www.bbc.co.uk	BBC
www.bloomberg.com	Bloomberg
www.ft.com	*Financial Times*
www.reuters.com	Reuters
money.cnn.com	CNN and *Fortune* magazine
yahoo finance	Helpful here, also

And there are books. Again, the choice is large, but again, the list below contains those the author has found particularly helpful.

- **Baxter, Martin and Rennie, Andrew, Financial Calculus: Cambridge University Press**

  A joy. The book starts from basics and moves apparently seamlessly through binomial trees to continuous models to an interesting presentation of interest rate models. This book is both profound and hugely enjoyable.
- **Benth, Fred Espen, Theory with Stochastic Analysis. An Introduction to Mathematical Finance: Springer**

  A good, approachable introduction to the next stage in the subject; deals with stochastic integration and martingales. Shows some VBA programs (Monte Carlo simulation and numerical methods).

- **Choudhry, Moorad, An Introduction to Credit Derivatives: Elsevier (Butterworth-Heinemann)**
  A straightforward introduction to credit derivatives. Case studies, diagrams and pictures of computer screens are helpful.
- **Etheridge, Alison, A Course in Financial Calculus: Cambridge University Press**
  Mathematically much more advanced, but very readable in the introductory sections. For anyone wanting to take the subject further and develop the more mathematical approach, this is a terrific book. (See Benth's book as a possible introduction.)
- **Hull, John C., Options, Futures and Other Derivatives: Prentice Hall (Pearson Education International)**
  Described by many as 'The Bible' of the subject. A first-class book for beginners and experienced practitioners alike.
- **Luenberger, David G., Investment Science: Oxford University Press**
  A very good and broad introduction to the subject.
- **Meissner, Gunter, Credit derivatives: Blackwell Publishing**
  Credit derivatives in a discrete setting.
- **Neftci, Salih N., An Introduction to the Mathematics of Financial Derivatives: Academic Press**
  An excellent introduction to the subject and to the harder mathematics that will follow if the reader wishes to take the subject further.
- **Schönbucher, Philipp J., Credit Derivatives Pricing Models: John Wiley**
  Very good indeed. The early part of the book is more descriptive, with concrete examples; later sections involve more advanced mathematical ideas. The book focuses mainly on credit risk and credit derivatives (Chapter 8).
- **Servigny, Arnaud de and Renault, Olivier, Measuring and Managing Credit Risk: McGraw-Hill**
  Very readable, vast in scope, mainly concerned with credit risk and credit derivatives (Chapter 8).
- **Shreve, Steven E., Stochastic Calculus for Finance I and II: Springer**
  Volume I focuses on the binomial tree method. *Volume II* considers continuous models. Very readable, interesting and beautifully presented. *Volume I* complements and extends Chapter 7.
- **van der Hoek, John and Elliott, Robert J., Binomial Models in Finance: Springer**
  A very clear description of how binomial trees can be used in more advanced modelling. Topics include assets paying dividends, exchange rate

contracts and interest rate derivatives. The authors show how binomial tree models can be constructed to calculate values consistent with market prices.

- **Wilmott, Paul, Derivatives: The Theory and Practice of Financial Engineering: John Wiley**

  A first-class introduction to the subject; very well organised and extremely readable. The Excel diagrams make reader participation almost a certainty.
- **Wilmott, Paul, Howison, Sam and Dewynne, Jeff, The Mathematics of Financial Derivatives: Cambridge University Press**

  Although mathematically more advanced, the book is genuinely an 'introduction' to the subject. Very clear and readable. Probably the best introduction to the subject using the differential equation approach.

For Excel:

Advanced Modelling in Finance using Excel and VBA, Mary Jackson and Mike Staunton: Wiley Finance – an excellent book and really useful for performing calculations in Excel. Includes VBA programming.

Excel 2000/2003 VBA Programmer's Reference, John Green: Wrox – for serious programmers.

Excel 2003, Steve Johnson: Pearson, Prentice Hall

Excel 2003 in Easy Steps, Stephen Copestake: Computer Step

Maran Illustrated Excel 2003: Maran

Microsoft Office, Excel 2003 Quick Steps, John Cronan: McGraw-Hill

Visual Basic 2005 Demystified, Jeff Kent: McGraw-Hill

Assumptions

We assume (almost) throughout the book that the price an asset can be bought for is the same as the price the asset can be sold for. This gives the asset 'one price' which is convenient for pricing theory.

We assume no transaction costs. So when a commodity is bought or sold, there is no charge made by the agent handling the sale. Again, this is not wholly realistic, but this assumption does make it easier to see what is happening in the theory without becoming embroiled in administration.

We assume no costs of storage. This applies mainly in Chapter 2 with forward contracts on commodities.

Short selling is a way to derive profit from a fall in the value of a share or some other security. A 'short seller' will borrow a security and immediately

sell it. The short seller hopes the security will then fall in value. If this happens, the short seller will buy the security (at a lower price) and replace what he has borrowed. The difference in prices becomes the short seller's profit.

If this sounds easy, there are strict regulations governing 'short sales'. Two of the non-legal regulations are that any dividends that are paid during the period in which the security has been borrowed must be paid to the rightful owner by the short seller, and if the security is required by the rightful owner, the short seller has to replace the security immediately. There are also dangerous overtones in short selling. There is the potential for unlimited loss. If the security rises in value, the short seller must purchase at the higher price (perhaps the considerably higher price) to replace the security.

We certainly are not advocating that the reader becomes involved in short sales. This is completely a game for the professionals. But we do use the idea of short selling in some of the pricing arguments.

Names of companies, firms and organisations

These are almost entirely fictitious. We have used the names BT and BP: all other names bear no resemblance to any organisation existing now or in the past. The reason for this is that, of course, the share price and the financial standing of a company change over time. So what might be a realistic share price today will almost certainly not be realistic by the time this is being read. Using real companies would mean that with high probability, the data would be inaccurate and possibly misleading. There are no such problems with a fictitious company.

To the reader

But what really matters is that the reader has the enthusiasm and determination to try out the examples and exercises for themselves. Put pen to paper. Try to see what is actually happening. Financial mathematics is not a spectator sport. You need to get involved. And involvement means doing. If the algebra is looking taxing, there are Excel spreadsheets to help you through the worst of it. We hope you enjoy the ideas that lie ahead. This is a beautiful subject: it is important and potentially highly profitable. It is hard to think of more compelling reasons to read a book.

Acknowledgements

With grateful thanks to Gary Cook for his invaluable advice in matters of Excel. To Ian Hammond, who, with his many friendly enquiries – 'How's the book going?' – shamed me into writing, when I would have preferred to be doing other things. To the referee for his/her many helpful, perceptive, well-informed and extremely useful observations. I am grateful to David Buik and Andrew Garrood of Cantor Index for very helpful initial discussions on the content of the book. To the editorial and production staff of Cambridge University Press for all their considerable expertise and their unfailingly helpful and patient advice. To Mike and Tim for listening, being occasionally impressed but always totally supportive. And to Dorothy, for her help with writing and editing, for being calming and encouraging and for providing a reassuringly normal and happy world when the writing was going badly. To all, I offer heartfelt thanks. Theirs was the inspiration. The mistakes, alas, are all mine.

An introduction to Excel

All the calculations in this book can be performed on a calculator. However, they can be performed more efficiently and much faster on a spreadsheet in Excel. If a calculation has to be repeated, then with a spreadsheet in place the repeat calculation is almost immediate. With a calculator, you just have to start again.

In this chapter we present a short introduction to those features of Excel that are used to perform calculations in this book. We recommend, however, that the reader acquires a complete introduction to Excel and learns more about this remarkable facility. To go further with financial mathematics, a knowledge of VBA (Visual Basic for Applications and available through Excel) or some other high-level programming language is essential. (See the texts described in the Introduction.)

The starting point for an Excel calculation is an Excel worksheet (Figure 1).

We have indicated the menu bar, the tool bars and the formula bar. Also, for future reference, we have indicated the chart wizard, the paste function and the name box.

Each cell in a worksheet has a name or a reference. This is **letter** followed by **number**. In Figure 1, the cell B6 is illustrated. Observe that the letter identifies the column (B) and the number identifies the row (6).

To enter a number, words or a formula in a particular cell, left click the required cell. The heavy border indicates that this is now the active cell. (The cell has been 'highlighted').

For a number or words:

type in the number or the words

press Enter

For a formula:

type = followed by the formula

The formula bar now has the symbols shown in Figure 2.

To enter the formula, click the Enter button. Or, press Enter. To cancel, click the cross.

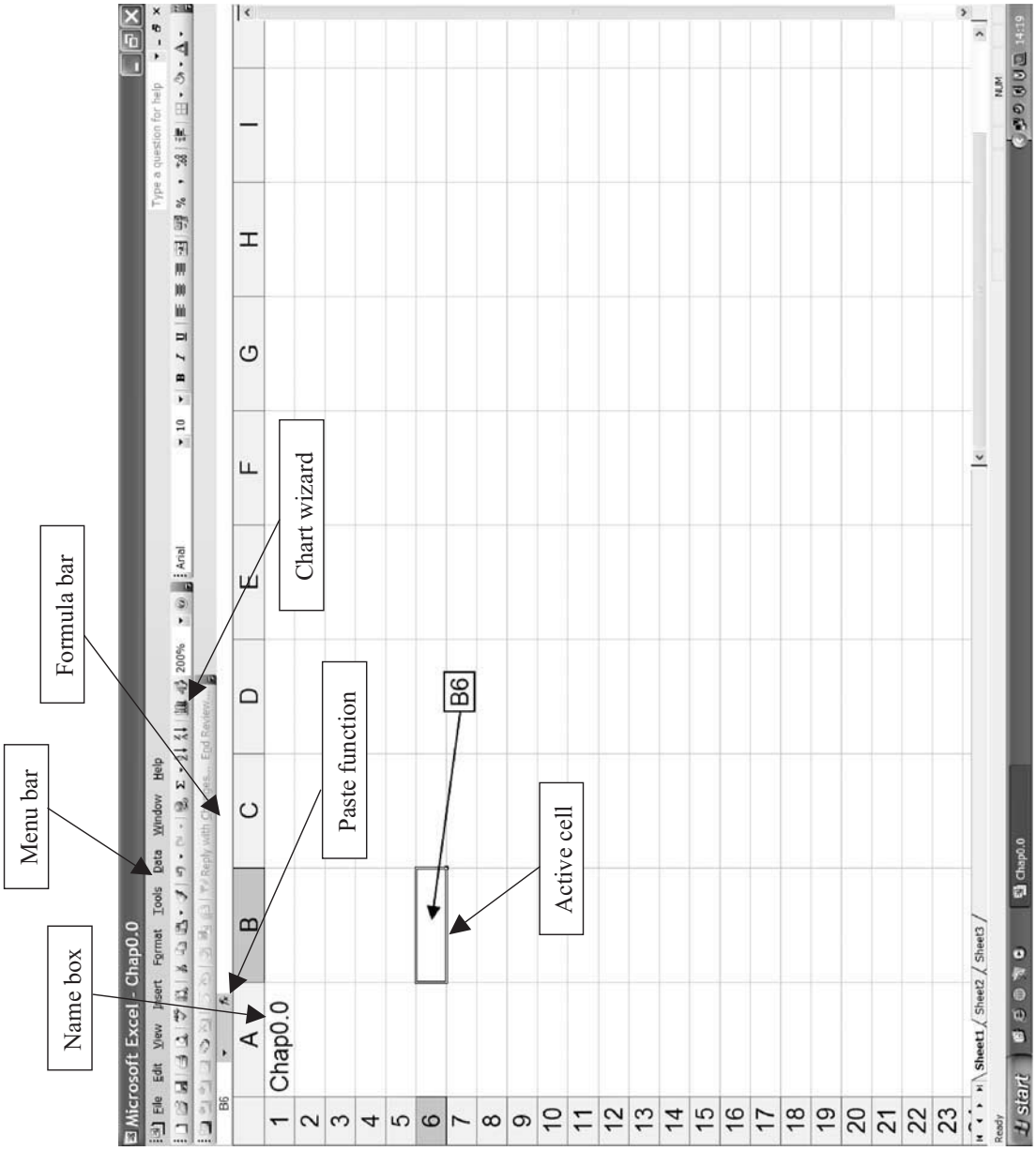

Figure 1