Cambridge University Press
978-0-521-86327-8 — Frege
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Excerpt
More Information

Prelude to the Afternoon of a Paradox

A LETTER ARRIVED. It was brought to the university, then taken from the mail cubicle to the mathematician-philosopher's office. He received it at his desk with the usual triage of correspondence. He may have brought it back to the house with him as particularly interesting and requiring more concentrated attention. He had a rough idea of what it was about. He looked it over later in the day in the garden or at his sun-streaked table where he sometimes wrote. He needed to think more about what exactly it was supposed to mean.

Aged fifty-four years, Gottlob Frege was on the threshold of publishing the crowning conclusion to his lifetime's research in the logicist foundations of arithmetic. He had the usual doubts about some parts, but had worked everything out vigilantly, and in the end the whole package seemed fully in order. It was a significant accomplishment, a contribution to mathematical knowledge and scientific philosophy. Mathematics was shown to be, as Plato thought, independent of the vagaries of subjective thought, but, as only Frege expected to prove, planted on rock solid foundations in pure logic. Having begun with arithmetic, and having devised an exact logical notation for the expression of basic mathematical ideas, he was satisfied that elementary arithmetic could be reduced to logic, that it was nothing more or less than pure logic applied to a purely logical concept of number.

True, he was publishing the second volume of his 1893 masterwork, *Grundgesetze der Arithmetik, begriffsschriftlich abgeleitet* (*Basic Laws of Arithmetic, Derived using Concept-Writing*), out of his own pocket in the next year, 1903. The demand for heavy-duty texts full of formal symbolisms that no one else working in logic or mathematics was interested in using or had any facility reading at the time was not encouraging. Printing books was a business like any other that must earn its keep. Technical writings no less than novels, travelogues, poetry, and libretti need at least to break even at the cash-drawer. The publisher, having already produced the author's first volume, and having been underwhelmed with sales, was unwilling to take the same risk again for the second volume without subsidy. A reasonable conclusion from a profit-driven rather than philosophical motive. Who would want volume two, having not already bought volume one?

Committed as he was to the philosophical project of presenting the complete reduction of arithmetic to logic, the scholar reluctantly paid the publisher's costs to see the work in print. He could do no more than present his ideas as he had rigorously painstakingly developed them. He was obligated to see the project through and have copies of his completed logic placed on library shelves. Perhaps he wanted to see the finished project in print, to hold in his hands what he had never doubted to be a major contribution to the foundations of knowledge. Then, too, his research had been supported by a large optical corporation, and they would want to see concrete results for their investment in his ideas, time, and labor. He could hope for a more appreciative reception from more knowledgeable discerning opinion when the whole work became available. It might take years, but the certainty of eventual success could not be doubted.

He knew, reflecting on the matter, that mathematical philosophy was ripe for a formally exact theory of the logical basis of arithmetic. There were already gratifying signs of positive recognition, of respect for and interest in what he had done, if not quite the revolution in philosophy of mathematics he endeavored to ignite. Unfortunately, recognition came so late, when certain important things could no longer be done. There were many misunderstandings. There was much to correct in how philosophy thinks of mathematics in its relation to a minimally adequate pure symbolic logic for the formally exact expression of arithmetical truths. Mathematical proofs are always matters of logical inference. It was a technical problem to discover and fit the right logic of functions to the inferences made in mathematical reasoning. Firmly convinced of the rightness of the approach, the author knew that it might nevertheless take time, many years perhaps, for the work to find its proper place at the end of a line of predecessors who had also sought to define the most basic concepts of mathematics and reconstruct the logic of mathematical reasoning.

What was required of the foundations of mathematics was first and foremost an adequate language for the expression of mathematical ideas. Frege's *Begriffsschrift, Concept-Script* or *Concept-Writing*, was designed to provide a syntactically univocal notation that in meaningful expressions referred and could only refer to individual existent objects, dynamic or abstract, belonging to the logic's reference domain, to the extensions of predicates in the language by which instantiated properties are truly or falsely ascribed to existent entities. The logic secondly introduced deductively valid inference mechanisms, the syntax and formal semantics of proof, whereby the theorems of mathematics, beginning with all the essentials of elementary arithmetic, could be rigorously derived as logical consequences of basic intuitive logically supportable axioms. If it were successful, then in this sense, for the first time in the history of mathematics and philosophy, everything in arithmetic from top to bottom would be tightly fastened down to a surveyable choice of formal logical principles. The same principles differently expressed turned out to be essentially those needed for what today is known as an algebraic second-order propositional and predicate-quantificational logic or general functional calculus. Such a system, the mathematician-philosopher's *Begriffsschrift*, differently expressed in equivalent syntax and with essentially the same existence-presuppositional extensional semantics, remains in contemporary applications one of the most widely adopted and adapted logics. It is the logic that in more linear sentential notation is still considered the standard classical first- and second-order logic. More, it is the basis, the *Urgrundplan*, for almost all variations into greater logical exotica for specialized applications of nonclassical logics, the port of call from which they depart and with which they are inevitably formally compared.

The postmark was from England. He did not recognize at first glance the handwriting of the correspondent. He had heard before, sometimes in critical but always most respectful terms, of the aristocratic philosopher Bertrand Arthur William Russell. He knew him to be working in a similar way on the logical foundations of mathematics. Russell was more directly inspired by the logical arithmetic of Giuseppe Peano, and as a result more comfortable with logic written in linear sentences, like most formalisms, rather than in Frege's diagrammatic two-dimensional *Begriffsschrift* notation. Russell read everything through the lens of Peano's mathematical logic. Peano was interesting, important. A good technician, which was not said lightly. Turning the envelope from front to back and back to front again, satisfied that the author had been identified, Frege withdrew two pages written in Russell's near native-perfect German, acquired from the German-speaking nanny of his privileged childhood. Frege opened the envelope again, unfolded the pages. He began to read the letter that had taken only a few days to cross the English Channel, to reach him in the heart of Thüringen, marked Friday's Hill, Haslemere, 16 June 1902.