OPTIMIZATION METHODS IN FINANCE

Optimization models are playing an increasingly important role in financial decisions. This is the first textbook devoted to explaining how recent advances in optimization models, methods and software can be applied to solve problems in computational finance ranging from asset allocation to risk management, from option pricing to model calibration more efficiently and more accurately. Chapters discussing the theory and efficient solution methods for all major classes of optimization problems alternate with chapters illustrating their use in modeling problems of mathematical finance.

The reader is guided through the solution of asset/liability cash flow matching using linear programming techniques, which are also used to explain asset pricing and arbitrage. Volatility estimation is discussed using nonlinear optimization models. Quadratic programming formulations are provided for portfolio optimization problems based on a mean-variance model, for returns-based style analysis and for risk-neutral density estimation. Conic optimization techniques are introduced for modeling volatility constraints in asset management and for approximating covariance matrices. For constructing an index fund, the authors use an integer programming model. Option pricing is presented in the context of dynamic programming and so is the problem of structuring asset backed securities. Stochastic programming is applied to asset/liability management, and in this context the notion of Conditional Value at Risk is described. The final chapters are devoted to robust optimization models in finance.

The book is based on Master's courses in financial engineering and comes with worked examples, exercises and case studies. It will be welcomed by applied mathematicians, operational researchers and others who work in mathematical and computational finance and who are seeking a text for self-learning or for use with courses.

Gerard Cornuejols is an IBM University Professor of Operations Research at the Tepper School of Business, Carnegie Mellon University

Reha Tütüncü is a Vice President in the Quantitative Resources Group at Goldman Sachs Asset Management, New York

Already published or forthcoming
1. *The Concepts and Practice of Mathematical Finance,* by Mark S. Joshi
2. *C++ Design Patterns and Derivatives Pricing,* by Mark S. Joshi
3. *Volatility Perturbations for Equity, Fixed Income and Credit Derivative,* by Jean-Pierre Fouque, George Papanicolaou, Ronnie Sircar and Knut Solna
4. *Continuous Time Approach to Financial Volatility,* by Ole Barndorff-Nielsen and Neil Shephard
5. *Optimization Methods in Finance,* by Gerard Cornuejols and Reha Tütüncü
6. *Modern Interest Rate Theory,* by D.C. Brody and L.P. Hughston

OPTIMIZATION METHODS IN FINANCE

GERARD CORNUEJOLS
Carnegie Mellon University

REHA TÜTÜNCÜ
Goldman Sachs Asset Management

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521861700

First published 2007

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

ISBN-13 978-0-521-86170-0 hardback

To Julie
and
to Paz

Contents

Foreword

The use of sophisticated mathematical tools in modern finance is now commonplace. Researchers and practitioners routinely run simulations or solve differential equations to price securities, estimate risks, or determine hedging strategies. Some of the most important tools employed in these computations are optimization algorithms. Many computational finance problems ranging from asset allocation to risk management, from option pricing to model calibration, can be solved by optimization techniques. This book is devoted to explaining how to solve such problems efficiently and accurately using recent advances in optimization models, methods, and software.

Optimization is a mature branch of applied mathematics. Typical optimization problems have the objective of allocating limited resources to alternative activities in order to maximize the total benefit obtained from these activities. Through decades of intensive and innovative research, fast and reliable algorithms and software have become available for many classes of optimization problems. Consequently, optimization is now being used as an effective management and decision-support tool in many industries, including the financial industry.

This book discusses several classes of optimization problems encountered in financial models, including linear, quadratic, integer, dynamic, stochastic, conic, and robust programming. For each problem class, after introducing the relevant theory (optimality conditions, duality, etc.) and efficient solution methods, we discuss several problems of mathematical finance that can be modeled within this problem class. The reader is guided through the solution of asset/liability cash-flow matching using linear programming techniques, which are also used to explain asset pricing and arbitrage. Volatility estimation is discussed using nonlinear optimization models. Quadratic programming formulations are provided for portfolio optimization problems based on a mean-variance model for returns-based style analysis and for risk-neutral density estimation. Conic optimization techniques are introduced for modeling volatility constraints in asset management and for approximating covariance matrices. For constructing an index fund, we use an integer programming model. Option pricing is presented in the context of dynamic programming and so is the problem of structuring asset-backed securities. Stochastic programming is applied to asset/liability management, and in this context the notion of Conditional Value at Risk is described. Robust optimization models for portfolio selection and option pricing are also discussed.

This book is intended as a textbook for Master's programs in financial engineering, finance, or computational finance. In addition, the structure of chapters, alternating between optimization methods and financial models that employ these methods, allows the use of this book as a primary or secondary text in upper level undergraduate or introductory graduate courses in operations research, management science, and applied mathematics.

Optimization algorithms are sophisticated tools and the relationship between their inputs and outputs is sometimes opaque. To maximize the value one gets from these tools and to understand how they work, users often need a significant amount of guidance and practical experience with them. This book aims to provide this guidance and serve as a reference tool for the finance practitioners who use or want to use optimization techniques.

This book has its origins in courses taught at Carnegie Mellon University in the Masters program in Computational Finance and in the MBA program at the Tepper School of Business (Gérard Cornuéjols), and at the Tokyo Institute of Technology, Japan, and the University of Coimbra, Portugal (Reha Tütüncü). We thank the attendants of these courses for their feedback and for many stimulating discussions. We would also like to thank the colleagues who provided the initial impetus for this project or collaborated with us on various research projects that are reflected in the book, especially Rick Green, Raphael Hauser, John Hooker, Mark Koenig, Masakazu Kojima, Vijay Krishnamurthy, Yanjun Li, Ana Margarida Monteiro, Mustafa Pınar, Sanjay Srivastava, Michael Trick, and Luís Vicente. Various drafts of this book were experimented with in class by Javier Peña, François Margot, Miguel Lejeune, Miroslav Karamanov, and Kathie Cameron, and we thank them for their comments. Initial drafts of this book were completed when the second author was on the faculty of the Department of Mathematical Sciences at Carnegie Mellon University; he gratefully acknowledges their financial support.