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Where the discussion of a topic runs over two consecutive pages, reference is made only to the first of these. For discussions spread over three or more pages the first and last page numbers are given; these references are usually to the major treatment of the corresponding topic. Isolated refences to a topic, including those appearing on consecutive pages, are listed individually. Some long topics are split, e.g. 'Fourier transforms' and 'Fourier transforms, examples'. The letter 'n' after a page number indicates that the topic is discussed in a footnote on the relevant page.

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