

The Rock Physics Handbook, Second Edition

Tools for Seismic Analysis of Porous Media

The science of rock physics addresses the relationships between geophysical observations and the underlying physical properties of rocks, such as composition, porosity, and pore fluid content. The *Rock Physics Handbook* distills a vast quantity of background theory and laboratory results into a series of concise, self-contained chapters, which can be quickly accessed by those seeking practical solutions to problems in geophysical data interpretation.

In addition to the wide range of topics presented in the First Edition (including wave propagation, effective media, elasticity, electrical properties, and pore fluid flow and diffusion), this Second Edition also presents major new chapters on granular material and velocity–porosity–clay models for clastic sediments. Other new and expanded topics include anisotropic seismic signatures, nonlinear elasticity, wave propagation in thin layers, borehole waves, models for fractured media, poroelastic models, attenuation models, and cross-property relations between seismic and electrical parameters. This new edition also provides an enhanced set of appendices with key empirical results, data tables, and an atlas of reservoir rock properties expanded to include carbonates, clays, and gas hydrates.

Supported by a website hosting MATLAB routines for implementing the various rock physics formulas presented in the book, the Second Edition of *The Rock Physics Handbook* is a vital resource for advanced students and university faculty, as well as in-house geophysicists and engineers working in the petroleum industry. It will also be of interest to practitioners of environmental geophysics, geomechanics, and energy resources engineering interested in quantitative subsurface characterization and modeling of sediment properties.

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Frontmatter

[More information](#)

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Contents

<i>Preface</i>	<i>page xi</i>
----------------	----------------

1	Basic tools	1
1.1	The Fourier transform	1
1.2	The Hilbert transform and analytic signal	6
1.3	Statistics and probability	9
1.4	Coordinate transformations	18
2	Elasticity and Hooke’s law	21
2.1	Elastic moduli: isotropic form of Hooke’s law	21
2.2	Anisotropic form of Hooke’s law	23
2.3	Thomsen’s notation for weak elastic anisotropy	35
2.4	Tsvankin’s extended Thomsen parameters for orthorhombic media	39
2.5	Third-order nonlinear elasticity	40
2.6	Effective stress properties of rocks	43
2.7	Stress-induced anisotropy in rocks	47
2.8	Strain components and equations of motion in cylindrical and spherical coordinate systems	54
2.9	Deformation of inclusions and cavities in elastic solids	56
2.10	Deformation of a circular hole: borehole stresses	68
2.11	Mohr’s circles	74
2.12	Static and dynamic moduli	76
3	Seismic wave propagation	81
3.1	Seismic velocities	81
3.2	Phase, group, and energy velocities	83
v		

vi	Contents	
3.3	NMO in isotropic and anisotropic media	86
3.4	Impedance, reflectivity, and transmissivity	93
3.5	Reflectivity and amplitude variations with offset (AVO) in isotropic media	96
3.6	Plane-wave reflectivity in anisotropic media	105
3.7	Elastic impedance	115
3.8	Viscoelasticity and Q	121
3.9	Kramers–Kronig relations between velocity dispersion and Q	127
3.10	Waves in layered media: full-waveform synthetic seismograms	129
3.11	Waves in layered media: stratigraphic filtering and velocity dispersion	134
3.12	Waves in layered media: frequency-dependent anisotropy, dispersion, and attenuation	138
3.13	Scale-dependent seismic velocities in heterogeneous media	146
3.14	Scattering attenuation	150
3.15	Waves in cylindrical rods: the resonant bar	155
3.16	Waves in boreholes	160
4	Effective elastic media: bounds and mixing laws	169
4.1	Hashin–Shtrikman–Walpole bounds	169
4.2	Voigt and Reuss bounds	174
4.3	Wood’s formula	175
4.4	Voigt–Reuss–Hill average moduli estimate	177
4.5	Composite with uniform shear modulus	178
4.6	Rock and pore compressibilities and some pitfalls	179
4.7	Kuster and Toksöz formulation for effective moduli	183
4.8	Self-consistent approximations of effective moduli	185
4.9	Differential effective medium model	190
4.10	Hudson’s model for cracked media	194
4.11	Eshelby–Cheng model for cracked anisotropic media	203
4.12	T -matrix inclusion models for effective moduli	205
4.13	Elastic constants in finely layered media: Backus average	210
4.14	Elastic constants in finely layered media: general layer anisotropy	215
4.15	Poroelastic Backus average	216
4.16	Seismic response to fractures	219
4.17	Bound-filling models	224
5	Granular media	229
5.1	Packing and sorting of spheres	229
5.2	Thomas–Stieber model for sand–shale systems	237

vii	Contents	
5.3	Particle size and sorting	242
5.4	Random spherical grain packings: contact models and effective moduli	245
5.5	Ordered spherical grain packings: effective moduli	264
6	Fluid effects on wave propagation	266
6.1	Biot’s velocity relations	266
6.2	Geertsma–Smit approximations of Biot’s relations	272
6.3	Gassmann’s relations: isotropic form	273
6.4	Brown and Korringa’s generalized Gassmann equations for mixed mineralogy	282
6.5	Fluid substitution in anisotropic rocks	284
6.6	Generalized Gassmann’s equations for composite porous media	287
6.7	Generalized Gassmann equations for solid pore-filling material	290
6.8	Fluid substitution in thinly laminated reservoirs	292
6.9	BAM: Marion’s bounding average method	295
6.10	Mavko–Jizba squirt relations	297
6.11	Extension of Mavko–Jizba squirt relations for all frequencies	298
6.12	Biot–squirt model	302
6.13	Chapman <i>et al.</i> squirt model	304
6.14	Anisotropic squirt	306
6.15	Common features of fluid-related velocity dispersion mechanisms	310
6.16	Dvorkin–Mavko attenuation model	315
6.17	Partial and multiphase saturations	320
6.18	Partial saturation: White and Dutta–Odé model for velocity dispersion and attenuation	326
6.19	Velocity dispersion, attenuation, and dynamic permeability in heterogeneous poroelastic media	331
6.20	Waves in a pure viscous fluid	338
6.21	Physical properties of gases and fluids	339
7	Empirical relations	347
7.1	Velocity–porosity models: critical porosity and Nur’s modified Voigt average	347
7.2	Velocity–porosity models: Geertsma’s empirical relations for compressibility	350
7.3	Velocity–porosity models: Wyllie’s time-average equation	350
7.4	Velocity–porosity models: Raymer–Hunt–Gardner relations	353

viii	Contents	
7.5	Velocity–porosity–clay models: Han’s empirical relations for shaley sandstones	355
7.6	Velocity–porosity–clay models: Tosaya’s empirical relations for shaley sandstones	357
7.7	Velocity–porosity–clay models: Castagna’s empirical relations for velocities	358
7.8	V_P – V_S –density models: Brocher’s compilation	359
7.9	V_P – V_S relations	363
7.10	Velocity–density relations	380
7.11	Eaton and Bowers pore-pressure relations	383
7.12	Kan and Swan pore-pressure relations	383
7.13	Attenuation and quality factor relations	384
7.14	Velocity–porosity–strength relations	386
8	Flow and diffusion	389
8.1	Darcy’s law	389
8.2	Viscous flow	394
8.3	Capillary forces	396
8.4	Kozeny–Carman relation for flow	401
8.5	Permeability relations with S_{wi}	407
8.6	Permeability of fractured formations	410
8.7	Diffusion and filtration: special cases	411
9	Electrical properties	414
9.1	Bounds and effective medium models	414
9.2	Velocity dispersion and attenuation	418
9.3	Empirical relations	421
9.4	Electrical conductivity in porous rocks	424
9.5	Cross-property bounds and relations between elastic and electrical parameters	429
	Appendices	437
A.1	Typical rock properties	437
A.2	Conversions	452
A.3	Physical constants	456
A.4	Moduli and density of common minerals	457
A.5	Velocities and moduli of ice and methane hydrate	457

Cambridge University Press
978-0-521-86136-6 - The Rock Physics Handbook, Second Edition: Tools for Seismic Analysis of Porous Media
Gary Mavko, Tapan Mukerji and Jack Dvorkin
Frontmatter
[More information](#)

ix	Contents	
A.6	Physical properties of common gases	468
A.7	Velocity, moduli, and density of carbon dioxide	474
A.8	Standard temperature and pressure	474
	<i>References</i>	479
	<i>Index</i>	503

Cambridge University Press
978-0-521-86136-6 - The Rock Physics Handbook, Second Edition: Tools for Seismic Analysis of Porous Media
Gary Mavko, Tapan Mukerji and Jack Dvorkin
Frontmatter
[More information](#)

Preface to the Second Edition

In the decade since publication of the *Rock Physics Handbook*, research and use of rock physics has thrived. We hope that the First Edition has played a useful role in this era by making the scattered and eclectic mass of rock physics knowledge more accessible to experts and nonexperts, alike.

While preparing this Second Edition, our objective was still to summarize in a convenient form many of the commonly needed theoretical and empirical relations of rock physics. Our approach was to present *results*, with a few of the key assumptions and limitations, and almost never any derivations. Our intention was to create a quick reference and not a textbook. Hence, we chose to encapsulate a broad range of topics rather than to give in-depth coverage of a few. Even so, there are many topics that we have not addressed. While we have summarized the assumptions and limitations of each result, we hope that the brevity of our discussions does not give the impression that application of any rock physics result to real rocks is free of pitfalls. We assume that the reader will be generally aware of the various topics, and, if not, we provide a few references to the more complete descriptions in books and journals.

The handbook contains 101 sections on basic mathematical tools, elasticity theory, wave propagation, effective media, elasticity and poroelasticity, granular media, and pore-fluid flow and diffusion, plus overviews of dispersion mechanisms, fluid substitution, and V_P – V_S relations. The book also presents empirical results derived from reservoir rocks, sediments, and granular media, as well as tables of mineral data and an atlas of reservoir rock properties. The emphasis still focuses on elastic and seismic topics, though the discussion of electrical and cross seismic-electrical relations has grown. An associated website (<http://srb.stanford.edu/books>) offers MATLAB codes for many of the models and results described in the Second Edition.

In this Second Edition, Chapter 2 has been expanded to include new discussions on elastic anisotropy including the Kelvin notation and eigenvalues for stiffnesses, effective stress behavior of rocks, and stress-induced elasticity anisotropy. Chapter 3 includes new material on anisotropic normal moveout (NMO) and reflectivity, amplitude variation with offset (AVO) relations, plus a new section on elastic impedance (including anisotropic forms), and updates on wave propagation in stratified media, and borehole waves. Chapter 4 includes updates of inclusion-based effective media models, thinly layered media, and fractured rocks. Chapter 5 contains

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Gary Mavko, Tapan Mukerji and Jack Dvorkin

Frontmatter

[More information](#)

extensive new sections on granular media, including packing, particle size, sorting, sand–clay mixture models, and elastic effective medium models for granular materials. Chapter 6 expands the discussion of fluid effects on elastic properties, including fluid substitution in laminated media, and models for fluid-related velocity dispersion in heterogeneous poroelastic media. Chapter 7 contains new sections on empirical velocity–porosity–mineralogy relations, V_P – V_S relations, pore-pressure relations, static and dynamic moduli, and velocity–strength relations. Chapter 8 has new discussions on capillary effects, irreducible water saturation, permeability, and flow in fractures. Chapter 9 includes new relations between electrical and seismic properties. The Appendices has new tables of physical constants and properties for common gases, ice, and methane hydrate.

This *Handbook* is complementary to a number of other excellent books. For in-depth discussions of specific rock physics topics, we recommend *Fundamentals of Rock Mechanics*, 4th Edition, by Jaeger, Cook, and Zimmerman; *Compressibility of Sandstones*, by Zimmerman; *Physical Properties of Rocks: Fundamentals and Principles of Petrophysics*, by Schon; *Acoustics of Porous Media*, by Bourbié, Coussy, and Zinszner; *Introduction to the Physics of Rocks*, by Guéguen and Palciauskas; *A Geoscientist's Guide to Petrophysics*, by Zinszner and Pellerin; *Theory of Linear Poroelasticity*, by Wang; *Underground Sound*, by White; *Mechanics of Composite Materials*, by Christensen; *The Theory of Composites*, by Milton; *Random Heterogeneous Materials*, by Torquato; *Rock Physics and Phase Relations*, edited by Ahrens; and *Offset Dependent Reflectivity – Theory and Practice of AVO Analysis*, edited by Castagna and Backus. For excellent collections and discussions of classic rock physics papers we recommend *Seismic and Acoustic Velocities in Reservoir Rocks*, Volumes 1, 2 and 3, edited by Wang and Nur; *Elastic Properties and Equations of State*, edited by Shankland and Bass; *Seismic Wave Attenuation*, by Toksöz and Johnston; and *Classics of Elastic Wave Theory*, edited by Pelissier *et al.*

We wish to thank the students, scientific staff, and industrial affiliates of the Stanford Rock Physics and Borehole Geophysics (SRB) project for many valuable comments and insights. While preparing the Second Edition we found discussions with Tiziana Vanorio, Kaushik Bandyopadhyay, Ezequiel Gonzalez, Youngseuk Keehm, Robert Zimmermann, Boris Gurevich, Juan-Mauricio Florez, Anyela Marcote-Rios, Mike Payne, Mike Batzle, Jim Berryman, Pratap Sahay, and Tor Arne Johansen, to be extremely helpful. Li Teng contributed to the chapter on anisotropic AVOZ, and Ran Bachrach contributed to the chapter on dielectric properties. Dawn Burgess helped tremendously with editing, graphics, and content. We also wish to thank the readers of the First Edition who helped us to track down and fix errata.

And as always, we are indebted to Amos Nur, whose work, past and present, has helped to make the field of rock physics what it is today.

Gary Mavko, Tapan Mukerji, and Jack Dvorkin.