The science of rock physics addresses the relationships between geophysical observations and the underlying physical properties of rocks, such as composition, porosity, and pore fluid content. The *Rock Physics Handbook* distills a vast quantity of background theory and laboratory results into a series of concise, self-contained chapters, which can be quickly accessed by those seeking practical solutions to problems in geophysical data interpretation.

In addition to the wide range of topics presented in the First Edition (including wave propagation, effective media, elasticity, electrical properties, and pore fluid flow and diffusion), this Second Edition also presents major new chapters on granular material and velocity–porosity–clay models for clastic sediments. Other new and expanded topics include anisotropic seismic signatures, nonlinear elasticity, wave propagation in thin layers, borehole waves, models for fractured media, poroelastic models, attenuation models, and cross-property relations between seismic and electrical parameters. This new edition also provides an enhanced set of appendices with key empirical results, data tables, and an atlas of reservoir rock properties expanded to include carbonates, clays, and gas hydrates.

Supported by a website hosting MATLAB routines for implementing the various rock physics formulas presented in the book, the Second Edition of *The Rock Physics Handbook* is a vital resource for advanced students and university faculty, as well as in-house geophysicists and engineers working in the petroleum industry. It will also be of interest to practitioners of environmental geophysics, geomechanics, and energy resources engineering interested in quantitative subsurface characterization and modeling of sediment properties.

**Gary Mavko** received his Ph.D. in Geophysics from Stanford University in 1977 where he is now Professor (Research) of Geophysics. Professor Mavko co-directs the Stanford Rock Physics and Borehole Geophysics Project (SRB), a group of approximately 25 researchers working on problems related to wave propagation in earth materials. Professor Mavko is also a co-author of *Quantitative Seismic Interpretation* (Cambridge University Press, 2005), and has been an invited instructor for numerous industry courses on rock physics for seismic reservoir characterization. He received the Honorary Membership award from the Society of Exploration Geophysicists (SEG) in 2001, and was the SEG Distinguished Lecturer in 2006.

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Tools for Seismic Analysis of Porous Media

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Preface to the Second Edition

In the decade since publication of the *Rock Physics Handbook*, research and use of rock physics has thrived. We hope that the First Edition has played a useful role in this era by making the scattered and eclectic mass of rock physics knowledge more accessible to experts and nonexperts, alike.

While preparing this Second Edition, our objective was still to summarize in a convenient form many of the commonly needed theoretical and empirical relations of rock physics. Our approach was to present results, with a few of the key assumptions and limitations, and almost never any derivations. Our intention was to create a quick reference and not a textbook. Hence, we chose to encapsulate a broad range of topics rather than to give in-depth coverage of a few. Even so, there are many topics that we have not addressed. While we have summarized the assumptions and limitations of each result, we hope that the brevity of our discussions does not give the impression that application of any rock physics result to real rocks is free of pitfalls. We assume that the reader will be generally aware of the various topics, and, if not, we provide a few references to the more complete descriptions in books and journals.

The handbook contains 101 sections on basic mathematical tools, elasticity theory, wave propagation, effective media, elasticity and poroelasticity, granular media, and pore-fluid flow and diffusion, plus overviews of dispersion mechanisms, fluid substitution, and $V_P-V_S$ relations. The book also presents empirical results derived from reservoir rocks, sediments, and granular media, as well as tables of mineral data and an atlas of reservoir rock properties. The emphasis still focuses on elastic and seismic topics, though the discussion of electrical and cross seismic-electrical relations has grown. An associated website (http://srb.stanford.edu/books) offers MATLAB codes for many of the models and results described in the Second Edition.

In this Second Edition, Chapter 2 has been expanded to include new discussions on elastic anisotropy including the Kelvin notation and eigenvalues for stiffnesses, effective stress behavior of rocks, and stress-induced elasticity anisotropy. Chapter 3 includes new material on anisotropic normal moveout (NMO) and reflectivity, amplitude variation with offset (AVO) relations, plus a new section on elastic impedance (including anisotropic forms), and updates on wave propagation in stratified media, and borehole waves. Chapter 4 includes updates of inclusion-based effective media models, thinly layered media, and fractured rocks. Chapter 5 contains
extensive new sections on granular media, including packing, particle size, sorting, sand–clay mixture models, and elastic effective medium models for granular materials. Chapter 6 expands the discussion of fluid effects on elastic properties, including fluid substitution in laminated media, and models for fluid-related velocity dispersion in heterogeneous poroelastic media. Chapter 7 contains new sections on empirical velocity–porosity–mineralogy relations, $V_p–V_s$ relations, pore-pressure relations, static and dynamic moduli, and velocity–strength relations. Chapter 8 has new discussions on capillary effects, irreducible water saturation, permeability, and flow in fractures. Chapter 9 includes new relations between electrical and seismic properties. The Appendices has new tables of physical constants and properties for common gases, ice, and methane hydrate.


We wish to thank the students, scientific staff, and industrial affiliates of the Stanford Rock Physics and Borehole Geophysics (SRB) project for many valuable comments and insights. While preparing the Second Edition we found discussions with Tiziana Vanorio, Kaushik Bandyopadhyay, Ezequiel Gonzalez, Youngseuk Keehm, Robert Zimmermann, Boris Gurevich, Juan-Mauricio Florez, Anyela Marcote-Rios, Mike Payne, Mike Batzle, Jim Berryman, Pratap Sahay, and Tor Arne Johansen, to be extremely helpful. Li Teng contributed to the chapter on anisotropic AVOZ, and Ran Bachrach contributed to the chapter on dielectric properties. Dawn Burgess helped tremendously with editing, graphics, and content. We also wish to thank the readers of the First Edition who helped us to track down and fix errata.

And as always, we are indebted to Amos Nur, whose work, past and present, has helped to make the field of rock physics what it is today.

Gary Mavko, Tapan Mukerji, and Jack Dvorkin.