STRING THEORY AND M-THEORY

A MODERN INTRODUCTION

String theory is one of the most exciting and challenging areas of modern theoretical physics. This book guides the reader from the basics of string theory to very recent developments at the frontier of string theory research.

The book begins with the basics of perturbative string theory, world-sheet supersymmetry, space-time supersymmetry, conformal field theory and the heterotic string, and moves on to describe modern developments, including D-branes, string dualities and M-theory. It then covers string geometry (including Calabi–Yau compactifications) and flux compactifications, and applications to cosmology and particle physics. One chapter is dedicated to black holes in string theory and M-theory, and the microscopic origin of black-hole entropy. The book concludes by presenting matrix theory, AdS/CFT duality and its generalizations.

This book is ideal for graduate students studying modern string theory, and it will make an excellent textbook for a 1-year course on string theory. It will also be useful for researchers interested in learning about developments in modern string theory. The book contains about 120 solved exercises, as well as about 200 homework problems, solutions of which are available for lecturers on a password protected website at www.cambridge.org/9780521860697.

KATRIN BECKER is a Professor of physics at Texas A & M University. She was awarded the Radcliffe Fellowship from Harvard University in 2006 and received the Alfred Sloan Fellowship in 2003.

MELANIE BECKER is a Professor of physics at Texas A & M University. In 2006 she was awarded an Edward, Frances and Shirley B. Daniels Fellowship from the Radcliffe Institute for Advanced Studies at Harvard University. In 2001 she received the Alfred Sloan Fellowship.

JOHN H. SCHWARZ is the Harold Brown Professor of Theoretical Physics at the California Institute of Technology. He is a MacArthur Fellow and a member of the National Academy of Sciences.

This is the first comprehensive textbook on string theory to also offer an up-to-date picture of the most important theoretical developments of the last decade, including the AdS/CFT correspondence and flux compactifications, which have played a crucial role in modern efforts to make contact with experiment. An excellent resource for graduate students as well as researchers in high-energy physics and cosmology.

Nima Arkani-Hamed, Harvard University

An exceptional introduction to string theory that contains a comprehensive treatment of all aspects of the theory, including recent developments. The clear pedagogical style and the many excellent exercises should provide the interested student or researcher a straightforward path to the frontiers of current research.

David Gross, Director of the Kavli Institute for Theoretical Physics, University of California, Santa Barbara and winner of the Nobel Prize for Physics in 2004

Masterfully written by pioneers of the subject, comprehensive, up-to-date and replete with illuminating problem sets and their solutions, *String Theory and M-theory: A Modern Introduction* provides an ideal preparation for research on the current forefront of the fundamental laws of nature. It is destined to become the standard textbook in the subject.

Andrew Strominger, Harvard University

This book is a magnificient resource for students and researchers alike in the rapidly evolving field of string theory. It is unique in that it is targeted for students without any knowledge of string theory and at the same time it includes the very latest developments of the field, all presented in a very fluid and simple form. The lucid description is nicely complemented by very instructive problems. I highly recommend this book to all researchers interested in the beautiful field of string theory.

Cumrun Vafa, Harvard University

This elegantly written book will be a valuable resource for students looking for an entry-way to the vast and exciting topic of string theory. The authors have skillfully made a selection of topics aimed at helping the beginner get up to speed. I am sure it will be widely read.

Edward Witten, Institute for Advanced Study, Princeton, winner of the Fields Medal in 1990

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KATRIN BECKER,
Texas A & M University

MELANIE BECKER,
Texas A & M University

and

JOHN H. SCHWARZ
California Institute of Technology

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521860697

First published 2007

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

ISBN-13 978-0-521-86069-7 hardback
ISBN-10 0-521-86069-5 hardback

To our parents

An Ode to the Unity of Time and Space

Time, ah, time,
how you go off like this!

Physical things, ah, things,
so abundant you are!

The Ruo's waters are three thousand,
how can they not have the same source?

Time and space are one body,
mind and things sustain each other.

Time, o time,
does not time come again?

Heaven, o heaven,
how many are the appearances of heaven!

From ancient days constantly shifting on,
black holes flaring up.

Time and space are one body,
is it without end?

Great indeed
is the riddle of the universe.

Beautiful indeed
is the source of truth.

To quantize space and time
the smartest are nothing.

To measure the Great Universe with a long thin tube
the learning is vast.

Shing-Tung Yau

Contents

Preface

String theory is one of the most exciting and challenging areas of modern theoretical physics. It was developed in the late 1960s for the purpose of describing the strong nuclear force. Problems were encountered that prevented this program from attaining complete success. In particular, it was realized that the spectrum of a fundamental string contains an undesired massless spin-two particle. Quantum chromodynamics eventually proved to be the correct theory for describing the strong force and the properties of hadrons. New doors opened for string theory when in 1974 it was proposed to identify the massless spin-two particle in the string's spectrum with the graviton, the quantum of gravitation. String theory became then the most promising candidate for a quantum theory of gravity unified with the other forces and has developed into one of the most fascinating theories of high-energy physics.

The understanding of string theory has evolved enormously over the years thanks to the efforts of many very clever people. In some periods progress was much more rapid than in others. In particular, the theory has experienced two major revolutions. The one in the mid-1980s led to the subject achieving widespread acceptance. In the mid-1990s a second superstring revolution took place that featured the discovery of nonperturbative dualities that provided convincing evidence of the uniqueness of the underlying theory. It also led to the recognition of an eleven-dimensional manifestation, called M-theory. Subsequent developments have made the connection between string theory, particle physics phenomenology, cosmology, and pure mathematics closer than ever before. As a result, string theory is becoming a mainstream research field at many universities in the US and elsewhere.

Due to the mathematically challenging nature of the subject and the above-mentioned rapid development of the field, it is often difficult for someone new to the subject to cope with the large amount of material that needs to be learned before doing actual string-theory research. One could spend several years studying the requisite background mathematics and physics, but by the end of that time, much more would have already been developed, and one still wouldn't be up to date. An alternative approach is to shorten the learning process so that the student can jump into research more quickly. In this spirit, the aim of this book is to guide the student through the fascinating subject of string theory in one academic year. This book starts with the basics of string theory in the first few chapters and then introduces the reader to some of the main topics of modern research. Since the subject is enormous, it is only possible to introduce selected topics. Nevertheless, we hope that it will provide a stimulating introduction to this beautiful subject and that the dedicated student will want to explore further.

The reader is assumed to have some familiarity with quantum field theory and general relativity. It is also very useful to have a broad mathematical background. Group theory is essential, and some knowledge of differential geometry and basics concepts of topology is very desirable. Some topics in geometry and topology that are required in the later chapters are summarized in an appendix.

The three main string-theory textbooks that precede this one are by Green, Schwarz and Witten (1987), by Polchinski (1998) and by Zwiebach (2004). Each of these was also published by Cambridge University Press. This book is somewhat shorter and more up-to-date than the first two, and it is more advanced than the third one. By the same token, those books contain much material that is not repeated here, so the serious student will want to refer to them, as well. Another distinguishing feature of this book is that it contains many exercises with worked out solutions. These are intended to be helpful to students who want problems that can be used to practice and assimilate the material.

This book would not have been possible without the assistance of many people. We have received many valuable suggestions and comments about the entire manuscript from Rob Myers, and we have greatly benefited from the assistance of Yu-Chieh Chung and Guangyu Guo, who have worked diligently on many of the exercises and homework problems and have carefully read the whole manuscript. Moreover, we have received extremely useful feedback from many colleagues including Keshav Dasgupta, Andrew Frey, Davide Gaiotto, Sergei Gukov, Michael Haack, Axel Krause, Hong Lu, Juan Maldacena, Lubos Motl, Hirosi Ooguri, Patricia Schwarz, Eric Sharpe, James Sparks, Andy Strominger, Ian Swanson, Xi Yin and especially Cumrun Vafa. We have further received great comments and suggestions from many graduate students at Caltech and Harvard University. We thank Ram Sriharsha for his assistance with some of the homework problems and Ketan Vyas for writing up solutions to the homework problems, which will be made available to instructors. We thank Sharlene Cartier and Carol Silber-

stein of Caltech for their help in preparing parts of the manuscript, Simon Capelin of Cambridge U. Press, whose help in coordinating the different aspects of the publishing process has been indispensable, Elisabeth Krause for help preparing some of the figures and Kovid Goyal for his assistance with computer-related issues. We thank Steven Owen for translating from Chinese the poem that precedes the preface.

During the preparation of the manuscript KB and MB have enjoyed the warm hospitality of the Radcliffe Institute for Advanced Studies at Harvard University, the physics department at Harvard University and the Perimeter Institute for theoretical physics. They would like to thank the Radcliffe Institute for Advanced Study at Harvard University, which through its Fellowship program made the completion of this project possible. Special thanks go to the Dean of Science, Barbara Grosz. Moreover, KB would also like to thank the University of Utah for awarding a teaching grant to support the work on this book. JHS is grateful to the Rutgers high-energy theory group, the Aspen Center for Physics and the Kavli Institute for Theoretical Physics for hospitality while he was working on the manuscript.

KB and MB would like to give their special thanks to their mother, Ingrid Becker, for her support and encouragement, which has always been invaluable, especially during the long journey of completing this manuscript. Her artistic talents made the design of the cover of this book possible. JHS thanks his wife Patricia for love and support while he was preoccupied with this project.

Katrin Becker
Melanie Becker
John H. Schwarz

Cambridge University Press
978-0-521-86069-7 - String Theory and M-Theory: A Modern Introduction
Katrin Becker, Melanie Becker, and John H. Schwarz
Frontmatter
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NOTATION AND CONVENTIONS

A	area of event horizon
AdS_D	D-dimensional anti-de Sitter space-time
A_3	three-form potential of $D = 11$ supergravity
b, c	fermionic world-sheet ghosts
b_n	Betti numbers
b_r^μ, $r \in \mathbb{Z} + 1/2$	fermionic oscillator modes in NS sector
B_2 or B	NS–NS two-form potential
c	central charge of CFT
$c_1 = [\mathcal{R}/2\pi]$	first Chern class
C_n	R–R n-form potential
d_m^μ, $m \in \mathbb{Z}$	fermionic oscillator modes in R sector
D	number of space-time dimensions
$F = dA + A \wedge A$	Yang–Mills curvature two-form (antihermitian)
$F = dA + iA \wedge A$	Yang–Mills curvature two-form (hermitian)
$F_4 = dA_3$	four-form field strength of $D = 11$ supergravity
F_m, $m \in \mathbb{Z}$	odd super-Virasoro generators in R sector
$F_{n+1} = dC_n$	$(n + 1)$-form R–R field strength
$g_s = \langle \exp \Phi \rangle$	closed-string coupling constant
G_r, $r \in \mathbb{Z} + 1/2$	odd super-Virasoro generators in NS sector
G_D	Newton's constant in D dimensions
$H_3 = dB_2$	NS–NS three-form field strength
$h^{p,q}$	Hodge numbers
$j(\tau)$	elliptic modular function
$J = ig_{a\bar{b}} dz^a \wedge d\bar{z}^{\bar{b}}$	Kähler form
$\mathcal{J} = J + iB$	complexified Kähler form
k	level of Kac–Moody algebra
K	Kaluza–Klein excitation number
$\mathcal{K}$	Kähler potential
$l_p = 1.6 \times 10^{-33}\,\text{cm}$	Planck length for $D = 4$
ℓ_p	Planck length for $D = 11$
$l_s = \sqrt{2\alpha'}$, $\ell_s = \sqrt{\alpha'}$	string length scale
L_n, $n \in \mathbb{Z}$	generators of Virasoro algebra
$m_p = 1.2 \times 10^{19}\text{GeV}/c^2$	Planck mass for $D = 4$
$M_p = 2.4 \times 10^{18}\text{GeV}/c^2$	reduced Planck mass $m_p/\sqrt{8\pi}$
$M, N, \ldots$	space-time indices for $D = 11$
$\mathcal{M}$	moduli space

N_L, N_R	left- and right-moving excitation numbers
$Q_{\rm B}$	BRST charge
$R = d\omega + \omega \wedge \omega$	Riemann curvature two-form
$R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu}$	Ricci tensor
$\mathcal{R} = R_{a\bar{b}} dz^a \wedge d\bar{z}^{\bar{b}}$	Ricci form
S	entropy
S^a	world-sheet fermions in light-cone gauge GS formalism
$T_{\alpha\beta}$	world-sheet energy–momentum tensor
T_p	tension of p-brane
W	winding number
x^μ, $\mu = 0, 1, \ldots D-1$	space-time coordinates
X^μ, $\mu = 0, 1, \ldots D-1$	space-time embedding functions of a string
$x^{\pm} = (x^0 \pm x^{D-1})/\sqrt{2}$	light-cone coordinates in space-time
x^I, $I = 1, 2, \ldots, D-2$	transverse coordinates in space-time
Z	central charge
α^μ_m, $m \in \mathbb{Z}$	bosonic oscillator modes
α'	Regge-slope parameter
β, γ	bosonic world-sheet ghosts
γ_μ	Dirac matrices in four dimensions
Γ_M	Dirac matrices in 11 dimensions
$\Gamma_{\mu\nu}{}^{\rho}$	affine connection
$\eta(\tau)$	Dedekind eta function
Θ^{Aa}	world-volume fermions in covariant GS formalism
λ^A	left-moving world-sheet fermions of heterotic string
$\Lambda \sim 10^{-120} M_{\rm p}^4$	observed vacuum energy density
σ^α, $\alpha = 0, 1, \ldots, p$	world-volume coordinates of a p-brane
$\sigma^0 = \tau$, $\sigma^1 = \sigma$	world-sheet coordinates of a string
$\sigma^{\pm} = \tau \pm \sigma$	light-cone coordinates on the world sheet
$\sigma^\mu_{\alpha\dot{\beta}}$	Dirac matrices in two-component spinor notation
Φ	dilaton field
$\chi(M)$	Euler characteristic of M
ψ^μ	world-sheet fermion in RNS formalism
Ψ_M	gravitino field of $D = 11$ supergravity
$\omega_\mu{}^{\alpha}{}_{\beta}$	spin connection
Ω	world-sheet parity transformation
Ω_n	holomorphic n-form

- $\hbar = c = 1$.
- The signature of any metric is 'mostly +', that is, $(-, +, \ldots, +)$.
- The space-time metric is $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$.
- In Minkowski space-time $g_{\mu\nu} = \eta_{\mu\nu}$.
- The world-sheet metric tensor is $h_{\alpha\beta}$.
- A hermitian metric has the form $ds^2 = 2g_{a\bar{b}} dz^a d\bar{z}^{\bar{b}}$.
- The space-time Dirac algebra in $D = d+1$ dimensions is $\{\Gamma_\mu, \Gamma_\nu\} = 2g_{\mu\nu}$.
- $\Gamma^{\mu_1\mu_2\cdots\mu_n} = \Gamma^{[\mu_1}\Gamma^{\mu_2}\cdots\Gamma^{\mu_n]}$.
- The world-sheet Dirac algebra is $\{\rho_\alpha, \rho_\beta\} = 2h_{\alpha\beta}$.
- $|F_n|^2 = \frac{1}{n!} g^{\mu_1\nu_1} \cdots g^{\mu_n\nu_n} F_{\mu_1\ldots\mu_n} F_{\nu_1\ldots\nu_n}$.
- The Levi–Civita tensor $\varepsilon^{\mu_1\cdots\mu_D}$ is totally antisymmetric with $\varepsilon^{01\cdots d} = 1$.