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0521859794 - Mechanics of Solids and Materials
Robert J. Asaro and Vlado A. Lubarda
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MECHANICS OF SOLIDS AND MATERIALS

Mechanics of Solids and Materials intends to provide a modern and integrated treatment of the foundations of solid mechanics as applied to the mathematical description of material behavior. The book blends both innovative topics (e.g., large strain, strain rate, temperature, time-dependent deformation and localized plastic deformation in crystalline solids, and deformation of biological networks) and traditional topics (e.g., elastic theory of torsion, elastic beam and plate theories, and contact mechanics) in a coherent theoretical framework. This, and the extensive use of transform methods to generate solutions, makes the book of interest to structural, mechanical, materials, and aerospace engineers. Plasticity theories, micromechanics, crystal plasticity, thin films, energetics of elastic systems, and an overall review of continuum mechanics and thermodynamics are also covered in the book.

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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
40 West 20th Street, New York, NY 10011-4211, USA
www.cambridge.org
Information on this title: www.cambridge.org/9780521859790

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First published 2006

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication Data

Asaro, Robert J.
Mechanics of solids and materials / Robert J. Asaro and Vlado A. Lubarda.
p. cm.

Includes bibliographical references and index.

ISBN 0-521-85979-4 (hardback)

1. Mechanics, Applied. I. Lubarda, Vlado A. II. Title.

TA350.A735 2006

620.1 – dc22 2005025722

ISBN-13 978-0-521-85979-0 hardback

ISBN-10 0-521-85979-4 hardback

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Cambridge University Press
0521859794 - Mechanics of Solids and Materials
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Preface

This book is written for graduate students in solid mechanics and materials science and should also be useful to researchers in these fields. The book consists of eight parts. Part 1 covers the mathematical preliminaries used in later chapters. It includes an introduction to vectors and tensors, basic integral theorems, and Fourier series and integrals. The second part is an introduction to nonlinear continuum mechanics. This incorporates kinematics, kinetics, and thermodynamics of a continuum and an application to nonlinear elasticity. Part 3 is devoted to linear elasticity. The governing equations of the three-dimensional elasticity with appropriate specifications for the two-dimensional plane stress and plane strain problems are given. The applications include the analyses of bending of beams and plates, torsion of prismatic rods, contact problems, semi-infinite media, and three-dimensional isotropic and anisotropic elastic problems. Part 4 is concerned with micromechanics, which includes the analyses of dislocations and cracks in isotropic and anisotropic media, the well-known Eshelby elastic inclusion problem, energy analyses of imperfections and configurational forces, and micropolar elasticity. In Part 5 we analyze dislocations in bimetals and thin films, with an application to the study of strain relaxation in thin films and stability of planar interfaces. Part 6 is devoted to mathematical and physical theories of plasticity and viscoplasticity. The phenomenological or continuum theory of plasticity, single crystal, polycrystalline, and laminate plasticity are presented. The micromechanics of crystallographic slip is addressed in detail, with an analysis of the nature of crystalline deformation, embedded in its tendency toward localized plastic deformation. Part 7 is an introduction to biomechanics, particularly the formulation of governing equations of the mechanics of solids with a growing mass and constitutive relations for biological membranes. Part 8 is a collection of 180 solved problems covering all chapters of the book. This is included to provide additional development of the basic theory and to further illustrate its application.

The book is transcribed from lecture notes we have used for various courses in solid mechanics and materials science, as well as from our own published work. We have also consulted and used major contributions by other authors, their research work and written books, as cited in the various sections. As such, this book can be used as a textbook for a sequence of solid mechanics courses at the graduate level within mechanical, structural, aerospace, and materials science engineering programs. In particular, it can be used for

the introduction to continuum mechanics, linear and nonlinear elasticities, theory of dislocations, fracture mechanics, theory of plasticity, and selected topics from thin films and biomechanics. At the end of each chapter we offer a list of recommended references for additional reading, which aid further study and mastering of the particular subject.

Standard notations and conventions are used throughout the text. Symbols in bold, both Latin and Greek, denote tensors or vectors, the order of which is indicated by the context. Typically the magnitude of a vector will be indicated by the name symbol unbolded. Thus, for example, \mathbf{a} or \mathbf{b} indicate two vectors or tensors. If \mathbf{a} and \mathbf{b} are vectors, then the scalar product, *i.e.*, the dot product between them is indicated by a single dot, as $\mathbf{a} \cdot \mathbf{b}$. Since \mathbf{a} and \mathbf{b} are vectors in this context, the scalar product is also $ab \cos \theta$, where θ is the angle between them. If \mathbf{A} is a higher order tensor, say second-order, then the dot product of \mathbf{A} and \mathbf{a} produces another vector, *viz.*, $\mathbf{A} \cdot \mathbf{a} = \mathbf{b}$. In the index notation this is expressed as $A_{ij}a_j = b_i$. Unless explicitly stated otherwise, the summation convention is adopted whereby a repeated index implies summation over its full range. This means, accordingly, that the scalar product of two vectors as written above can also be expressed as $a_j b_j = \phi$, where ϕ is the scalar result. Two additional operations are introduced and defined in the text involving double dot products. For example, if \mathbf{A} and \mathbf{B} are two second-rank tensors, then $\mathbf{A} : \mathbf{B} = A_{ij} B_{ij}$ and $\mathbf{A} \cdot \mathbf{B} = A_{ij} B_{ji}$. For higher order tensors, similar principles apply. If \mathbf{C} is a fourth-rank tensor, then $\mathbf{C} : \mathbf{e} \Rightarrow C_{ijkl} e_{kl} = \{\dots\}_{ij} \cdot \cdot$.

In finite vector spaces we assume the existence of a convenient set of *basis vectors*. Most commonly these are taken to be orthogonal and such that an arbitrary vector, say \mathbf{a} , can be expressed *wrt* its components along these base vectors as $\mathbf{a} = a_1 \mathbf{e}_1 + a_2 \mathbf{e}_2 + a_3 \mathbf{e}_3$, where $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ are the orthogonal set of base vectors in question. Other more or less standard notations are used, *e.g.*, the left- or right-hand side of an equation is referred to as the *lhs*, or *rhs*, respectively. The commonly used phrase with respect is abbreviated as *wrt*, and so on.

We are grateful to many colleagues and students who have influenced and contributed to our work in solid mechanics and materials science over a long period of time and thus directly or indirectly contributed to our writing of this book. Specifically our experiences at Stanford University, Brown University, UCSD, Ford Motor Company (RJA), Ohio State University (RJA), University of Montenegro (VAL), and Arizona State University (VAL) have involved collaborations that have been of great professional value to us. Research funding by NSF, the U.S. Army, the U.S. Air Force, the U.S. Navy, DARPA, the U.S. DOE, Alcoa Corp., and Ford Motor Co. over the past several decades has greatly facilitated our research in solid mechanics and materials science. We are also most grateful to our families and friends for their support during the writing of this book

La Jolla, California
 July, 2005

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