Ocean Engineering Mechanics provides an introduction to water waves and wave-structure interactions for fixed and floating bodies. The author provides a foundation in wave mechanics, including a thorough discussion of linear and nonlinear regular waves, and he presents methods for determining the averaged properties of random waves. He then explains applications to engineering situations in coastal zones. This introduction to the coastal engineering aspects of wave mechanics includes an introduction to shore protection. The book also covers the basics of wave-structure interactions for situations involving rigid structures, compliant structures, and floating bodies in regular and random seas. The final chapters deal with the various analytical methods available for the engineering analyses of wave-induced forces and motions of floating and compliant structures in regular and random seas. An introduction to soil-structure interactions is also included. This book can be used for both introductory and advanced courses in ocean engineering mechanics.

Michael E. McCormick is currently the Corbin A. McNeill Professor in the Department of Naval Architecture and Ocean Engineering at the U.S. Naval Academy. He is a Fellow of the Marine Technology Society, the American Society of Mechanical Engineers, and the American Society of Civil Engineers. In 1976, he became the co-editor of the journal Ocean Engineering and remained so for thirty years. Prior to that, he was the editor of the Marine Technology Society Journal. Professor McCormick was the first recipient of the U.S. Naval Academy Alumni Award for Research Excellence, and he was also awarded the U.S. Navy Meritorious Civilian Service Award and the U.S. Navy Superior Civilian Service Award. He is the author of Ocean Engineering Wave Mechanics and Ocean Wave Energy Conversion. Professor McCormick received a Ph.D. in mechanical engineering from the Catholic University of America and a Ph.D. in civil engineering and a Sc.D. in engineering science from Trinity College, Dublin.
Ocean Engineering Mechanics

WITH APPLICATIONS

Michael E. McCormick
United States Naval Academy
I dedicate this book to my dear wife, Mary Ann, and to my family for their love and support, and to my dear friend and colleague, Professor Rameswar Bhattacharyya, for his never-ending support and encouragement, and to the late Professor Manley St. Denis for all he taught me in the early days of my career.
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Preface

It has been more than three decades since my first book on ocean engineering, *Ocean Engineering Wave Mechanics*. My purpose in writing that book was to give ocean engineering students and ocean technologists an introduction to the mechanics of water waves, and to present and demonstrate the analytical techniques used in wave-structure interaction problems. Since the 1973 publication of that book, ocean technology has been one of the most rapidly advancing engineering fields. The purpose of this book is to present both fundamental and advanced techniques in the analyses of both water waves and wave-structure interactions. The classical analytical works in the areas of wave mechanics are discussed in detail so that the reader can follow the lines of thought of the masters who produced these classic analyses.

Most of the material presented herein is for readers with a basic education in applied mechanics, including fluid mechanics or hydraulics and applied mathematics. The material is presented so that the reader can immediately apply the various analytical techniques to problems of interest. To this end, examples are presented in each section. Certain topics, such as the cnoidal theory, are of an advanced analytical nature and, as such, are more appropriate for postgraduate education. Following these topics are examples designed to demonstrate the application of these advanced analytical methods.

I would like to express my thanks to Dr. David R. B. Kraemer of the University of Wisconsin, Platteville, for his help and advice during the preparation of much of this book. Dr. Kraemer’s expertise in computational techniques in fluid and applied mechanics and his willingness to share that expertise were of great value. In addition, my sincere appreciation goes to Mr. Jeffrey Cerquetti of Johnson, Mirmiran & Thompson, Inc. and Dr. Patrick J. Hudson of the Applied Physics Laboratory of Johns Hopkins University for their advice and expertise in numerous areas of hydrodynamics.

Two of my friends and former students at the U.S. Naval Academy have made my situation conducive to book writing. Those are Mr. Robert Murtha and Mr. Bernard Bailey. Each of these fine gentlemen knows how they have contributed.

A special thanks is given to Professor Jacek Mostwin of Johns Hopkins Medical Institutions. Because of his consideration and skills, I was able to complete this book.
My sincere appreciation is given to my long-time friends and colleagues, Dr. Ronald Gularte and Mrs. Alice Gularte, for proofreading the manuscript. They provided guidance and editorial comments that were invaluable.

Finally, I would like to express my appreciation to my dear friend and colleague, Professor Rameswar Bhattacharyya of the U.S. Naval Academy, for his suggestions and encouragement. It has been my good fortune to be able to work closely with Professor Bhattacharyya for more than thirty-eight years, and I have profited greatly from the experience.

Michael E. McCormick
Annapolis, Maryland
December 2008
Notation

General

\( a_w \)  
added mass (kg)

\( a \)  
cylindrical radius (m)

\( A_w \)  
added-mass moment of inertia (N-m-s\(^2\)/rad)

\( A \)  
area (m\(^2\))

\( b_{t,p,v} \)  
linear radiation, power take-off, and viscous damping coefficients (N-s/m)

\( b \)  
half-breadth and crest width (m)

\( b_v \)  
nonlinear viscous damping coefficient (N-s\(^2\)/m\(^2\))

\( B \)  
breadth of a structure into the page, or beam of a floating body (m)

\( c \)  
celerity vector (m/s)

\( c_g \)  
group velocity (m/s)

\( C_d \)  
drag coefficient

\( D \)  
diameter (m)

\( d \)  
draft of a fixed or floating structure (m)

\( e \)  
2.7182818 . . .

\( f \)  
frequency (Hz)

\( F \)  
force (N)

\( g \)  
gravitational constant (\( \simeq \) 9.81 m/s\(^2\))

\( h \)  
water depth (m)

\( H \)  
traveling wave height (m)

\( H_0^{(1,2)}(\cdot) \)  
Hankel function of the first and second kinds

\( H \)  
standing wave height (m)

\( i \)  
\((-1)^{1/2}\)

\( i,j,k \)  
\( x,y,z \)-unit vectors

\( I_e \)  
second moment of area with respect to the e-axis (m\(^4\))

\( I_e \)  
body mass-moment of inertia with respect to the e-axis (N-m-s\(^2\)/rad)

\( I_n \)  
modified Bessel function of the first kind

\( J_n(\cdot) \)  
Bessel function of the first kind

\( K_C \)  
Keulegan-Carpenter number

\( K_n(\cdot) \)  
modified Bessel function of the second kind

\( K_r \)  
refraction coefficient in eq. 6.85

\( K_R \)  
reflection coefficient in eq. 6.23
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<tr>
<td>$k$</td>
<td>wave number, $2\pi/\lambda$ (1/m)</td>
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<tr>
<td>$L$</td>
<td>body length (m)</td>
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<tr>
<td>$m$</td>
<td>body mass (kg)</td>
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<tr>
<td>$M$</td>
<td>moment (N-m)</td>
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<tr>
<td>$n$</td>
<td>order of Bessel function and index number</td>
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<td>$n$</td>
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<td>$p$</td>
<td>pressure (N/m$^2$)</td>
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<tr>
<td>$P$</td>
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<td>$r$</td>
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<td>SPM</td>
<td><em>Shore Protection Manual</em> (U.S. Army, 1984)</td>
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<tr>
<td>SWL</td>
<td>still-water level</td>
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<td>$T$</td>
<td>line tension (N)</td>
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<td>$T$</td>
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<tr>
<td>$u,v,w$</td>
<td>$x,y,z$-velocity components (m/s)</td>
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<td>$W$</td>
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<td>$x,y$</td>
<td>inertial horizontal coordinates (m)</td>
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<tr>
<td>$Y_n(\cdot)$</td>
<td>Bessel function of the second kind</td>
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<td>inertial vertical coordinate (m)</td>
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<tr>
<td>$Z$</td>
<td>local vertical coordinate (m)</td>
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<tr>
<td>$\Re$</td>
<td>real part of a quantity</td>
</tr>
<tr>
<td>$\Im$</td>
<td>imaginary part of a quantity</td>
</tr>
<tr>
<td>$\eta$</td>
<td>free-surface displacement (m)</td>
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<tr>
<td>$\theta$</td>
<td>angular coordinate (radians, degrees)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wavelength (m)</td>
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<tr>
<td>$\mu$</td>
<td>dynamic viscosity (N-s/m$^2$)</td>
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<tr>
<td>$\nu$</td>
<td>kinematic viscosity (m$^2$/s)</td>
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<tr>
<td>$\rho$</td>
<td>mass density (kg/m$^3$)</td>
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<tr>
<td>$\phi$</td>
<td>velocity potential (m$^2$/s)</td>
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<tr>
<td>$\psi$</td>
<td>two-dimensional stream function (m$^2$/s)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>circular wave frequency (rad/s)</td>
</tr>
</tbody>
</table>

**Subscripts**

- **avg**: average value
- **$o$**: amplitude
- **max**: maximum value
- **0**: deep-water wave properties
Chapter 1

\[ E(T_o) \] wave-energy spectral density (m^2/s)
\[ F \] fetch (m, km)
\[ F_{min} \] minimum fetch (m, km)
\[ T_o \] modal period (s)
\[ U \] wind speed (m/s, km/hr)
\[ X_D \] developing length of a wind-generated sea (m, km)

Chapter 2

\[ C_p \] pressure coefficient in Figure 2.11
\[ f(t) \] see eq. 2.70
\[ f_v \] vortex-shedding frequency (Hz)
\[ F_B \] buoyant force (N)
\[ F_r \] Froude number
\[ j \] index number
\[ L_{m,p} \] model and prototype lengths (m)
\[ m,n \] indices
\[ n_\zeta \] scale factor (\( \zeta = F, L, p, P, t, V \))
\[ M_{+,,-} \] three-dimensional source and sink strengths (m^3/s)
\[ M_{+,,-} \] line source and sink strengths (m^2/s)
\[ N \] maximum index number
\[ R_{oi} \] outer and inner diameters (m)
\[ R,\beta,\Theta \] spherical coordinates
\[ S_\ell \] Strouhal number based on length \( \ell \)
\[ \hat{S} \] safety factor
\[ V_0 \] upstream velocity (m/s)
\[ \gamma \] volume (m^3)
\[ \gamma \] weight density (N/m^3)
\[ \Gamma \] circulation (m^2/s)
\[ \sigma_{1,2,u} \] axial, hoop, and ultimate stresses (N/m^2)
\[ \Phi \] three-dimensional velocity potential (m^3/s)
\[ \Psi \] three-dimensional stream function (m^3/s)

Chapter 3

\[ \alpha,\beta \] arbitrary phase angles in eqs. 3.12 and 3.13
\[ C \] arbitrary constant
\[ E \] total energy (N-m)
\[ E_p \] potential energy (N-m)
\[ E_k \] kinetic energy (N-m)
\[ F_{A,B}(\lambda) \] wavelength functions in eq. 3.34
\[ K_S \] shoaling coefficient
\[ N \] sea-bed normal unit vector
\[ P \] energy-flux vector (N-m/s)
\[ T(t) \] time function in eq. 3.9
\[ U,W \] standing-wave horizontal and vertical particle velocity components (m/s)
\[ X(x) \] spatial function in eq. 3.9
## Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$\Delta F$</td>
<td>see eq. 3.35</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>power-conversion efficiency</td>
</tr>
<tr>
<td>$\xi(x,t)$</td>
<td>translating horizontal displacement (m)</td>
</tr>
<tr>
<td>$\zeta(x,t)$</td>
<td>translating vertical displacement (m)</td>
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<tr>
<td>$\Phi$</td>
<td>standing-wave velocity potential (m²/s)</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>standing-wave stream function (m³/s)</td>
</tr>
</tbody>
</table>

### Subscripts

- $wc$: water column

## Chapter 4

- A: integration constant in eq. 4.93
- B: perturbation constant in eq. 4.94
- C: free-surface constant value (m)
- $E'$: energy per crest width (N-m/m)
- $E()$: complete elliptic integral of the second kind
- $f_j, F_j$: see eqs. 4.77 and 4.78
- $K()$: complete elliptic integral of the first kind
- $K$: total energy per unit volume (N-m/m³)
- $t$: height of the trough above the sea (m)
- $S$: bottom coordinate ($= z + h$)
- $U_R$: Ursell parameter; see Figure 4.1
- $U,W$: Stokian horizontal and vertical velocity components (m/s)
- $U_{con}$: convective velocity (m/s)
- $\alpha$: crest angle from the vertical (radians, degrees)
- $\Gamma$: free-surface function in eq. 4.88
- $\varepsilon$: perturbation constant in eqs. 4.8 and 4.9
- $\eta$: free-surface displacement from the wave trough (m)
- $M$: parameter
- $\xi$: horizontal particle convection length (m)

### Subscripts

- $b$: breaking condition
- $c$: at a wave crest
- $\alpha$: angular component
- $R$: radial component
- $\theta$: properties at an origin of a coordinate system

## Chapter 5

- $a$: wave amplitude (m)
- $A$: Weibull parameter in eq. 5.27
- $A$: generic spectral parameter in eq. 5.42
- $A_o$: coefficient in eq. 5.89
- $B$: Weibull parameter in eq. 5.27
Notation

- $B$: generic spectral parameter in eq. 5.42
- $E$: energy per free-surface area (N-m/m²)
- $E$: energy intensity (N-m/m)
- $F$: fetch (m, km)
- $G()$: spreading function in eq. 5.92
- $I$: maximum wave period index
- $j$: wave height index
- $J$: maximum wave height index
- $M$: dimensionless wave height ratio
- $m$: Weibull parameter in eq. 5.27
- $m_H$: shape factor in eq. 5.102
- $m_T$: shape factor in eq. 5.103
- $m$: generic spectral parameter in eq. 5.42
- $n$: generic spectral parameter in eq. 5.42
- $n_{i,j}$: number of observed waves corresponding to the index $j$
- $N$: expected number of observed waves
- $p()$: probability density function
- $P()$: cumulative frequency of occurrence
- $P()$: cumulative probability of occurrence
- $R$: radial coordinate from a wave crest (m)
- $s$: spreading parameter
- $S(T)$: wave spectral density (m²/s)
- $t_D$: duration (hours)
- $U_{10,19.5}$: wind speed at heights of 10 m and 19.5 m above the still-water level (km/hr)
- $Z$: arbitrary variable
- $\alpha$: equivalent Mach angle in Figure 5.17 (radians, degrees)
- $\beta$: wind angle from onshore direction (radians, degrees)
- $\Gamma_2$: the gamma function evaluated at $(m+2)/m$
- $\delta$: boundary layer thickness in Figure 5.16 (m)
- $\theta$: angle from wind direction in the horizontal plane (radians, degrees)
- $\Theta$: angle from true north (radians, degrees)

Subscripts

- $avg$: averaged
- $B$: Bretschneider spectral density
- $DF$: critical duration
- $fds$: fully developed sea
- $h$: at a finite water depth
- $H_{j,J}$: property of the $j$ or $J$ wave height
- $I$: wave component index
- $J$: generic wave spectral density
- $J$: direction index
- $JON$: JONSWAP spectral density
- $LT$: long-term
- $rms$: root-mean-square
- $s$: significant wave
### Notation

- **$z$**: zero up-crossing period
- **$\pm$**: maximum and negative wave amplitudes

### Chapter 6

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<td>incident and reflected amplitude coefficients</td>
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<td>$A,B$</td>
<td>coefficients in eq. 6.132</td>
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<td>wave crest width (m)</td>
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<td>$B_B$</td>
<td>boundary value of amplitude function</td>
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<td>$B_n(\cdot)$</td>
<td>generic Bessel function in eq. 6.99d</td>
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<tr>
<td>$B$</td>
<td>breadth of structure (m)</td>
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<tr>
<td>$B_F$</td>
<td>complex coefficient in eq. 6.111</td>
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<tr>
<td>$B_G$</td>
<td>complex coefficient in eq. 6.113</td>
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<tr>
<td>$B_{FG}$, $B_{FBG}$</td>
<td></td>
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<td>$C$</td>
<td>line-integration path</td>
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<tr>
<td>$C,D$</td>
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<td>Fresnel integral</td>
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<td>energy per unit crest length over the $n$th step of the shoal in eq. 6.78</td>
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<td>$E(\cdot)$</td>
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<td>arbitrary spatial function in eq. 6.101</td>
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<td>$H'$</td>
<td>pure shoaling wave height (m)</td>
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<td>$K$</td>
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<td>$m$</td>
<td>slope of structural face</td>
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<td>$N$</td>
<td>wall porosity</td>
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<td>$N$</td>
<td>number of quasi-steps on the shoal in Figure 6.13</td>
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<td>normal unit vector on the sea bed</td>
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<td>$s$</td>
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<tr>
<td>ε</td>
<td>angle between wave direction and wall (radians, degrees)</td>
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<tr>
<td>ε</td>
<td>phase angles defined in eq. 6.140 (radians, degrees)</td>
</tr>
<tr>
<td>θ</td>
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<tr>
<td>Θ</td>
<td>angle measured from the normal on the leeward side of the seawall (radians, degrees), as in Figure 6.21</td>
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<td>Λ</td>
<td>alongshore component of the wavelength in eq. 6.20 (m)</td>
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<tr>
<td>ν</td>
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<tr>
<td>σ</td>
<td>phase angle in eq. 6.37 (radians, degrees)</td>
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<tr>
<td>Q_{sed}</td>
<td>volume-rate of sediment transport</td>
</tr>
<tr>
<td>Σ_N</td>
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</tr>
<tr>
<td>φ</td>
<td>spatially dependent velocity potential (m²/s)</td>
</tr>
<tr>
<td>Φ</td>
<td>spatially and temporally dependent velocity potential (m²/s)</td>
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**Subscripts**

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<th>Subscript</th>
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<td>properties downwave of the shoal (m³/s)</td>
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<tr>
<td>A</td>
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<td>B</td>
<td>property in Region B</td>
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<td>C</td>
<td>property in Region C</td>
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<td>m</td>
<td>summation index</td>
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<tr>
<td>R</td>
<td>reflected properties</td>
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<td>T</td>
<td>transmitted properties</td>
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**Chapter 7**

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<td>A</td>
<td>parameter in eq. 7.2</td>
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</tr>
<tr>
<td>B</td>
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<tr>
<td>C_{y,e}</td>
<td>constants associated with stresses</td>
</tr>
<tr>
<td>C_p</td>
<td>porosity factor in eq. 7.15, equal to (1 - \frac{\sqrt{\text{void}}}{\sqrt{\text{total}}})</td>
</tr>
<tr>
<td>D</td>
<td>local rate of energy dissipation (N\cdot m^{-1}\cdot s^{-1})</td>
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<tr>
<td>D_{50}</td>
<td>mean sediment diameter (m, cm, mm)</td>
</tr>
<tr>
<td>E_{1,2}</td>
<td>constants in eqs. 7.71 and 7.72</td>
</tr>
<tr>
<td>f_{\mu}</td>
<td>friction factor in eq. 7.58</td>
</tr>
<tr>
<td>H</td>
<td>pure shoaling wave height (m)</td>
</tr>
<tr>
<td>K</td>
<td>proportionality constant in eq. 7.41</td>
</tr>
<tr>
<td>m</td>
<td>slope of the sea bed</td>
</tr>
<tr>
<td>P_e</td>
<td>energy flux line intensity in eq. 7.51 (N/s)</td>
</tr>
<tr>
<td>R_u</td>
<td>runup in Figure 7.1 (m)</td>
</tr>
<tr>
<td>[s]</td>
<td>equivalent radiation stress matrix (N/m)</td>
</tr>
<tr>
<td>S_{XX,YY}</td>
<td>equivalent components of radiation stress in eq. 7.29 (N/m)</td>
</tr>
<tr>
<td>S_{XX}</td>
<td>principal component of radiation stress in eq. 7.20 (N/m)</td>
</tr>
</tbody>
</table>
### Notation

- $S_{YY}$: transverse component of radiation stress in eq. 7.23 (N/m)
- $[S]$: radiation stress matrix (N/m)
- $T_e$: function of the eddy viscosity in eq. 7.54 (N/m)
- $U, V, W$: particle velocity components with respect to the wave direction in Figure 7.7 (m/s)
- $V$: velocity vector in Figure 7.7
- $V_{l}$: alongshore (or longshore) velocity in Figure 7.6 (m/s)
- $x_s$: maximum set-up in eq. 7.48 (m)
- $X, Y, Z$: wave coordinate system in Figure 7.7 (m)
- $\gamma$: experimental proportionality constant in eq. 7.44
- $\kappa$: parametric constant in eq. 7.49 (1/m)
- $\mu_e$: eddy viscosity in eq. 7.59 (N-s/m²)
- $\xi$: surf similarity parameter in eq. 7.9
- $\tau_y$: time-averaged bed shear stress (N/m²)
- $\tau_{sys}$: radiation stress (N/m²)
- $\tau_e$: effective eddy shear stress in eq. 7.61 (N/m²)
- $x$: $x/x_b$
- $F$: momentum flux in eq. 7.35 (N/m)

### Subscripts

- $b$: breaking condition
- $ts$: longshore property
- $s$: surf-zone conditions
- $S$: maximum set-up
- $sed$: sediment property
- $\varepsilon$: eddy viscosity property

### Chapter 8

- $B_T$: cap width of the trunk (m)
- $f$: expected number of failures
- $F_r$: Froude number
- $H'$: pure shoaling wave height (m)
- $h_T$: height of the breakwater (m)
- $k_A$: layer coefficient in eq. 8.5
- $K_{DT}$: stability coefficient in eq. 8.1
- $L_{0,1,\ldots}$: alongshore separation distance between groins 0 and 1, 1 and 2, \ldots (m)
- $L_g$: groin length in Figure 8.2 (m)
- $m$: slope of the sea bed
- $m$: shape parameter in eq. 8.14
- $n$: number of primary stone layers
- $n_L$: length scale factor in eq. 8.11
- $n_t$: time scale factor in eq. 8.13
- $n_v$: velocity scale factor in eq. 8.13
- $N$: number of cap stones of the breakwater trunk
- $N$: number of armor stones of a breakwater
Notation

\( N_{100} \)  expected number of observed waves over 100 years
\( P \)  probability of failure (= 1 – \( R \), as in Example 8.4)
\( Q_{sed} \)  volume rate of sediment transport (m³/s)
\( r_T \)  total thickness of the primary armor stone layer (m)
\( R \)  reliability, as defined in Example 8.4
\( V_\ell \)  alongshore (or longshore) velocity in Figure 8.2 (m/s)
\( W_{1T} \)  armor stone weight (N)
\( W_{2T} \)  shield stone weight (N)
\( W_{3T} \)  foundation stone weight (N)
\( W_{4T} \)  toe stone weight (N)
\( \rho_{stone} \)  mass density of the stone material (kg/m³)
\( \varepsilon \)  angle of the breakwater weather face with respect to the horizontal (radians, degrees)

Subscripts

\( D \)  design condition
\( \text{avg} \)  average value
\( \text{max} \)  maximum value
\( \text{ref} \)  reference value
\( \text{rms} \)  root-mean-square value
\( T \)  breakwater trunk properties

Chapter 9

\( a \)  semi-length of a Lewis form (m)
\( a_e \)  radius of a circle or circular cylinder (m)
\( A \)  semi-length of a rectangular caisson (m)
\( A_d \)  projected area (m²)
\( A_m \)  see eq. 9.141
\( A_1 \)  Lewis transformation constant (m²)
\( A_3 \)  Lewis transformation constants (m⁴)
\( b \)  semi-width of a Lewis form (m)
\( B \)  semi-width of a rectangular caisson (m)
\( B_{mn} \)  see eq. 9.141
\( B_1 \)  \( 2Y_{\text{max}}|a=1 \)
\( C \)  contour enclosing an area \( S \) (m)
\( C_i \)  inertial coefficient in eq. 9.26
\( C_d \)  drag coefficient
\( C_M \)  mass coefficient defined in Figure 9.17 and eq. 9.79 for a circular caisson
\( C_M \)  mass coefficient defined in eq. 9.80 for a rectangular caisson
\( f(\cdot) \)  arbitrary function
\( E_m \)  constant associated with the index \( m \) in eq. 9.63
\( F_d \)  drag force (N)
\( F_w \)  wave-induced pressure force on the wall (N)
\( F_{\text{W}} \)  wave-induced pressure force on the wall, excluding higher-order terms in \( \eta_w \) in eq. 9.5 (N)
\( F^* \)  non-dimensional force defined in eq. 9.151
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Notation

**Subscripts**

- **B** B Bretschneider wave spectral density
- **brace** property on a cross-brace
- **cp** property at the center of pressure
- **CL** property on the centerline of the cylinder
- **G** Garrett force and moment
- **I** incident wave property
- **j,k,l** caisson indices
- **m,n** summation indices
- **MF** MacCamy-Fuchs property
- **PM** Pierson-Moskowitz wave spectral density
- **S** scattered wave property
- **s** significant wave property
- **w** property at the wall
- **X,Y** properties in the x- and y-directions
- **□** properties associated with the rectangular caisson

**Chapter 10**

- **a_p** damping plate radius in Figure 10.10 (m)
- **ALP** articulated-leg platform
- **A_d** projected area for drag (m²)
- **A_wp** waterplane area for drag (m²)
- **b_c** heaving critical damping coefficient (N-s/m)
- **b_p** power take-off damping coefficient (N-s/m)
- **b_r** radiation damping coefficient (N-s/m)
- **b_v** equivalent linear viscous damping coefficient (N-s/m)
- **b_z** combine damping coefficient (N-s/m) in eq. 10.25
- **b_n** nonlinear viscous damping coefficient (N-s²/m²)
- **C** constant in eq. 10.55
- **C_d** drag coefficient
- **H(ω)** amplitude response function in eq. 10.51
- **H(ω)*** complex conjugate of (ω)
- **k_s** spring constant (N/m)
- **l_s** relaxed mooring line length (m)
- **N** power of the velocity in eq. 10.1
- **N** number of mooring lines
- **N_M** number of observations over **M** years
- **P(Z)** probability density (1/m)
- **P(·)** probability of an event
- **P_z** power absorbed (N-m/s)
- **S_f(T)** wave spectral density in eq. 10.55 (m²-s)
- **S_x(T)** response spectral density in eq. 10.55 (m²-s)
- **T** time interval (s)
- **T_S** mooring line tension (N)
- **T_{nc}** natural heaving period (s)
- **V_z** heaving velocity vector of a body (m/s)
- **w_cw** capture width in eq. 10.46 (m)
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**Subscripts**

- ABS absolute value
- avg averaged property
- b damping property
- B Bretschneider spectral density
- dyn dynamic pressure
- j amplitude number
- J generic spectral density
- n natural frequency property
- N wave index in eq. 10.60
- rms root-mean-square value
- S property of mooring line spring effect
- \( x,y,z \) motion directions

**Chapter 11**

- \( a \) radius of a circle (m)
- \( a_w \) total added mass of a floating body (kg)
- \( A \) area in eq. 11.185 (m²)
- \( A_w \) added-mass moment of inertia (N-m-s²/rad)
- \( A_0 \) Lewis parameter (m⁰)
- \( A_1 \) Lewis parameter (m²)
- \( A_2 \) Lewis parameter (m³)
- \( A_3 \) Lewis parameter (m⁴)
- \( b_Z \) linear damping coefficient for heaving motions (N-s/m)
- \( b_wZ \) quasi-linear damping term in eqs. 11.82 and 11.84 (N-s/m)
- \( b_\xi \) waterline semi-breadth of a fixed or floating body at a distance \( \xi \) from the center of gravity (m)
- \( B \) center of buoyancy
- \( B' \) displaced center of buoyancy
- \( B_\eta \) linear damping coefficient for pitching motions (N-m-s/rad)
- \( B_\xi(\xi) \) breadth of a body at \( \xi \) (m)
- \( c \) linear restoring coefficient (N/m)
- \( c(\cdot) \) cosine integral
- \( C \) angular restoring coefficient (N-m/rad)
- \( C_{area} \) sectional area coefficient in eq. 11.43
- \( C_{max} \) (length) maxima coefficient in eq. 11.42
Notation

\( C_{SF} \) scale factor in eq. 11.40 when \( a = 1 \)
\( C_{SF,a} \) scale factor in eq. 11.40 when \( a \neq 1 \)
\( C_{smith} \) Smith correction factor in eq. 11.63b
\( d,e,f \) angular coupling terms in eq. 11.15 (see Table 11.1)
\( d_\xi \) draft at \( \xi \) (m)
\( D_1 \) diameter of a semicircular section at \( \xi \) (m)
\( D_{E,F} \) linear coupling terms in eq. 11.16 (see Table 11.1)
\( f() \) see eqs. 11.96 through 11.98
\( F(k) \) see eq. 11.159
\( F_a \) inertial reaction force in eq. 11.29 (N)
\( F_B \) buoyant force (N)
\( F_t \) radiation damping force in eq. 11.31 (N)
\( F_W \) total wave force in eq. 11.25 (N)
\( g() \) see eqs. 11.96 through 11.98
\( G \) center of gravity
\( G_M \) metacentric height (m)
\( k_2 \) shape parameter in eq. 11.144
\( k_4 \) frequency coefficient in Table 11.3
\( K \) keel
\( I_{x,y} \) second moment of area with respect to the \( x \)- or \( y \)-axis (m\(^4\))
\( L_{i,j} \) operators defined in eqs. 11.113 through 11.116, where \( i = 1,2 \) and \( j = 1,2 \)
\( L \) waterplane ship length (m)
\( \ell_{aft} \) distance from stern to \( G \) in the waterplane (m)
\( \ell_{fwd} \) distance from the bow to \( G \) in the waterplane (m) pitching motions (N-m-s/rad)
\( \ell \) freeboard of a floating body (m)
\( M \) metacenter
\( M_s \) two-dimensional source strength (m\(^2\)/s)
\( M_t \) inertial reaction moment in eq. 11.30 (N-m)
\( M_t \) radiation damping moment in eq. 11.31 (N-m)
\( M_W \) total wave moment in eq. 11.26 (N-m)
\( O \) origin of the ship coordinate system, \( X,Y,Z \)
\( P \) point on the strip in Figure 11.5
\( P \) point on the strip in Figure 11.24a
\( r \) radius of a circle (m)
\( r,\beta \) polar coordinates in the \( Y-Z \) plane, as in Figures 11.7b, 11.7c, and 11.12
\( R_Z \) amplitude ratio in eqs. 11.88 and 11.89
\( R_\xi \) radius of a semicircular section at \( \xi \) (m)
\( si() \) sine integral
\( s \) curvilinear coordinate in eq. 11.185 (m)
\( s_o \) amplitude in eq. 11.178 (m) \( S(Y,Z,t) \) strip envelope geometry in Figure 11.22 (m)
\( S_\xi \) strip area at \( \xi \) (m\(^2\))
\( S_{body} \) spatial portion of strip envelope geometry in eq. 11.178 (m)
\( T_e \) period of encounter (s)
\( U \) ship’s forward speed (m/s)
\( V_w(t) \) vertical speed of the free surface (m/s)
\( V_z \) heaving body speed (m/s)
\( V_{wz} \) vertical water particle velocity (m/s)
### Notation

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