This concise textbook is an introduction to econometrics from the Bayesian viewpoint. It begins with an explanation of the basic ideas of subjective probability and shows how subjective probabilities must obey the usual rules of probability to ensure coherency. It then turns to the definitions of the likelihood function, prior distributions, and posterior distributions. It explains how posterior distributions are the basis for inference and explores their basic properties. The Bernoulli distribution is used as a simple example. Various methods of specifying prior distributions are considered, with special emphasis on subject-matter considerations and exchange ability. The regression model is examined to show how analytical methods may fail in the derivation of marginal posterior distributions, which leads to an explanation of classical and Markov chain Monte Carlo (MCMC) methods of simulation. The latter is proceeded by a brief introduction to Markov chains. The remainder of the book is concerned with applications of the theory to important models that are used in economics, political science, biostatistics, and other applied fields. These include the linear regression model and extensions to Tobit, probit, and logit models; time series models; and models involving endogenous variables.

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Introduction to Bayesian Econometrics

EDWARD GREENBERG

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Preface

To Instructors and Students

THIS BOOK IS a concise introduction to Bayesian statistics and econometrics. It can be used as a supplement to a frequentist course by instructors who wish to introduce the Bayesian viewpoint or as a text in a course in Bayesian econometrics supplemented by readings in the current literature.

While the student should have had some exposure to standard probability theory and statistics, the book does not make extensive use of statistical theory. Indeed, because of its reliance on simulation techniques, it requires less background in statistics and probability than do most books that take a frequentist approach. It is, however, strongly recommended that the students become familiar with the forms and properties of the standard probability distributions collected in Appendix A.

Since the advent of Markov chain Monte Carlo (MCMC) methods in the early 1990s, Bayesian methods have been extended to a large and growing number of applications. This book limits itself to explaining in detail a few important applications. Its main goal is to provide examples of MCMC algorithms to enable students and researchers to design algorithms for the models that arise in their own research. More attention is paid to the design of algorithms for the models than to the specification and interpretation of the models themselves because we assume that the student has been exposed to these models in other statistics and econometrics classes.

The decision to keep the book short has also meant that we have taken a stand on some controversial issues rather than discuss a large number of alternative methods. In some cases, alternative approaches are discussed in end of chapter notes.

Exercises have been included at the end of the chapters, but the best way to learn the material is for students to apply the ideas to empirical applications of their choice. Accordingly, even though it is not explicitly stated, the first exercise at the end of every chapter in Part III should direct students to formulate a model; collect
Preface

data; specify a prior distribution on the basis of previous research design and, if necessary, program an algorithm; and present the results.

A link to the Web site for the course may be found at my Web site: http://edg.wustl.edu. The site contains errata, links to data sources, some computer code, and other information.

Acknowledgments

I would like to acknowledge and offer my sincere gratitude to some of the people who have helped me throughout my career. On the professional side, I start with my undergraduate years at the business school of New York University, where Abraham Gitlow awakened my interest in economics. My first statistics course was with F. J. Viser and my second with Ernest Kurnow, who encouraged me to continue my studies and guided me in the process.

At the University of Wisconsin–Madison, I was mentored by, among others, Peter Steiner and Guy Orcutt. Econometrics was taught by Jack Johnston, who was writing the first edition of his pathbreaking book, and I was fortunate to have Arthur Goldberger and Arnold Zellner as teachers and colleagues. My first mathematical statistics course was with Enders Robinson, and I later audited George Box’s class, where I received my first exposure to Bayesian ideas. Soon afterward, Zellner began to apply the methods to econometrics in a workshop that I attended.

My interest in Bayesian methods was deepened at Washington University first by E. T. Jaynes and then by Siddhartha Chib. Sid Chib has been my teacher, collaborator, and friend for the last 15 years. His contributions to Bayesian statistics, econometrics, and MCMC methods have had enormous impact. I have been extremely fortunate to have had the opportunity to work with him. The students in my courses in Bayesian econometrics contributed to my understanding of the material by their blank stares and penetrating questions. I am most grateful to them.

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In December 2005, my wife of more than 46 years passed away. I dedicate this book to Joan’s memory.