

## Dynamics of Particles and Rigid Bodies: A Systematic Approach

*Dynamics of Particles and Rigid Bodies: A Systematic Approach* is intended for undergraduate courses in dynamics. This work is a unique blend of conceptual, theoretical, and practical aspects of dynamics generally not found in dynamics books at the undergraduate level. In particular, in this book the concepts are developed in a highly rigorous manner and are applied to examples using a step-by-step approach that is completely consistent with the theory. In addition, for clarity, the notation used to develop the theory is identical to that used to solve example problems. The result of this approach is that a student is able to see clearly the connection between the theory and the application of theory to example problems. While the material is not new, instructors and their students will appreciate the highly pedagogical approach that aids in the mastery and retention of concepts. The approach used in this book teaches a student to develop a systematic approach to problem solving. The work is supported by a great range of examples and reinforced by numerous problems for student solution. An instructor's solutions manual is available.

**Anil V. Rao** earned his B.S. in mechanical engineering and A.B. in mathematics from Cornell University, his M.S.E. in aerospace engineering from the University of Michigan, and his M.A. and Ph.D. in mechanical and aerospace engineering from Princeton University. After earning his Ph.D., Dr. Rao joined the Flight Mechanics Department at The Aerospace Corporation in Los Angeles, where he was involved in mission support for U.S. Air Force launch vehicle programs and trajectory optimization software development. Subsequently, Dr. Rao joined The Charles Stark Draper Laboratory, Inc., in Cambridge, Massachusetts. Since joining Draper, Dr. Rao has been involved in numerous projects related to trajectory optimization, guidance, and navigation of aerospace vehicles. Concurrently, for the past several years Dr. Rao has been an Adjunct Professor of Aerospace and Mechanical Engineering at Boston University where he has taught the core undergraduate engineering dynamics course. Since joining BU, Dr. Rao has been voted AIAA/ASME Faculty Member of the Year Award by the Aerospace and Mechanical Engineering Department and has been voted Boston University College of Engineering Professor of the Year for outstanding teaching.

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Anil V. Rao

Frontmatter

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**DYNAMICS  
OF PARTICLES AND RIGID BODIES  
A SYSTEMATIC APPROACH**

**ANIL V. RAO**

Boston University



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*Vakratunda Mahaakaaya Soorya Koti Samaprabha  
Nirvighnam Kuru Mein Deva Sarva Kaaryashu Sarvadaa*

*I dedicate this book with love to Anita and Vikram  
and to my parents, Saroj and Rajeswara*

# Contents

<b>Preface</b>	<b>ix</b>
<b>Acknowledgments</b>	<b>xiii</b>
<b>Nomenclature</b>	<b>xv</b>
<b>1 Introductory Concepts</b>	<b>1</b>
1.1 Scalars . . . . .	2
1.2 Vectors . . . . .	3
1.3 Tensors . . . . .	10
1.4 Matrices . . . . .	14
1.5 Ordinary Differential Equations . . . . .	25
<b>2 Kinematics</b>	<b>27</b>
2.1 Reference Frames . . . . .	28
2.2 Coordinate Systems . . . . .	30
2.3 Rate of Change of Scalar and Vector Functions . . . . .	32
2.4 Position, Velocity, and Acceleration . . . . .	35
2.5 Degrees of Freedom of a Particle . . . . .	37
2.6 Relative Position, Velocity, and Acceleration . . . . .	39
2.7 Rectilinear Motion . . . . .	39
2.8 Using Noninertial Reference Frames to Describe Motion . . . . .	42
2.9 Rate of Change of a Vector in a Rotating Reference Frame . . . . .	42
2.10 Kinematics in a Rotating Reference Frame . . . . .	50
2.11 Common Coordinate Systems . . . . .	51
2.12 Kinematics in a Rotating and Translating Reference Frame . . . . .	85
2.13 Practical Approach to Computing Velocity and Acceleration . . . . .	88
2.14 Kinematics of a Particle in Continuous Contact with a Surface . . . . .	99
2.15 Kinematics of Rigid Bodies . . . . .	104
Summary of Chapter 2 . . . . .	126
Problems for Chapter 2 . . . . .	130
<b>3 Kinetics of Particles</b>	<b>145</b>
3.1 Forces Commonly Used in Dynamics . . . . .	146
3.2 Inertial Reference Frames . . . . .	156
3.3 Newton's Laws for a Particle . . . . .	157
3.4 Comments on Newton's Laws . . . . .	157
3.5 Examples of Application of Newton's Laws in Particle Dynamics . . . . .	158

3.6	Linear Momentum and Linear Impulse for a Particle . . . . .	176
3.7	Moment of a Force and Moment Transport Theorem for a Particle . . . . .	178
3.8	Angular Momentum and Angular Impulse for a Particle . . . . .	179
3.9	Instantaneous Linear and Angular Impulse . . . . .	187
3.10	Power, Work, and Energy for a Particle . . . . .	188
	Summary of Chapter 3 . . . . .	218
	Problems for Chapter 3 . . . . .	222
<b>4</b>	<b>Kinetics of a System of Particles</b>	<b>237</b>
4.1	Center of Mass and Linear Momentum of a System of Particles . . . . .	238
4.2	Angular Momentum of a System of Particles . . . . .	240
4.3	Newton's 2 <sup>nd</sup> Law for a System of Particles . . . . .	242
4.4	Moment of a System of Forces Acting on a System of Particles . . . . .	248
4.5	Rate of Change of Angular Momentum for a System of Particles . . . . .	249
4.6	Impulse and Momentum for a System of Particles . . . . .	266
4.7	Work and Energy for a System of Particles . . . . .	280
4.8	Collision of Particles . . . . .	289
	Summary of Chapter 4 . . . . .	305
	Problems for Chapter 4 . . . . .	310
<b>5</b>	<b>Kinetics of Rigid Bodies</b>	<b>321</b>
5.1	Center of Mass and Linear Momentum of a Rigid Body . . . . .	322
5.2	Angular Momentum of a Rigid Body . . . . .	323
5.3	Moment of Inertia Tensor of a Rigid Body . . . . .	325
5.4	Principal-Axis Coordinate Systems . . . . .	334
5.5	Actions on a Rigid Body . . . . .	349
5.6	Moment Transport Theorem for a Rigid Body . . . . .	353
5.7	Euler's Laws for a Rigid Body . . . . .	355
5.8	Systems of Rigid Bodies . . . . .	391
5.9	Rotational Dynamics of a Rigid Body Using Moment of Inertia . . . . .	410
5.10	Work and Energy for a Rigid Body . . . . .	419
5.11	Impulse and Momentum for a Rigid Body . . . . .	441
5.12	Collision of Rigid Bodies . . . . .	450
	Summary of Chapter 5 . . . . .	467
	Problems for Chapter 5 . . . . .	472
	<b>Appendices</b>	<b>486</b>
<b>A</b>	<b>Principle-Axis Moments of Inertia of Homogeneous Bodies</b>	<b>487</b>
<b>B</b>	<b>Identities, Derivatives, Integrals, and Gradient</b>	<b>493</b>
B.1	Identities . . . . .	493
B.2	Derivatives and Integrals . . . . .	493
B.3	Gradient of a Scalar Function . . . . .	494
<b>C</b>	<b>Answers to Selected Problems</b>	<b>497</b>
	<b>Bibliography</b>	<b>503</b>
	<b>Index</b>	<b>505</b>

## Preface

The subject of dynamics has been taught in engineering curricula for decades, traditionally as a second-semester course as part of a year-long sequence in engineering mechanics. This approach to teaching dynamics has led to a wide array of currently available engineering mechanics books, including Beer and Johnston (1997), Bedford and Fowler (2005), Hibbeler (2001), and Merriam and Kraige (1997). From my experience, the reasons these books are adopted for undergraduate courses in engineering mechanics are threefold. First, they include a wide variety of worked examples and have more than 1000 problems for the students to solve at the end of each chapter. The variety of problems provides instructors with the flexibility to assign different problems every semester for several years. Second, these books are generic enough that they can be used to teach undergraduates in virtually any branch of engineering. Third, they cover both statics and dynamics, thereby making it possible for a student to purchase a single book for a year-long engineering mechanics course. Using these empirical measures, it is hard to dispute that these books cover a tremendous amount of material and enable an instructor to tailor the material to the needs of a particular course. Given the vast array of undergraduate dynamics books already available, an obvious question that arises is, why write yet another book on the subject of undergraduate engineering dynamics? While it is clear that the availability of another book on the subject would clearly add to the *number* of choices available to instructors, it may be difficult at first glance to see how the addition of another book would add *value* to the existing literature. However, after my experience over the past several years of teaching dynamics, not only do I now believe that there is room for another book, but I feel strongly that the paradigm used to teach the subject of dynamics needs to be completely overhauled.

Before I ever taught undergraduate dynamics, I, too, believed that the existing books on engineering mechanics were more than adequate and that an additional book would add little to no value to the existing literature. Consequently, without giving it much thought, the first time I taught engineering dynamics (course EK302 at Boston University) I randomly chose one of the standard undergraduate textbooks. Given my notions at the time, it never occurred to me that the book I chose for my class would pose so many difficulties for my students. However, not more than a few weeks into my first semester of teaching, I was met by vehement complaints from my students regarding the textbook. Given their frustration and my sincere desire to keep them motivated, I began investigating more thoroughly why my students found the textbook so difficult to follow and what I could do to help them overcome their frustration.

My investigation began by carefully reading each of the aforementioned engineering mechanics textbooks. My conclusion from reading these books was that the frustration my students were experiencing emanated from two sources. First, I found an enormous

inconsistency between the presentation of the theory and the application of the theory to examples. Second, I found the approach to problem solving was highly formulaic and did not place an emphasis on understanding. Essentially, I concluded that these books lacked the pedagogy required for a student to master the key concepts and, instead, promoted an ad hoc approach to problem solving. More importantly, because of the inconsistency between the presentation and the application of the theory, I found that these books make it difficult for a student's understanding of the material to grow as the course progressed. Consequently, rather than solving problems systematically from first principles, my students were trying to solve homework problems either by emulating a problem solved in the book, by reverse engineering a solution using the answers at the back of the book (by analogy to a boundary-value problem, I call this approach the "shooting" method for finding a solution to a problem), or by searching for formulas from which they could "plug in" the information that they are given. The worst part was that, given a new problem (however similar it may appear to be to previous problems), they were at a loss as to how to proceed because they had not truly understood the key concepts.

My desire to write this book has grown out of my experience that undergraduate engineering dynamics needs to be taught in an extremely systematic and highly explicit manner. My approach has been put to the test over the past several years while teaching the core undergraduate engineering dynamics course at Boston University. I consider my approach to dynamics to be a significant departure from any of the existing books on undergraduate engineering dynamics. First, different from the aforementioned books, I have developed a highly rigorous presentation of the concepts. Second, the level of rigor in solving problems is *identical* to that used in developing the theory. Using my approach, it is possible for a student to see clearly the connection between the theory and the application of the theory. To this end, I have adopted a more advanced (but what I believe is a significantly more descriptive) notation than is commonly found in other undergraduate engineering dynamics books. Third, I have kept the material at the undergraduate level, i.e., the types of problems that are included share similarities with those found in many other engineering dynamics books. With regard to notation, with the exception of second-order tensors, the only mathematical prerequisite for this book is vector calculus (with regard to tensors, I believe that, given a few simple explanations and without losing a step, the basics of tensor algebra can be handled by a fourth-semester undergraduate student in mechanical or aerospace engineering). Fourth, in absolutely every topic covered in this book, I use a step-by-step vector mechanics approach to solving problems. I have found through experience that the approach I have chosen works extremely well in practice. In particular, I am able to see substantial growth in the thought process of my students from the first week of class to the final exam.

This book is intended for undergraduate students who want a systematic and rigorous approach to the subject of particle and rigid body dynamics. Because of the intended audience, certain topics in this book have been intentionally omitted. In particular, I do not cover the topics of systems where mass is gained or lost. Furthermore, I cover three-dimensional kinetics of rigid bodies in a relatively limited manner. In the case of systems that gain or lose mass, to teach this topic correctly requires a basic course on fluid mechanics, which many students do not have upon entering an undergraduate engineering dynamics course. With regard to three-dimensional kinetics of a rigid body, it is simply not possible to cover this entire topic in a one-semester undergraduate engineering dynamics course.

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Frontmatter

[More information](#)**Preface****xi**

The material presented in this book is not new. However, I believe strongly that my approach is highly pedagogical, truly aids in mastering the key concepts, and promotes retention of the material well beyond the duration of the course. As I have already said, my approach is a significant departure from approaches used in other books. To motivate my approach, I have attempted throughout the book to include a sufficient number of worked examples and have included a wide range of problems at the end of each chapter for a student to solve. Most of the problems are ones that I have constructed myself while others are based on problems from the beautifully written book by Greenwood (1988). Finally, the notation I have adopted for kinematics is based on the notation developed by Kane and Levinson (1985).

Finally, I would like to re-emphasize that this book has been written with the student in mind. To this end, everywhere possible I have attempted to provide explicit guidance so that the student is able to follow clearly both the theory and the examples. It is my sincere hope that students everywhere will benefit from this book.

Anil V. Rao

Boston, Massachusetts

## Acknowledgments

Writing a textbook is an arduous task and I have many people to acknowledge for their inspiration and support. First, I am indebted to all of the teachers I have had in my life, but particularly to my high school calculus teacher, Mr. David Bock, for giving me the inspiration to want to be a teacher, and to my Ph.D. thesis advisor, Dr. Kenneth D. Mease, for encouraging me to develop a rigorous approach to research and for teaching me by example the true value of expressing my thoughts in as clear a manner as possible.

With regard to the evolution of this book, I acknowledge my friend and colleague, Dr. Scott Ploen, for helping me greatly to improve both my perspective on the subject of dynamics and to develop pedagogical approaches to motivate students to learn the subject. I also acknowledge my former students, Theresia Becker and Kimberley Clarke, for taking enormous amounts of time and effort to carefully examine the manuscript for typographical errors and for providing helpful suggestions for improving the discussions in the text. Next, I would like to thank my friend, Mr. David Woffinden, and my teaching assistants, Christophe Lecomte and Josh Burnett, for providing me with valuable feedback about the content, style, and clarity of the manuscript. In addition, I would like to acknowledge Dr. John G. Papastivridis for his help in obtaining an accurate historical reference to the parallel-axis theorem. Finally, I gratefully acknowledge Dr. Donald T. Greenwood, Dr. Oliver M. O'Reilly, and Dr. David Geller for taking the time to carefully read and provide constructive criticism of the manuscript. I particularly thank Dr. O'Reilly for helping me gain insight into the Euler basis and the dual Euler basis and for helping me arrive at an accurate description of a conservative torque.

With regard to making this book a possibility, I owe a special acknowledgment to Dr. John Baillieul for giving me the opportunity to teach at Boston University. Without Dr. Baillieul's help, I would never have been able to do something that has turned out to be so fulfilling and would never have had the opportunity, let alone the inspiration, to write this book.

I also thank my dear parents, Saroj and Rajeswara, whose lifelong efforts made it possible for me to obtain a high quality education and have made a work such as this a reality. Finally, I thank my beloved wife, Anita, for her encouragement and support during the time when I was working on this manuscript. I realize only now just how much time my writing this book took from other things in our lives, and I am grateful for her patience throughout this long endeavor.

Anil V. Rao  
Boston, Massachusetts

# Nomenclature

## Symbols

$\otimes$	=	Tensor product between two vectors
$\odot$	=	Vector direction out of page
$\otimes$	=	Vector direction into page
$\nabla$	=	Gradient operator
$\mathbb{R}$	=	One-dimensional Euclidean space
$\mathbb{R}^3$	=	Three-dimensional Euclidean space
$\mathcal{A}$	=	General reference frame
$\mathcal{B}$	=	General reference frame
$\mathcal{F}$	=	Fixed inertial reference frame
$\mathcal{N}$	=	General inertial reference frame
$\mathcal{R}$	=	Rigid body
$\mathcal{R}_1$	=	Rigid body
$\mathcal{R}_2$	=	Rigid body
${}^{\mathcal{A}}\frac{d\mathbf{b}}{dt}$	=	Rate of change of $\mathbf{b}$ as viewed by an observer in reference frame $\mathcal{A}$

## Scalars

$E$	=	Total energy
$g$	=	Magnitude of acceleration due to gravity
$G$	=	Universal constant of gravitation
$\ell$	=	Length of linear spring
$\ell_0$	=	Unstretched length of linear spring
$m$	=	Mass
$M$	=	Mass
$r$	=	Magnitude of position or radius
$R$	=	Magnitude of reaction force or radius
$s$	=	Arc-length
$t$	=	Time
$T$	=	Kinetic energy
$u$	=	Dummy variable of integration
$U$	=	Potential energy
$v$	=	Speed
$x$	=	First component of Cartesian position

$y$	=	Second component of Cartesian position
$z$	=	Third component of Cartesian position
$\beta$	=	Angle
$\kappa$	=	Curvature of trajectory
$\mu$	=	Coefficient of friction
$\mu_d$	=	Coefficient of dynamic friction
$\mu_s$	=	Coefficient of static friction
$\theta$	=	Angle
$\dot{\theta}$	=	Angular rate
$\omega_1$	=	First component of angular velocity
$\omega_2$	=	Second component of angular velocity
$\omega_3$	=	Third component of angular velocity
$\phi$	=	Angle
$\dot{\phi}$	=	Angular rate
$\tau$	=	Torsion of trajectory

## Vectors and Tensors

${}^{\mathcal{A}}\mathbf{a}$	=	Acceleration as viewed by an observer in reference frame $\mathcal{A}$
${}^{\mathcal{A}}\bar{\mathbf{a}}$	=	Acceleration of center of mass as viewed by an observer in reference frame $\mathcal{A}$
$\mathbf{a}$	=	General vector
$\mathbf{b}$	=	General vector
$\mathbf{c}$	=	Constant vector
$\mathbf{e}_1$	=	First basis vector
$\mathbf{e}_2$	=	Second basis vector
$\mathbf{e}_3$	=	Third basis vector
$\mathbf{e}_x$	=	First Cartesian basis vector
$\mathbf{e}_y$	=	Second Cartesian basis vector
$\mathbf{e}_z$	=	Third Cartesian basis vector
$\mathbf{e}_t$	=	Unit tangent vector
$\mathbf{e}_n$	=	Principle unit normal vector
$\mathbf{e}_b$	=	Principle unit bi-normal vector
$\mathbf{f}_{ij}$	=	Force exerted by particle $j$ on particle $i$
$\mathbf{g}$	=	Local acceleration due to gravity
$\mathbf{n}$	=	Unit normal to surface
$\mathbf{r}$	=	Position
$\bar{\mathbf{r}}$	=	Position of center of mass
$\mathbf{u}$	=	Unit tangent vector
${}^{\mathcal{A}}\mathbf{v}$	=	Velocity as viewed by an observer in reference frame $\mathcal{A}$
${}^{\mathcal{A}}\bar{\mathbf{v}}$	=	Velocity of center of mass as viewed by an observer in reference frame $\mathcal{A}$
${}^{\mathcal{A}}\bar{\mathbf{v}}_C^{\mathcal{R}}$	=	Velocity of point $C$ on rigid body $\mathcal{R}$ as viewed by an observer in reference frame $\mathcal{A}$
$\mathbf{w}$	=	Unit tangent vector
$\mathbf{E}_1$	=	First basis vector
$\mathbf{E}_2$	=	Second basis vector
$\mathbf{E}_3$	=	Third basis vector

## Nomenclature

xvii

$\mathbf{E}_x$	=	First Cartesian basis vector
$\mathbf{E}_y$	=	Second Cartesian basis vector
$\mathbf{E}_z$	=	Third Cartesian basis vector
${}^{\mathcal{N}}\mathbf{G}$	=	Linear momentum in inertial reference frame $\mathcal{N}$
$\frac{{}^{\mathcal{N}}d}{{}^{\mathcal{N}}dt}({}^{\mathcal{N}}\mathbf{G})$	=	Rate of change of linear momentum inertial reference frame $\mathcal{N}$
${}^{\mathcal{N}}\mathbf{H}_Q$	=	Angular momentum in inertial reference frame $\mathcal{N}$ relative to point $Q$
${}^{\mathcal{N}}\mathbf{H}_O$	=	Angular momentum in inertial reference frame $\mathcal{N}$ relative to point $O$ fixed in $\mathcal{N}$
${}^{\mathcal{N}}\bar{\mathbf{H}}$	=	Angular momentum in inertial reference frame $\mathcal{N}$ relative to center of mass
$\frac{{}^{\mathcal{N}}d}{{}^{\mathcal{N}}dt}({}^{\mathcal{N}}\mathbf{H}_Q)$	=	Rate of change of ${}^{\mathcal{N}}\mathbf{H}_Q$ in inertial reference frame $\mathcal{N}$ relative to point $Q$
$\frac{{}^{\mathcal{N}}d}{{}^{\mathcal{N}}dt}({}^{\mathcal{N}}\mathbf{H}_O)$	=	Rate of change of angular momentum in inertial reference frame $\mathcal{N}$ relative to point $O$ fixed in $\mathcal{N}$
$\frac{{}^{\mathcal{N}}d}{{}^{\mathcal{N}}dt}({}^{\mathcal{N}}\bar{\mathbf{H}})$	=	Rate of change of angular momentum in inertial reference frame $\mathcal{N}$ relative to center of mass
$\mathbf{I}^{\mathcal{R}}$	=	Moment of inertia tensor of a rigid body $\mathcal{R}$
$\mathbf{I}_Q^{\mathcal{R}}$	=	Moment of inertia tensor of a rigid body $\mathcal{R}$ relative to point $Q$
$\bar{\mathbf{I}}^{\mathcal{R}}$	=	Moment of inertia tensor of a rigid body $\mathcal{R}$ relative to center of mass of $\mathcal{R}$
$\mathbf{M}$	=	Moment
$\mathbf{M}_O$	=	Moment relative to point $O$
$\mathbf{M}_Q$	=	Moment relative to point $Q$
$\bar{\mathbf{M}}$	=	Moment relative to center of mass
$\mathbf{N}$	=	Reaction force
$\mathbf{S}$	=	Symmetric tensor
$\mathbf{T}$	=	General tensor
$\mathbf{U}$	=	Identity tensor
$\mathbf{R}$	=	Reaction force
${}^{\mathcal{A}}\boldsymbol{\alpha}^{\mathcal{B}}$	=	Angular acceleration of reference frame $\mathcal{B}$ as viewed by an observer in reference frame $\mathcal{A}$
${}^{\mathcal{A}}\boldsymbol{\omega}^{\mathcal{B}}$	=	Angular velocity of reference frame $\mathcal{B}$ as viewed by an observer in reference frame $\mathcal{A}$
$\boldsymbol{\rho}$	=	Relative position
$\boldsymbol{\tau}$	=	Pure torque