Complex Social Networks

The book provides a systematic and self-contained account of the fast-developing theory of complex social networks. Social networks are central to the understanding of most socio-economic phenomena in the modern world. The classical approach to studying them, however, relies on a methodology that abstracts from their size and complexity and is thus hardly suitable for the task. In contrast, the approach taken here keeps complexity at the core but aims at integrating it fruitfully with the incentive considerations that are preeminent in traditional economic analysis. The treatment starts with a detailed discussion of the basic models that act as “benchmarks” for the complex-network literature: random networks, small worlds, and scale-free networks. Then, it separately studies the three different forces that underlie, with varying intensities, almost all interesting network phenomena in social contexts: diffusion, search, and play. Finally, these forces are combined into a unified framework that is brought to bear on the important issue of network formation and the coevolution of agents’ behavior and their pattern of interaction.

Fernando Vega-Redondo is Professor of Economics at the University of Alicante, Spain, and the University of Essex, UK. He also taught at the Autonomous University of Barcelona and has held visiting positions at Harvard University, Hebrew University of Jerusalem, University of California–San Diego, Boston University, and Cornell University. Professor Vega-Redondo is the author of Economics and the Theory of Games (Cambridge University Press, 2003) and Evolution, Games, and Economic Behavior (1996). He has written more than 70 articles in professional journals, including the Journal of Economic Theory, Econometrica, International Economic Review, and Games and Economic Behavior. Professor Vega-Redondo received his Ph.D. from the University of Minnesota in 1984.
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Preface

The aim of this book is to provide a systematic account of a recent body of theoretical research that lies at the intersection of two fertile strands of literature. One of these strands, the study of complex networks, is a new field that has been developing at a fast pace during the last decade. The other one, social network analysis, has been an active area of research in sociology and economics for quite some time now – only lately, however, has it started to be seriously concerned with the implications of complexity. There is, I believe, much potential in bringing these two approaches together to shed light on network-based phenomena in complex social environments. This monograph is written with the intention of helping both the social scientist and other network researchers in this fascinating endeavor.

For the social scientist, the monograph may be used, inter alia, as a self-contained introduction to some of the main issues and techniques that mark the modern literature on complex networks. Since this literature has largely developed as an outgrowth of statistical physics, some of the powerful methodology being used is often alien to researchers from other disciplines. On the other hand, for the network theorist who lacks an economic background, the present monograph can fulfill a reciprocal role. Specifically, it may serve as an illustration of the questions and concerns that inform the economists’ approach to the study of socioeconomic networks. I hope that this twin, admittedly ambitious, objective is at least partly achieved in the following pages.

Writing this book has truly been a team effort. This concerns, in particular, the help provided by some of my (ex-)students, who have read through all the material and made a large number of useful suggestions. I want to highlight, specifically, the role played by José Antonio García-Martínez, Dunia López-Pintado, Miguel Ángel Meléndez-Jiménez, and Arnold Polanski. But a special mention must go to Constanza Fosco, whose thorough review of every detail has been most important. Some of my colleagues and coauthors, such as V. Bhaskar, Coralio Ballester, Antonio Cabrales, Andrea Galeotti, Sanjeev Goyal, and Juana Santamaría-García, have also contributed specific comments or general points on different parts of the manuscript.
My original interest in the field of complex networks has been substantially bolstered through numerous collaborations with statistical physicists over the last few years. Alex Arenas, Albert Díaz-Guilera, George Ehrhardt, Roger Guimerà, Enrique Louis, Conrad Pérez, Frantisek Slanina, and, in a particularly intense and sustained way, Matteo Marsili have been providing both valuable stimuli and expert help all along. Despite our quite different backgrounds, it has generally been surprisingly easy to “talk science” with them, since we share a similar view and excitement of what is interesting in research. Finally, I also want to express my sincere thanks to Matthew Jackson, for his advice and support as coeditor of the *Econometric Society Monograph Series*, to which I am privileged to contribute this work.

The completion of the book has imposed quite a toll on some people, and I am very grateful to all of them for their indulgent understanding. These certainly include my coauthors, who have seen how some of the ongoing projects advanced more slowly than desired. My students and colleagues, at both Alicante and Essex, have also experienced some occasional blackouts, as I concentrated on the task of finishing different parts of the manuscript. But those who have probably been most affected, and certainly deserve my highest gratitude, are my family. In my view, inevitably subjective, they represent a vivid example that a “complex” and coevolving network of immensely rich relationships can also arise on the small scale!
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