1

Nuclei in the Cosmos

> There is a coherent plan in the universe, though I don't know what it's a plan for.
>
> *Fred Hoyle*

In order to understand about the composition of stars and how they produce energy, we need to know about nuclei, and about the reactions which they undergo. This chapter provides an introduction to the description of nuclei, and surveys the range of scenarios in which important reactions occur. We begin with the Big Bang, then discuss energy production cycles in stars, and finish with an outline of some of the processes by which we think that heavy elements are produced in supernovae and other stellar enviroments. The more detailed discussion of nuclear physics begins in Chapter 2, to which the more advanced student is directed.

1.1 Nuclei

1.1.1 Properties of nuclei

Each isotope (A, Z), characterized by *mass number* A and charge Z, has in its ground state a rest mass $m_{A,Z}$. This total mass is less than the sum of the masses of the constituent protons and neutrons due to the binding energy of the system. Energy is released when the bound state is formed. The binding energy may be calculated by

$$B(A, Z) = (Zm_p + Nm_n - m_{A,Z})c^2, \tag{1.1.1}$$

and is the energy required to break up the nucleus into its A constituent nucleons. The number of neutrons is $N = A - Z$. A unit atomic mass (1 u) has rest energy $mc^2 = 931.494$ MeV.

The *binding energy per nucleon* B/A dictates whether energy must be supplied or will be released in the fusion of two nuclei to form their composite. The values of $B(A, Z)/A$ are shown in Fig. 1.1 for all the long-lived isotopes. The larger the energy one needs to supply to release a nucleon, the more stable is the nucleus. The most

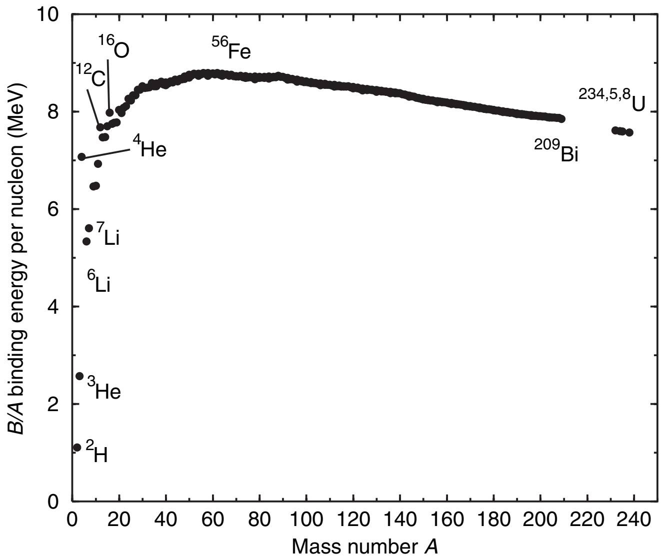

Fig. 1.1. Binding energies per nucleon, $B(A,Z)/A$, for all naturally occurring long-lived isotopes of A nucleons.

stable isotope is near ^{56}Fe, as seen from Fig. 1.1. If two nuclei A_1 and A_2 fuse to form $A = A_1 + A_2$, then the reaction is typically exothermic and energy is released if $A \lesssim 56$. If $A \gtrsim 56$ then fusion reactions are typically endothermic – energy is required – so we might expect the opposite process, *fission*, to be more likely. Fission occurs spontaneously for many nuclei $Z \gtrsim 90$, called the *actinides*.

The most stable nuclear isotopes for $Z \lesssim 20$ have $N \approx Z$, whereas heavier nuclei tend to have more neutrons, to compensate for the increased Coulomb repulsion. If we make a plot with N as the horizontal axis and Z as the vertical axis, we have the Segré chart of Fig. 1.2. Each row is a distinct chemical element, and the stable isotopes are the dark squares lying roughly along the diagonal. The naturally-occurring nuclei, with the longest lifetimes, are said to occupy the *valley of stability*. Neutron-rich nuclei are shown below, to the right of the valley of stability, out to the *neutron dripline*, the point beyond which one cannot form bound states, no matter how many neutrons are added to the system. There is a large gulf between observed isotopes and the predicted neutron dripline, especially for heavy elements.

Conversely, proton-rich nuclei, although they are not so numerous, can be seen above the central valley out to the dripline where proton emission (proton radioactivity) occurs. Most proton-rich nuclei for $A < 200$ have been observed. Nuclei between the driplines have ground states that are stable to nucleon emission, but may still slowly β-decay (see Section 2.2 for timescales) by the weak interaction (see for example the reactions 1.2.4), or radioactively decay (also slowly) by fission or α-particle emission.

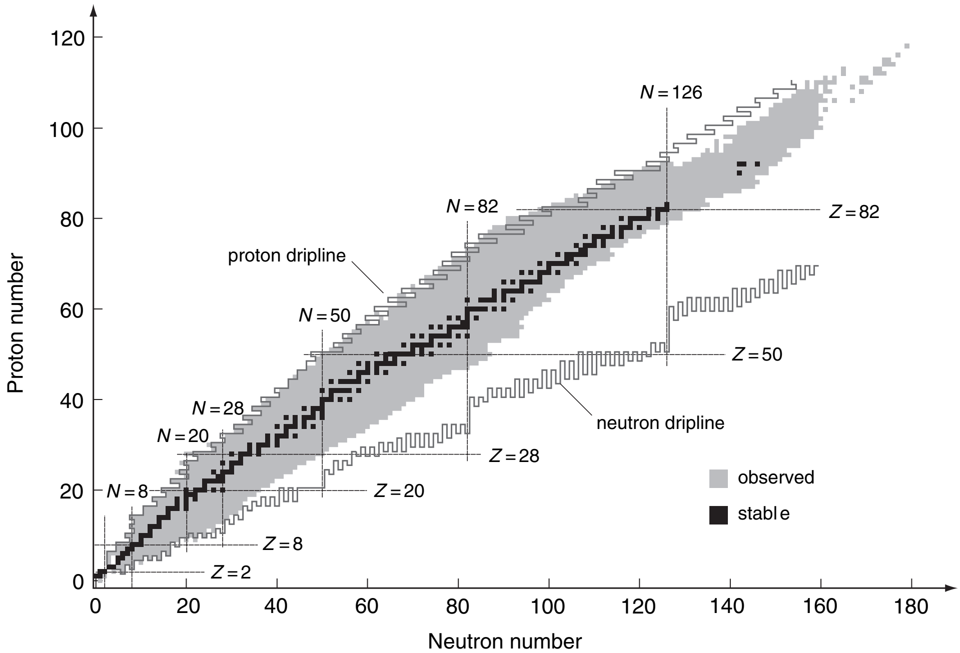

Fig. 1.2. Chart of stable and radioactive isotopes. Vertical and horizontal lines represent magic numbers. Figure courtesy of Marc Hausmann.

1.1.2 Nuclear reactions

If a nuclear reaction is performed in a laboratory, let the *projectile* be called A, the *target* called B, and the *residual nuclei* be C and D. The combination of A and B is called the *entrance channel*, and that of C and D is called an *exit channel* (more than one final channel may be possible). Then the reaction is labeled B(A,C)D, which is the common way of writing

$$\mathrm{A} + \mathrm{B} \rightarrow \mathrm{C} + \mathrm{D}. \tag{1.1.2}$$

Given all the isotopic mass values, we may calculate the energy required or released. This energy, called the Q-value for the reaction, is

$$Q = (m_\mathrm{A} + m_\mathrm{B} - m_\mathrm{C} - m_\mathrm{D})c^2. \tag{1.1.3}$$

Exothermic reactions have $Q > 0$, whereas a $Q < 0$ reaction is endothermic.

1.1.3 Forces in nuclei

There are four forces in nature: the *strong*, *electromagnetic*, *weak* and *gravitational* forces. The strong or *nuclear* forces are dominant in binding nuclei, but the other forces still have important roles to play in nuclear astrophysics. The *electromagnetic* force is responsible for the Coulomb repulsion between protons in nuclei, and the decrease in binding for heavy nuclei seen in Fig. 1.1. The *weak* interaction plays a role whenever reactions involve neutrinos; we will see some examples of this later in this chapter (Eqs. (1.2.1) and (1.2.4)). The *gravitational* attraction is not significant inside nuclei, but is responsible for creating galaxies and stars in the first place, and then compressing them to the stage where nuclear reactions begin.

1.1.4 The Coulomb barrier

In order that a nuclear reaction takes place, the nuclei involved have to be close to each other, but this is hindered by the Coulomb repulsion between the protons, which acts at longer distances compared with the nuclear force of short range. The overall potential energy between two charged nuclei separated by a distance R therefore follows the pattern shown in Fig. 1.3. There is a repulsive *Coulomb barrier* of height V_B, and scattering at energies $E < V_B$ still exists because of quantum tunneling through the barrier.

The exponential reduction of reaction rates for charged particles reacting at low relative energies will be extremely important in all astrophysical scenarios,

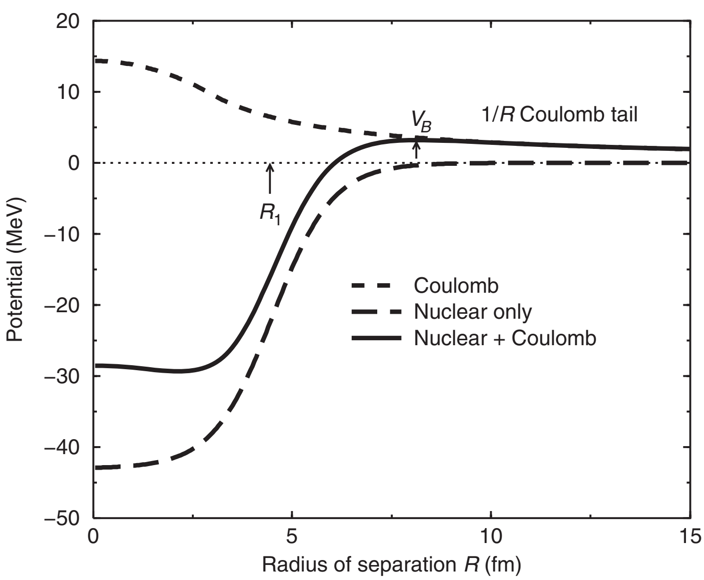

Fig. 1.3. The nuclear and Coulomb potential energies between a proton and ^{40}Ca as a function of the distance R between their centers, where R_1 is the radius of ^{40}Ca. The combined potential (solid line) has a maximum height of V_B, forming the Coulomb barrier.

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and will very often be the limiting factor for nuclear reactions. We will see (Section 2.4) that reaction rates are defined by the quantity σ, called the *cross section*. Because cross sections $\sigma(E)$ drop rapidly with decreasing center-of-mass energy E, due to the Coulomb repulsion, we factorize out a simple energy dependence according to

$$\sigma(E) = \frac{1}{E} e^{-2\pi\eta} S(E) \tag{1.1.4}$$

to define an *astrophysical S-factor* $S(E)$ which should vary less strongly with energy. The $1/E$ geometrical factor is associated with the wavelength of the incoming particle, and the exponential factor represents the penetrability through the Coulomb barrier. It depends on η, the *Sommerfeld parameter*, defined as $\eta = Z_1 Z_2 e^2/(\hbar v)$ (Eq. (3.1.71)) where $Z_1 Z_2 e^2$ is the product of charges and v the relative incident velocity. In Fig. 1.4 we show, in the upper panel, the cross section for the α capture on ^{3}He to synthesize ^{7}Be. The reaction cross section falls off rapidly as the energy decreases, whereas the S-factor, shown in the lower panel, is nearly constant.

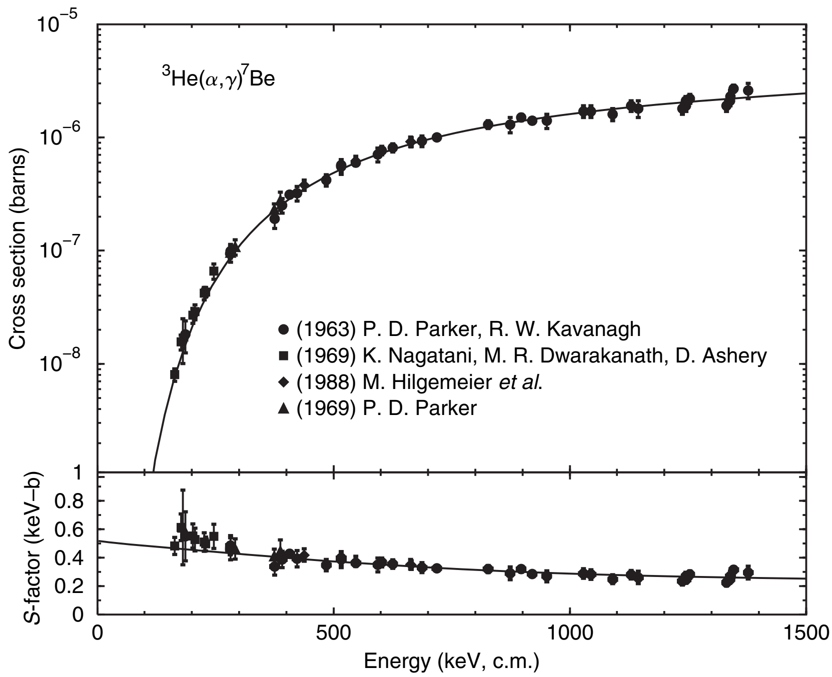

Fig. 1.4. Dependence of cross section and $S(E)$ on energy, for the reaction ^{3}He$(\alpha, \gamma)^7$Be. The solid curve is a calculation to be discussed in Appendix B.

1.2 Primordial nucleosynthesis

Having seen how nuclei and their reactions can be characterized, we now look at a range of nuclear reactions in astrophysics, starting at the beginning. A schematic illustration of the evolution of the Universe immediately after the Big Bang is presented in Fig. 1.5 and briefly described in this section.

Following the Big Bang, the Universe expanded and cooled with a temperature of $T_9 \approx 15/\sqrt{t}$ for time t in seconds and temperature T_9 in units of GK $= 10^9$ K. According to thermodynamics, this temperature corresponds to an energy of $E = k_B T$, where the Boltzmann constant is $k_B = 1.38 \times 10^{-23}$ J K^{-1} $=$ 0.0861 MeV GK^{-1}. This means that the material in the expanding Universe had an average thermal energy of $E \approx 1.3/\sqrt{t}$ MeV.

For very early times, $t < 1$ s, the thermal energy E was greater than 1 MeV. In particular, E was greater than $(m_n - m_p)c^2 = 1.24$ MeV, the difference in the rest energies of the neutron and proton. At these times, therefore, there was enough radiative energy available to easily convert neutrons to protons, and back again, in

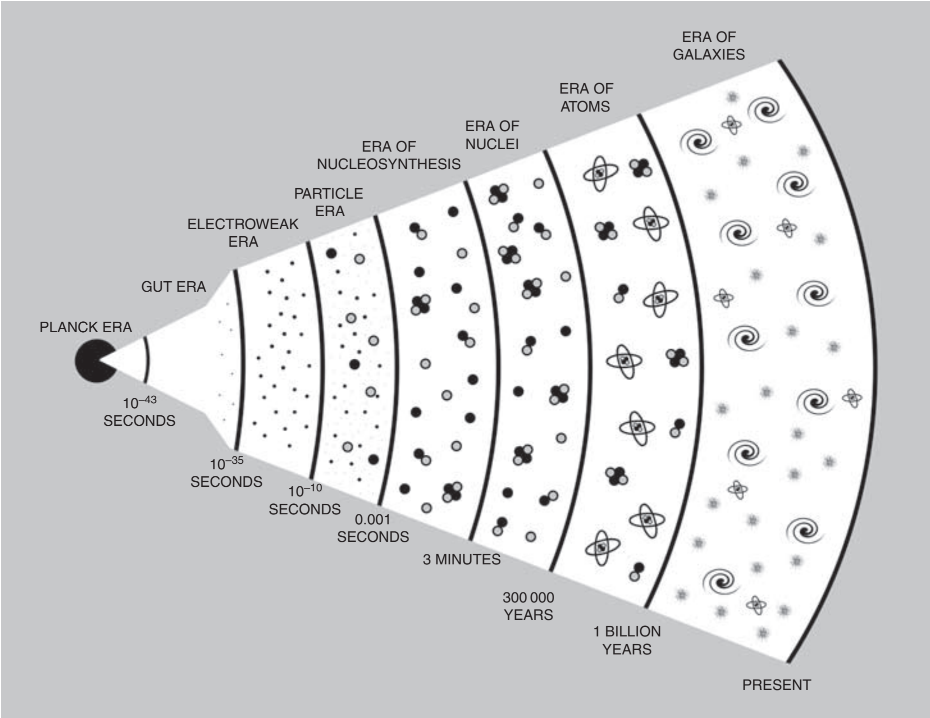

Fig. 1.5. History of the Universe, from the Big Bang to present times. Figure courtesy of Jon Whiting.

a statistical equilibrium by processes such as

$$\begin{aligned} \mathrm{n} + \mathrm{e}^+ &\rightarrow \mathrm{p} + \overline{\nu}_\mathrm{e} \\ \mathrm{p} + \mathrm{e}^- &\rightarrow \mathrm{n} + \nu_\mathrm{e} \\ \mathrm{e}^- + \mathrm{e}^+ &\leftrightarrow \gamma + \gamma \end{aligned} \tag{1.2.1}$$

where γ are photons, and ν_e and $\overline{\nu}_e$ are electron neutrinos and anti-neutrinos. The electrons e^- and positrons e^+ are commonly called *beta* (β) particles. The third reaction of (1.2.1) is the annihilation/production of electron-positron pairs in the high-temperature radiation environment.

Only after two seconds did the Universe cool down enough to enable the neutrons and protons to retain their identities ($E \lesssim 1\,\mathrm{MeV}$); this was the beginning of the astrophysics of nuclei. At this point there begun a period of about 250 s in which a *primordial nucleosynthesis* took place, and neutrons and protons combined to form hydrogen and helium isotopes, and perhaps a few lithium nuclei. At the beginning of this period there were only neutrons and protons, with a relative number density determined by their mass difference according to the Saha equation

$$\frac{n_n}{n_p} \approx \exp\left[-\frac{(m_n - m_p)c^2}{k_B T}\right], \tag{1.2.2}$$

an equation that will be derived in Chapter 12. When $k_B T \gg (m_n - m_p)c^2$, we have $n_n \approx n_p$. As the temperature dropped, there was a *freeze-out* in which the small $m_n - m_p$ difference led to the residual neutron and proton ratio of $n_n/n_p \sim 1/8$. This ratio may be found from a calculation that balances the cooling rate with the actual transition rates of the reactions (1.2.1) above.

Two protons or two neutrons cannot form a bound state, but a neutron and a proton may collide and form a deuteron, abbreviated d or ^{2}H. This reaction releases energy ($Q = 2.226\,\mathrm{MeV}$) in the form of a photon and the recoil energy of the deuteron:

$$\mathrm{n} + \mathrm{p} \rightarrow \mathrm{d} + \gamma. \tag{1.2.3}$$

This is what we call a *capture reaction*. These deuterons react easily with other protons and neutrons, giving rise to a series of reactions, the dominant ones of which are illustrated in Fig. 1.6. Tritons ^{3}H (t) can be formed, the hydrogen isotope with one proton and two neutrons, as well as helium isotopes with two protons and either one neutron (^{3}He) or two neutrons (^{4}He). The binding energy of the deuteron is one of the most fortunate coincidences in the Universe. The Universe took about 7 minutes to cool down to 2.226 MeV, and the neutron lifetime is slightly above

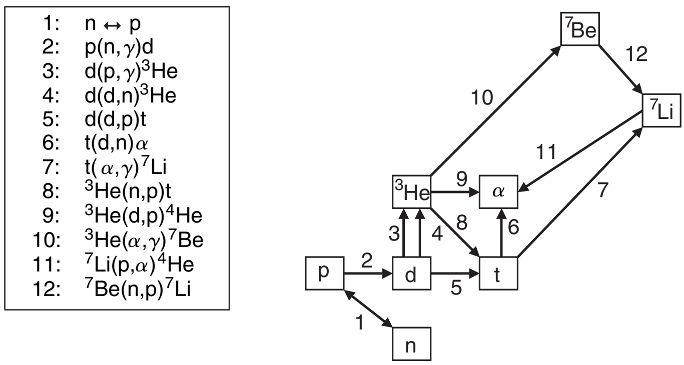

Fig. 1.6. The dominant reactions in primordial nucleosynthesis, after Kawano [1].

7 minutes. Had these two numbers not properly matched, there would have been no neutrons to initiate the whole primordial nucleosynthesis.

By time $t \approx 250$ s, the thermal energy E was 0.1 MeV, and all these primordial reactions came to a stop, except for the decays of neutrons, tritons and ^{7}Be. These last three nuclei were produced in primordial nucleosynthesis, but are not themselves stable, as they decay with lifetimes of 10.3 minutes, 12.3 years and 53 days, respectively, by *weak interactions* in what is called *β-decay*:

$$
\begin{aligned}
\mathrm{n} &\to \mathrm{p} + \mathrm{e}^- + \overline{\nu}_\mathrm{e}, \\
\mathrm{t} &\to {}^3\mathrm{He} + \mathrm{e}^- + \overline{\nu}_\mathrm{e}, \\
{}^7\mathrm{Be} &\to {}^7\mathrm{Li} + \mathrm{e}^+ + \nu_\mathrm{e}.
\end{aligned} \tag{1.2.4}
$$

Eventually, all the neutrons and radioactive nuclei transmuted into stable nuclei, such that only very small fractions of ^{7}Li, and practically no ^{6}Li remained. As a consequence, the initial composition of the Universe was almost entirely p, d, ^{3}He, ^{4}He, e$^-$, γ particles and neutrinos. This primeval ratio of abundances, listed in Table 1.1, can still be observed if we avoid regions where further reactions have taken place, such as in low-metal stars.

Very few nuclei heavier than helium are formed at this stage. One reason for this is that there are no stable nuclei with 5 nucleons, nor with 8 nucleons. The longest-lived isotopes with 5 nucleons are ^{5}He and ^{5}Li, but these emit neutrons and protons respectively. For element production there are thus bottlenecks at mass numbers of 5 and 8 that had to be later bridged by other means.

Very little further happened until the Universe reached time $t = 3.8 \times 10^5$ y, when the temperature and energies were low enough ($T \sim 4 \times 10^3$ K and $E \sim 0.4$ eV) for electrons to remain bound to nuclei in atoms. At that point, the atomic era started. After $t \sim 10^9$ y, stars and galaxies were formed, giving way to stellar

Table 1.1. *Isotopic abundances Y_i from primordial nucleosynthesis [2], defined by the fraction of nuclides i to the number of all nucleons. The nucleon number density is then $X_i = A_i Y_i$ of nucleons in that isotope of mass A_i. Normalization is $\sum_i X_i = 1$.*

Isotope	Nuclide fraction Y_i	Nucleon fraction X_i
^{1}H = p	0.75	0.75
^{2}H = d	2.44×10^{-5}	4.88×10^{-5}
^{3}He	1.0×10^{-5}	3.0×10^{-5}
^{4}He	0.062	0.2481
^{6}Li	1.1×10^{-14}	6.6×10^{-14}
^{7}Li	4.9×10^{-10}	34.3×10^{-14}

nucleosynthesis. Eventually some stars collapsed, heated up, and completely new cycles of nuclear reactions took place. This was how many heavier nuclei were produced. These processes continued to repeat themselves until the present day.

1.3 Reactions in light stars

After stars are formed by gravitational attraction, their continued contraction compresses the constituent gases and raises their temperature. If the star has a mass above a minimum of about 0.1 solar masses ($0.1\,M_\odot$), then the temperature rises to $T \sim 10{-}15 \times 10^6$ K and the density to $\rho \sim 10^2$ g cm^{-3}, and *nuclear hydrogen burning* can start. The release of energy in the resulting nuclear reactions is sufficient to stop further gravitational collapse, and the star remains in a phase of hydrostatic equilibrium. The compressive gravitational pressure is balanced by the expansive gas pressure of material heated by the nuclear reactions. Different initial stellar masses give rise, in this phase, to the range of main sequence stars represented in Fig. 1.7. The Hertzsprung-Russell diagram (H-R diagram) is a standard representation of stars in terms of their surface temperature and luminosity. Many stars are aligned roughly according to Stefan's law of $L \propto R^2 T^4$, and form what is called the main sequence. This corresponds to stars in their hydrogen-burning phase.

1.3.1 Proton-proton chains

The first series of nuclear reactions in a new star with mass $M < 1.5\,M_\odot$ is the *proton-proton chain*. This has the overall effect of converting 4 protons into one

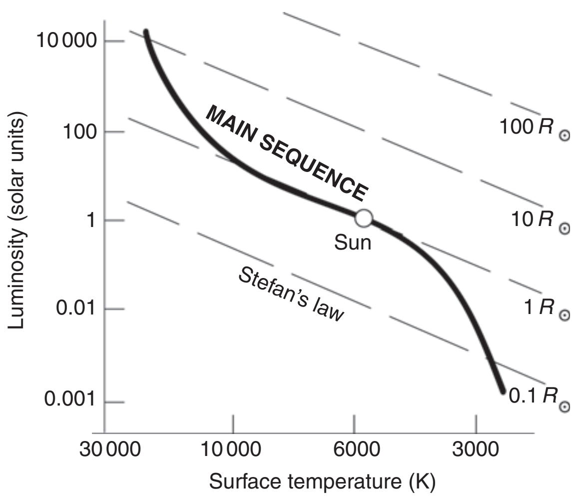

Fig. 1.7. H-R diagram: main sequence stars are represented by the thick curve, whereas the thin lines represent Stefan's law. Figure courtesy of Jon Whiting.

α particle (a ^{4}He nucleus), along with two positrons (2 e^+), two neutrinos (2 ν), and 26 MeV of released energy. It does not do this in one step, but via a chain that starts with

$$\mathrm{p} + \mathrm{p} \to \mathrm{d} + \mathrm{e}^+ + \nu_e \qquad (Q = 1.44\,\mathrm{MeV}). \tag{1.3.1}$$

This reaction, involving neutrinos, proceeds by the *weak interaction*, and has a very low reaction rate. In fact, its rate is so low that it has never been measured directly. The slowness of this initial step is what is responsible for the long lifetime of stars in their hydrogen-burning phase.

Following the formation of the deuteron d (^{2}H), a subsequent proton capture reaction

$$\mathrm{d} + \mathrm{p} \to {}^3\mathrm{He} + \gamma \qquad (Q = 5.49\,\mathrm{MeV}) \tag{1.3.2}$$

may readily occur. The reaction $\mathrm{d} + \mathrm{d} \to {}^4\mathrm{He} + \gamma$ may also occur, but is less likely since protons are much more abundant than deuterons at this stage: about 1 deuteron for every 10^{18} protons.

A second proton capture on ^{3}He cannot succeed because ^{4}Li is unbound, but other possible reactions involving ^{3}He are (in Chain I):

$${}^3\mathrm{He} + {}^3\mathrm{He} \to {}^4\mathrm{He} + 2\mathrm{p} + \gamma \qquad (Q = 12.86\,\mathrm{MeV}), \tag{1.3.3}$$

or

$${}^3\mathrm{He} + {}^4\mathrm{He} \to {}^7\mathrm{Be} + \gamma \qquad (Q = 1.59\,\mathrm{MeV}). \tag{1.3.4}$$