An Introduction to General Relativity and Cosmology

General relativity is a cornerstone of modern physics, and is of major importance in its applications to cosmology. Experts in the field Plebański and Krasiński provide a thorough introduction to general relativity to guide the reader through complete derivations of the most important results.

An Introduction to General Relativity and Cosmology is a unique text that presents a detailed coverage of cosmology as described by exact methods of relativity and inhomogeneous cosmological models. Geometric, physical and astrophysical properties of inhomogeneous cosmological models and advanced aspects of the Kerr metric are all systematically derived and clearly presented so that the reader can follow and verify all details. The book contains a detailed presentation of many topics that are not found in other textbooks.

This textbook for advanced undergraduates and graduates of physics and astronomy will enable students to develop expertise in the mathematical techniques necessary to study general relativity.

An Introduction to General Relativity and Cosmology

Jerzy Plebański
Centro de Investigación y de Estudios Avanzados
Instituto Politécnico Nacional
Apartado Postal 14-740, 07000 México D.F., Mexico

Andrzej Krasiński
Centrum Astronomiczne im. M. Kopernika,
Polska Akademia Nauk, Bartycka 18, 00 716 Warszawa,
Poland

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The scope of this text

General relativity is the currently accepted theory of gravitation. Under this heading one could include a huge amount of material. For the needs of this theory an elaborate mathematical apparatus was created. It has partly become a self-standing sub-discipline of mathematics and physics, and it keeps developing, providing input or inspiration to physical theories that are being newly created (such as gauge field theories, super-gravitation, and, more recently, the brane-world theories). From the gravitation theory, descriptions of astronomical phenomena taking place in strong gravitational fields and in large-scale sub-volumes of the Universe are derived. This part of gravitation theory develops in connection with results of astronomical observations. For the needs of this area, another sophisticated formalism was created (the Parametrised Post-Newtonian formalism). Finally, some tests of the gravitational theory can be carried out in laboratories, either terrestrial or orbital. These tests, their improvements and projects of further tests have led to developments in mathematical methods and in technology that are by now an almost separate branch of science – as an example, one can mention here the (monumentally expensive) search for gravitational waves and the calculations of properties of the wave signals to be expected.

In this situation, no single textbook can attempt to present the whole of gravitation theory, and the present text is no exception. We made the working assumption that relativity is part of physics (this view is not universally accepted!). The purpose of this course is to present those results that are most interesting from the point of view of a physicist, and were historically the most important. We are going to lead the reader through the mathematical part of the theory by a rather short route, but in such a way that the reader does not have to take anything on our word, is able to verify every detail, and, after reading the whole text, will be prepared to solve several problems by him/herself. Further help in this should be provided by the exercises in the text and the literature recommended for further reading.

The introductory part (Chapters 1–7), although assembled by J. Plebański long ago, has never been published in book form.[1] It differs from other courses on relativity in that it introduces differential geometry by a top-down method. We begin with general manifolds,

[1] A part of that material had been semi-published as copies of typewritten notes (Plebański, 1964).

on which no structures except tensors are defined, and discuss their basic properties. Then we add the notion of the covariant derivative and affine connection, without introducing the metric yet, and again proceed as far as possible. At that level we define geodesics via parallel displacement and we present the properties of curvature. Only at this point do we introduce the metric tensor and the (pseudo-)Riemannian geometry and specialise the results derived earlier to this case. Then we proceed to the presentation of more detailed topics, such as symmetries, the Bianchi classification and the Petrov classification.

Some of the chapters on classical relativistic topics contain material that, to the best of our knowledge, has never been published in any textbook. In particular, this applies to Chapter 8 (on symmetries) and to Chapter 16 (on cosmology with general geometry). Chapters 18 and 19 (on inhomogeneous cosmologies) are entirely based on original papers. Parts of Chapters 18 and 19 cover the material introduced in A. K.'s monograph on inhomogeneous cosmological models (Krasiński, 1997). However, the presentation here was thoroughly rearranged, extended, and brought up to date. We no longer briefly mention all contributions to the subject; rather, we have placed the emphasis on complete and clear derivations of the most important results. That material has so far existed only in scattered journal papers and has been assembled into a textbook for the first time (A. K.'s monograph (Krasiński, 1997) was only a concise review). Taken together, this collection of knowledge constitutes an important and interesting part of relativistic cosmology whose meaning has, unfortunately, not yet been appreciated properly by the astronomical community.

Most figures for this text, even when they look the same as the corresponding figures in the papers cited, were newly generated by A. K. using the program Gnuplot, sometimes on the basis of numerical calculations programmed in Fortran 90. The only figures taken verbatim from other sources are those that illustrated the joint papers by C. Hellaby and A. K.

J. Plebański kindly agreed to be included as a co-author of this text – having done his part of the job more than 30 years ago. Unfortunately, he was not able to participate actively in the writing up and proofreading. He died while the book was being edited. Therefore, the second author (A. K.) is exclusively responsible for any errors that may be found in this book.

Note for the reader. Some parts of this book may be skipped on first reading, since they are not necessary for understanding the material that follows. They are marked by asterisks. Chapters 18 and 19 are expected to be the highlights of this book. However, they go far beyond standard courses of relativity and may be skipped by those readers who wish to remain on the well-beaten track. Hesitating readers may read on, but can skip the sections marked by asterisks.

Andrzej Krasiński
Warsaw, September 2005

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