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978-0-521-85623-2 - An Introduction to General Relativity and Cosmology

Jerzy Plebanski and Andrzej Krasinski

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An Introduction to General Relativity and Cosmology

General relativity is a cornerstone of modern physics, and is of major importance in its applications to cosmology. Experts in the field Plebański and Kasiński provide a thorough introduction to general relativity to guide the reader through complete derivations of the most important results.

An Introduction to General Relativity and Cosmology is a unique text that presents a detailed coverage of cosmology as described by exact methods of relativity and inhomogeneous cosmological models. Geometric, physical and astrophysical properties of inhomogeneous cosmological models and advanced aspects of the Kerr metric are all systematically derived and clearly presented so that the reader can follow and verify all details. The book contains a detailed presentation of many topics that are not found in other textbooks.

This textbook for advanced undergraduates and graduates of physics and astronomy will enable students to develop expertise in the mathematical techniques necessary to study general relativity.

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An Introduction to General Relativity
and Cosmology

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Contents

<i>List of figures</i>	<i>page</i> xiii
<i>The scope of this text</i>	xvii
<i>Acknowledgements</i>	xix
1 How the theory of relativity came into being (a brief historical sketch)	1
1.1 Special versus general relativity	1
1.2 Space and inertia in Newtonian physics	1
1.3 Newton’s theory and the orbits of planets	2
1.4 The basic assumptions of general relativity	4
Part I Elements of differential geometry	7
2 A short sketch of 2-dimensional differential geometry	9
2.1 Constructing parallel straight lines in a flat space	9
2.2 Generalisation of the notion of parallelism to curved surfaces	10
3 Tensors, tensor densities	13
3.1 What are tensors good for?	13
3.2 Differentiable manifolds	13
3.3 Scalars	15
3.4 Contravariant vectors	15
3.5 Covariant vectors	16
3.6 Tensors of second rank	16
3.7 Tensor densities	17
3.8 Tensor densities of arbitrary rank	18
3.9 Algebraic properties of tensor densities	18
3.10 Mappings between manifolds	19
3.11 The Levi-Civita symbol	22
3.12 Multidimensional Kronecker deltas	23
3.13 Examples of applications of the Levi-Civita symbol and of the multidimensional Kronecker delta	24
3.14 Exercises	25

vi	<i>Contents</i>	
4	Covariant derivatives	26
4.1	Differentiation of tensors	26
4.2	Axioms of the covariant derivative	28
4.3	A field of bases on a manifold and scalar components of tensors	29
4.4	The affine connection	30
4.5	The explicit formula for the covariant derivative of tensor density fields	31
4.6	Exercises	32
5	Parallel transport and geodesic lines	33
5.1	Parallel transport	33
5.2	Geodesic lines	34
5.3	Exercises	35
6	The curvature of a manifold; flat manifolds	36
6.1	The commutator of second covariant derivatives	36
6.2	The commutator of directional covariant derivatives	38
6.3	The relation between curvature and parallel transport	39
6.4	Covariantly constant fields of vector bases	43
6.5	A torsion-free flat manifold	44
6.6	Parallel transport in a flat manifold	44
6.7	Geodesic deviation	45
6.8	Algebraic and differential identities obeyed by the curvature tensor	46
6.9	Exercises	47
7	Riemannian geometry	48
7.1	The metric tensor	48
7.2	Riemann spaces	49
7.3	The signature of a metric, degenerate metrics	49
7.4	Christoffel symbols	51
7.5	The curvature of a Riemann space	51
7.6	Flat Riemann spaces	52
7.7	Subspaces of a Riemann space	53
7.8	Flat Riemann spaces that are globally non-Euclidean	53
7.9	The Riemann curvature versus the normal curvature of a surface	54
7.10	The geodesic line as the line of extremal distance	55
7.11	Mappings between Riemann spaces	56
7.12	Conformally related Riemann spaces	56
7.13	Conformal curvature	58
7.14	Timelike, null and spacelike intervals in a 4-dimensional spacetime	61
7.15	Embeddings of Riemann spaces in Riemann spaces of higher dimension	63
7.16	The Petrov classification	70
7.17	Exercises	72

	<i>Contents</i>	vii
8	Symmetries of Riemann spaces, invariance of tensors	74
8.1	Symmetry transformations	74
8.2	The Killing equations	75
8.3	The connection between generators and the invariance transformations	77
8.4	Finding the Killing vector fields	78
8.5	Invariance of other tensor fields	79
8.6	The Lie derivative	80
8.7	The algebra of Killing vector fields	81
8.8	Surface-forming vector fields	81
8.9	Spherically symmetric 4-dimensional Riemann spaces	82
8.10	* Conformal Killing fields and their finite basis	86
8.11	* The maximal dimension of an invariance group	89
8.12	Exercises	91
9	Methods to calculate the curvature quickly – Cartan forms and algebraic computer programs	94
9.1	The basis of differential forms	94
9.2	The connection forms	95
9.3	The Riemann tensor	96
9.4	Using computers to calculate the curvature	98
9.5	Exercises	98
10	The spatially homogeneous Bianchi type spacetimes	99
10.1	The Bianchi classification of 3-dimensional Lie algebras	99
10.2	The dimension of the group versus the dimension of the orbit	104
10.3	Action of a group on a manifold	105
10.4	Groups acting transitively, homogeneous spaces	105
10.5	Invariant vector fields	106
10.6	The metrics of the Bianchi-type spacetimes	108
10.7	The isotropic Bianchi-type (Robertson–Walker) spacetimes	109
10.8	Exercises	112
11	* The Petrov classification by the spinor method	113
11.1	What is a spinor?	113
11.2	Translating spinors to tensors and vice versa	114
11.3	The spinor image of the Weyl tensor	116
11.4	The Petrov classification in the spinor representation	116
11.5	The Weyl spinor represented as a 3×3 complex matrix	117
11.6	The equivalence of the Penrose classes to the Petrov classes	119
11.7	The Petrov classification by the Debever method	120
11.8	Exercises	122

viii	<i>Contents</i>	
Part II	The theory of gravitation	123
12	The Einstein equations and the sources of a gravitational field	125
12.1	Why Riemannian geometry?	125
12.2	Local inertial frames	125
12.3	Trajectories of free motion in Einstein's theory	126
12.4	Special relativity versus gravitation theory	129
12.5	The Newtonian limit of relativity	130
12.6	Sources of the gravitational field	130
12.7	The Einstein equations	131
12.8	Hilbert's derivation of the Einstein equations	132
12.9	The Palatini variational principle	136
12.10	The asymptotically Cartesian coordinates and the asymptotically flat spacetime	136
12.11	The Newtonian limit of Einstein's equations	136
12.12	Examples of sources in the Einstein equations: perfect fluid and dust	140
12.13	Equations of motion of a perfect fluid	143
12.14	The cosmological constant	144
12.15	An example of an exact solution of Einstein's equations: a Bianchi type I spacetime with dust source	145
12.16	* Other gravitation theories	149
12.16.1	The Brans–Dicke theory	149
12.16.2	The Bergmann–Wagoner theory	150
12.16.3	The conformally invariant Canuto theory	150
12.16.4	The Einstein–Cartan theory	150
12.16.5	The bi-metric Rosen theory	151
12.17	Matching solutions of Einstein's equations	151
12.18	The weak-field approximation to general relativity	154
12.19	Exercises	160
13	The Maxwell and Einstein–Maxwell equations and the Kaluza–Klein theory	161
13.1	The Lorentz-covariant description of electromagnetic field	161
13.2	The covariant form of the Maxwell equations	161
13.3	The energy-momentum tensor of an electromagnetic field	162
13.4	The Einstein–Maxwell equations	163
13.5	* The variational principle for the Einstein–Maxwell equations	164
13.6	* The Kaluza–Klein theory	164
13.7	Exercises	167
14	Spherically symmetric gravitational fields of isolated objects	168
14.1	The curvature coordinates	168
14.2	Symmetry inheritance	172

<i>Contents</i>		ix
14.3	Spherically symmetric electromagnetic field in vacuum	172
14.4	The Schwarzschild and Reissner–Nordström solutions	173
14.5	Orbits of planets in the gravitational field of the Sun	176
14.6	Deflection of light rays in the Schwarzschild field	183
14.7	Measuring the deflection of light rays	186
14.8	Gravitational lenses	189
14.9	The spurious singularity of the Schwarzschild solution at $r = 2m$	191
14.10	* Embedding the Schwarzschild spacetime in a flat Riemannian space	196
14.11	Interpretation of the spurious singularity at $r = 2m$; black holes	200
14.12	The Schwarzschild solution in other coordinate systems	202
14.13	The equation of hydrostatic equilibrium	203
14.14	The ‘interior Schwarzschild solution’	206
14.15	* The maximal analytic extension of the Reissner–Nordström solution	207
14.16	* Motion of particles in the Reissner–Nordström spacetime with $e^2 < m^2$	217
14.17	Exercises	219
15	Relativistic hydrodynamics and thermodynamics	222
15.1	Motion of a continuous medium in Newtonian mechanics	222
15.2	Motion of a continuous medium in relativistic mechanics	224
15.3	The equations of evolution of θ , $\sigma_{\alpha\beta}$, $\omega_{\alpha\beta}$ and \dot{u}^α ; the Raychaudhuri equation	228
15.4	Singularities and singularity theorems	230
15.5	Relativistic thermodynamics	231
15.6	Exercises	234
16	Relativistic cosmology I: general geometry	235
16.1	A continuous medium as a model of the Universe	235
16.2	Optical observations in the Universe – part I	237
16.2.1	The geometric optics approximation	237
16.2.2	The redshift	239
16.3	The optical tensors	240
16.4	The apparent horizon	242
16.5	* The double-null tetrad	243
16.6	* The Goldberg–Sachs theorem	245
16.7	* Optical observations in the Universe – part II	253
16.7.1	The area distance	253
16.7.2	The reciprocity theorem	256
16.7.3	Other observable quantities	259
16.8	Exercises	260

17	Relativistic cosmology II: the Robertson–Walker geometry	261
17.1	The Robertson–Walker metrics as models of the Universe	261
17.2	Optical observations in an R–W Universe	263
17.2.1	The redshift	263
17.2.2	The redshift–distance relation	265
17.2.3	Number counts	265
17.3	The Friedmann equations and the critical density	266
17.4	The Friedmann solutions with $\Lambda = 0$	269
17.4.1	The redshift–distance relation in the $\Lambda = 0$ Friedmann models	270
17.5	The Newtonian cosmology	271
17.6	The Friedmann solutions with the cosmological constant	273
17.7	Horizons in the Robertson–Walker models	277
17.8	The inflationary models and the ‘problems’ they solved	282
17.9	The value of the cosmological constant	286
17.10	The ‘history of the Universe’	287
17.11	Invariant definitions of the Robertson–Walker models	290
17.12	Different representations of the R–W metrics	291
17.13	Exercises	293
18	Relativistic cosmology III: the Lemaître–Tolman geometry	294
18.1	The comoving–synchronous coordinates	294
18.2	The spherically symmetric inhomogeneous models	294
18.3	The Lemaître–Tolman model	296
18.4	Conditions of regularity at the centre	300
18.5	Formation of voids in the Universe	301
18.6	Formation of other structures in the Universe	303
18.6.1	Density to density evolution	304
18.6.2	Velocity to density evolution	306
18.6.3	Velocity to velocity evolution	308
18.7	The influence of cosmic expansion on planetary orbits	309
18.8	* Apparent horizons in the L–T model	311
18.9	* Black holes in the evolving Universe	316
18.10	* Shell crossings and necks/wormholes	321
18.10.1	$E < 0$	325
18.10.2	$E = 0$	327
18.10.3	$E > 0$	327
18.11	The redshift	328
18.12	The influence of inhomogeneities in matter distribution on the cosmic microwave background radiation	330
18.13	Matching the L–T model to the Schwarzschild and Friedmann solutions	332

<i>Contents</i>		xi
18.14	* General properties of the Big Bang/Big Crunch singularities in the L–T model	332
18.15	* Extending the L–T spacetime through a shell crossing singularity	337
18.16	* Singularities and cosmic censorship	339
18.17	Solving the ‘horizon problem’ without inflation	347
18.18	* The evolution of $R(t, M)$ versus the evolution of $\rho(t, M)$	348
18.19	* Increasing and decreasing density perturbations	349
18.20	* L&T curio shop	353
18.20.1	Lagging cores of the Big Bang	353
18.20.2	Strange or non-intuitive properties of the L–T model	353
18.20.3	Chances to fit the L–T model to observations	357
18.20.4	An ‘in one ear and out the other’ Universe	357
18.20.5	A ‘string of beads’ Universe	359
18.20.6	Uncertainties in inferring the spatial distribution of matter	359
18.20.7	Is the matter distribution in our Universe fractal?	362
18.20.8	General results related to the L–T models	362
18.21	Exercises	363
19	Relativistic cosmology IV: generalisations of L–T and related geometries	367
19.1	The plane- and hyperbolically symmetric spacetimes	367
19.2	G_3/S_2 -symmetric dust solutions with $R_{,r} \neq 0$	369
19.3	G_3/S_2 -symmetric dust in electromagnetic field, the case $R_{,r} \neq 0$	369
19.3.1	Integrals of the field equations	369
19.3.2	Matching the charged dust metric to the Reissner–Nordström metric	375
19.3.3	Prevention of the Big Crunch singularity by electric charge	377
19.3.4	* Charged dust in curvature and mass-curvature coordinates	379
19.3.5	Regularity conditions at the centre	382
19.3.6	* Shell crossings in charged dust	383
19.4	The Datt–Ruban solution	384
19.5	The Szekeres–Szafron family of solutions	387
19.5.1	The $\beta_{,z} = 0$ subfamily	388
19.5.2	The $\beta_{,z} \neq 0$ subfamily	392
19.5.3	Interpretation of the Szekeres–Szafron coordinates	394
19.5.4	Common properties of the two subfamilies	396
19.5.5	* The invariant definitions of the Szekeres–Szafron metrics	397
19.6	The Szekeres solutions and their properties	399
19.6.1	The $\beta_{,z} = 0$ subfamily	399
19.6.2	The $\beta_{,z} \neq 0$ subfamily	400
19.6.3	* The $\beta_{,z} = 0$ family as a limit of the $\beta_{,z} \neq 0$ family	401
19.7	Properties of the quasi-spherical Szekeres solutions with $\beta_{,z} \neq 0 = \Lambda$	403
19.7.1	Basic physical restrictions	403
19.7.2	The significance of \mathcal{E}	404

xii	<i>Contents</i>	
	19.7.3 Conditions of regularity at the origin	407
	19.7.4 Shell crossings	410
	19.7.5 Regular maxima and minima	413
	19.7.6 The apparent horizons	414
	19.7.7 Szekeres wormholes and their properties	418
	19.7.8 The mass-dipole	419
19.8	* The Goode–Wainwright representation of the Szekeres solutions	421
19.9	Selected interesting subcases of the Szekeres–Szafron family	426
	19.9.1 The Szafron–Wainwright model	426
	19.9.2 The toroidal Universe of Senin	428
19.10	* The discarded case in (19.103)–(19.112)	431
19.11	Exercises	435
20	The Kerr solution	438
20.1	The Kerr–Schild metrics	438
20.2	The derivation of the Kerr solution by the original method	441
20.3	Basic properties	447
20.4	* Derivation of the Kerr metric by Carter’s method – from the separability of the Klein–Gordon equation	452
20.5	The event horizons and the stationary limit hypersurfaces	459
20.6	General geodesics	464
20.7	Geodesics in the equatorial plane	466
20.8	* The maximal analytic extension of the Kerr spacetime	475
20.9	* The Penrose process	486
20.10	Stationary–axisymmetric spacetimes and locally nonrotating observers	487
20.11	* Ellipsoidal spacetimes	490
20.12	A Newtonian analogue of the Kerr solution	493
20.13	A source of the Kerr field?	494
20.14	Exercises	495
21	Subjects omitted from this book	498
	<i>References</i>	501
	<i>Index</i>	518

Figures

1.1	Real planetary orbits.	<i>page</i> 3
1.2	A vehicle flying across a light ray.	5
2.1	Parallel straight lines.	9
2.2	Parallel transport on a curved surface.	11
2.3	Parallel transport on a sphere.	11
6.1	One-parameter family of loops.	41
7.1	A light cone.	61
7.2	A non geodesic null line.	62
7.3	The Petrov classification.	71
8.1	A mapping of a manifold.	74
8.2	Surface-forming vector fields.	82
11.1	The Penrose–Petrov classification.	117
12.1	Fermi coordinates.	127
12.2	Gravitational field of a finite body.	157
14.1	Deflection of light rays.	185
14.2	Measuring the deflection of light, Eddington’s method.	187
14.3	Measuring the deflection of microwaves.	188
14.4	A gravitational lens.	189
14.5	Graph of $\xi(r) = r + 2m \ln \left \frac{r}{2m} - 1 \right $.	193
14.6	The Kruskal diagram.	195
14.7	The surface $\{t = \text{const}, \vartheta = \pi/2\}$ in the Schwarzschild spacetime.	197
14.8	Embedding of the Schwarzschild spacetime in six dimensions projected onto (Z_1, Z_2, Z_3) .	198
14.9	Embedding of the Schwarzschild spacetime in six dimensions projected onto (Z_3, Z_4, Z_5) .	199
14.10	The maximally extended Reissner–Nordström spacetime, $e^2 < m^2$.	211
14.11	The ‘throat’ in the Schwarzschild and in the R–N spacetime.	213
14.12	Embeddings of the $v = 0$ surface.	214
14.13	Surfaces of Fig. 14.12 placed in correct positions.	214
14.14	Maximal extension of the extreme R–N metric.	216

xiv	<i>Figures</i>	
14.15	Embeddings of the $\{t = \text{const}, \vartheta = \pi/2\}$ surface of the extreme R–N metric.	217
15.1	An everywhere concave function.	231
16.1	Refocussing of light in the Universe.	255
16.2	Reciprocity theorem.	256
17.1	$R(t)$ in Friedmann models.	270
17.2	Curves $\dot{R} = 0$ in the (R, λ) plane.	274
17.3	Recollapsing Friedmann models.	275
17.4	$\lambda = \lambda_E$ Friedmann models.	276
17.5	Remaining Friedmann models.	277
17.6	Illustration to (17.62).	280
17.7	The ‘horizon problem’ in R–W.	283
18.1	Black hole in the $E < 0$ L–T model.	318
18.2	3-d graph of black hole formation.	319
18.3	Contours of constant R -value.	320
18.4	The compactified diagram of Fig. 18.1.	322
18.5	The event horizon in the frame of Fig. 18.1.	323
18.6	A neck.	326
18.7	Radial rays in around central singularity.	335
18.8	A shell crossing in comoving coordinates.	339
18.9	A shell crossing in Gautreau coordinates.	340
18.10	A naked shell crossing.	343
18.11	Solutions of $s = S - sS'$.	346
18.12	Solution of the ‘horizon problem’ in L–T.	348
18.13	Evolution of the (t, r) subspace in (18.198).	356
18.14	The model of (18.202)–(18.205).	358
18.15	A ‘string of beads’ Universe.	360
19.1	Stereographic projection to Szekeres–Szafron coordinates.	396
19.2	Circles C1 and C2 projected as disjoint.	417
19.3	Circles C1 and C2 projected one inside the other.	417
19.4	A Szekeres wormhole as a handle.	419
19.5	Szafron–Wainwright model.	428
19.6	A 2-torus.	428
19.7	The 3-torus with the metric (19.311).	429
20.1	Ellipsoids and hyperboloids.	449
20.2	A surface of constant φ .	450
20.3	Space $t = \text{const}$ in the Kerr metric, case $a^2 < m^2$.	460
20.4	Space $t = \text{const}$ in the Kerr metric, case $a^2 = m^2$.	461
20.5	Space $t = \text{const}$ in the Kerr metric, case $a^2 > m^2$.	462
20.6	Light cones in the Kerr spacetime.	463
20.7	$E_{\min}(r)/\mu_0 - 1$ for different values of L_z .	468
20.8	Analogue of Fig. 20.7 for null geodesics.	470

	<i>Figures</i>	xv
20.9	Allowed ranges of λ and ρ for null geodesics, case $a^2 < m^2$.	472
20.10	Allowed ranges of λ and ρ for null geodesics, case $a^2 = m^2$.	473
20.11	Allowed ranges of λ and ρ for null geodesics, case $a^2 > m^2$.	473
20.12	Allowed ranges of λ and ρ for timelike geodesics.	474
20.13	Extending $r > r_+$ along ℓ -field and k -field.	478
20.14	Maximally extended Kerr spacetime.	482
20.15	Axial cross-section through (20.154).	485
20.16	Maximally extended extreme Kerr spacetime.	486
20.17	A discontinuous time coordinate.	489

The scope of this text

General relativity is the currently accepted theory of gravitation. Under this heading one could include a huge amount of material. For the needs of this theory an elaborate mathematical apparatus was created. It has partly become a self-standing sub-discipline of mathematics and physics, and it keeps developing, providing input or inspiration to physical theories that are being newly created (such as gauge field theories, super-gravitation, and, more recently, the brane-world theories). From the gravitation theory, descriptions of astronomical phenomena taking place in strong gravitational fields and in large-scale sub-volumes of the Universe are derived. This part of gravitation theory develops in connection with results of astronomical observations. For the needs of this area, another sophisticated formalism was created (the Parametrised Post-Newtonian formalism). Finally, some tests of the gravitational theory can be carried out in laboratories, either terrestrial or orbital. These tests, their improvements and projects of further tests have led to developments in mathematical methods and in technology that are by now an almost separate branch of science – as an example, one can mention here the (monumentally expensive) search for gravitational waves and the calculations of properties of the wave signals to be expected.

In this situation, no single textbook can attempt to present the whole of gravitation theory, and the present text is no exception. We made the working assumption that relativity is part of physics (this view is not universally accepted!). The purpose of this course is to present those results that are most interesting from the point of view of a physicist, and were historically the most important. We are going to lead the reader through the mathematical part of the theory by a rather short route, but in such a way that the reader does not have to take anything on our word, is able to verify every detail, and, after reading the whole text, will be prepared to solve several problems by him/herself. Further help in this should be provided by the exercises in the text and the literature recommended for further reading.

The introductory part (Chapters 1–7), although assembled by J. Plebański long ago, has never been published in book form.¹ It differs from other courses on relativity in that it introduces differential geometry by a top-down method. We begin with general manifolds,

¹ A part of that material had been semi-published as copies of typewritten notes (Plebański, 1964).

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Frontmatter

[More information](#)

xviii

The scope of this text

on which no structures except tensors are defined, and discuss their basic properties. Then we add the notion of the covariant derivative and affine connection, without introducing the metric yet, and again proceed as far as possible. At that level we define geodesics via parallel displacement and we present the properties of curvature. Only at this point do we introduce the metric tensor and the (pseudo-)Riemannian geometry and specialise the results derived earlier to this case. Then we proceed to the presentation of more detailed topics, such as symmetries, the Bianchi classification and the Petrov classification.

Some of the chapters on classical relativistic topics contain material that, to the best of our knowledge, has never been published in any textbook. In particular, this applies to Chapter 8 (on symmetries) and to Chapter 16 (on cosmology with general geometry). Chapters 18 and 19 (on inhomogeneous cosmologies) are entirely based on original papers. Parts of Chapters 18 and 19 cover the material introduced in A. K.'s monograph on inhomogeneous cosmological models (Kraśniński, 1997). However, the presentation here was thoroughly rearranged, extended, and brought up to date. We no longer briefly mention all contributions to the subject; rather, we have placed the emphasis on complete and clear derivations of the most important results. That material has so far existed only in scattered journal papers and has been assembled into a textbook for the first time (A. K.'s monograph (Kraśniński, 1997) was only a concise review). Taken together, this collection of knowledge constitutes an important and interesting part of relativistic cosmology whose meaning has, unfortunately, not yet been appreciated properly by the astronomical community.

Most figures for this text, even when they look the same as the corresponding figures in the papers cited, were newly generated by A. K. using the program Gnuplot, sometimes on the basis of numerical calculations programmed in Fortran 90. The only figures taken verbatim from other sources are those that illustrated the joint papers by C. Hellaby and A. K.

J. Plebański kindly agreed to be included as a co-author of this text – having done his part of the job more than 30 years ago. Unfortunately, he was not able to participate actively in the writing up and proofreading. He died while the book was being edited. Therefore, the second author (A. K.) is exclusively responsible for any errors that may be found in this book.

Note for the reader. Some parts of this book may be skipped on first reading, since they are not necessary for understanding the material that follows. They are marked by asterisks. Chapters 18 and 19 are expected to be the highlights of this book. However, they go far beyond standard courses of relativity and may be skipped by those readers who wish to remain on the well-beaten track. Hesitating readers may read on, but can skip the sections marked by asterisks.

Andrzej Kraśniński
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