Cambridge University Press
978-0-521-85512-9 - Mesoscopic Physics of Electrons and Photons
Eric Akkermans and Gilles Montambaux
Frontmatter
More information

# MESOSCOPIC PHYSICS OF ELECTRONS AND PHOTONS

Quantum mesoscopic physics covers a whole class of interference effects related to the propagation of waves in complex and random media. These effects are ubiquitous in physics, from the behavior of electrons in metals and semiconductors to the propagation of electromagnetic waves in suspensions such as colloids, and quantum systems like cold atomic gases. This book is a modern account of the problem of coherent wave propagation in random media.

As a solid introduction to quantum mesoscopic physics, this book provides a unified overview of the basic theoretical tools and methods. It highlights the common aspects of the various optical and electronic phenomena involved. With over 200 figures, and exercises throughout, the book is ideal for graduate students in physics, electrical engineering, optics, acoustics and astrophysics. It presents a large number of experimental results that cover a wide range of phenomena from semiconductors to optics, acoustics, and atomic physics. It will also be an important reference for researchers in this rapidly evolving field.

ERIC AKKERMANS is Professor of Physics in the Department of Physics at the Technion, Israel Institute of Technology, Israel. GILLES MONTAMBAUX is Directeur de Recherche at the CNRS, Laboratoire de Physique des Solides, Université Paris-Sud, France. Their research interests include the theory of condensed matter physics, mesoscopic quantum physics, and coherent effects in the propagation of waves in random media.

# Mesoscopic Physics of Electrons and Photons

*Eric Akkermans*
*Technion, Israel Institute of Technology*

*Gilles Montambaux*
*CNRS, Université Paris-Sud*

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521855129

First published 2007

Printed in the United Kingdom at the University Press, Cambridge

*A catalogue record for this publication is available from the British Library*

ISBN 978-0-521-85512-9 hardback

# Contents

Cambridge University Press
978-0-521-85512-9 - Mesoscopic Physics of Electrons and Photons
Eric Akkermans and Gilles Montambaux
Frontmatter
More information

Cambridge University Press
978-0-521-85512-9 - Mesoscopic Physics of Electrons and Photons
Eric Akkermans and Gilles Montambaux
Frontmatter
More information

Cambridge University Press
978-0-521-85512-9 - Mesoscopic Physics of Electrons and Photons
Eric Akkermans and Gilles Montambaux
Frontmatter
More information

Cambridge University Press
978-0-521-85512-9 - Mesoscopic Physics of Electrons and Photons
Eric Akkermans and Gilles Montambaux
Frontmatter
More information

Cambridge University Press
978-0-521-85512-9 - Mesoscopic Physics of Electrons and Photons
Eric Akkermans and Gilles Montambaux
Frontmatter
More information

Cambridge University Press
978-0-521-85512-9 - Mesoscopic Physics of Electrons and Photons
Eric Akkermans and Gilles Montambaux
Frontmatter
More information

# Preface

Wave propagation in random media has been the subject of intense activity for more than two decades. This is now an important area of research, whose frontiers are still fuzzy, and which includes a variety of problems such as wave localization (weak and strong), mesoscopic physics, effects of electron–electron interactions in metals, etc. Moreover, since many disorder effects are not truly specific to a given kind of wave, various approaches have been developed independently in condensed matter physics, in optics, in atomic physics and in acoustics.

A large number of monographs or review articles already exist in the literature and they cover in detail various aspects of the field. Our aim is rather to present the basic common features of the effects of disorder on wave propagation and also to provide the non-specialist reader with the tools necessary to enter and practice this field of research.

Our first concern has been to give a description of the basic physical effects using a single formalism independent of the specific nature of the waves (electrons, electromagnetic waves, etc.). To this purpose, we have started with a detailed presentation of "single-particle" average quantities such as the density of states and elastic collision time using the framework of the so-called "Gaussian model" for the two most important examples of waves, namely Schrödinger and scalar Helmholtz wave equations. We have tried, as much as possible, to make precise the very basic notion of multiple scattering by an ensemble of independent effective scatterers whose scattering cross section may be obtained using standard one-particle scattering theory.

Nevertheless, the quantities of physical interest that are accessible experimentally and used to describe wave propagation in the multiple scattering regime depend essentially on the probability of quantum diffusion which describes the propagation of a wave packet. This probability thus plays a central role and Chapter 4 is devoted to its detailed study. We then see emerging notions such as classical (Diffuson) and coherent (Cooperon) contributions to the probability, which provide basic explanations of the observed physical phenomena such as weak localization corrections to electronic transport, negative magnetoresistance in a magnetic field, coherent backscattering of light, as well as universal conductance fluctuations, optical speckles and mesoscopic effects in orbital magnetism.

It thus happens that all these effects result from the behavior of a single quantity, namely the probability of quantum diffusion. However, in spite of the common background shared

Cambridge University Press
978-0-521-85512-9 - Mesoscopic Physics of Electrons and Photons
Eric Akkermans and Gilles Montambaux
Frontmatter
More information

by optics and electronics of random media, each of these domains has its own specificity which allows us to develop complementary approaches. For instance, the continuous change in the relative phases of electronic wave functions that can be achieved using a magnetic field or a vector potential has no obvious equivalent in optics. On the other hand, it is possible in optics to change directions of incident and outgoing beams and from this angular spectroscopy to trace back correlations between angular channels.

We have made a special effort to try to keep this book accessible to the largest audience, starting at a graduate level in physics with an elementary acquaintance of quantum mechanics as a prerequisite. We have also skipped a number of interesting but perhaps too specialized issues among which are the study of quantum dots, relations between spectral and transport quantities, strong localization and the Anderson metal–insulator transition, electronic ballistic billiards where "quantum complexity" does not result from disorder but instead from the boundary shape, and metal–superconductor interfaces. All these aspects reflect the richness of the field of "quantum mesoscopic physics" to which this book constitutes a first introduction.

A pleasant task in finishing the writing of a book is certainly the compilation of acknowledgments to all those who have helped us at various stages of the elaboration and writing, either through discussions, criticisms and especially encouragement and support: O. Assaf, H. Bouchiat, B. Huard, J. Cayssol, C. Cohen-Tannoudji, N. Dupuis, D. Estève, A. Georges, S. Guéron, M. Kouchnir, R. Maynard, F. Piéchon, H. Pothier, B. Reulet, B. Shapiro, B. van Tiggelen, D. Ullmo, J. Vidal, E. Wolf. We wish to single out the contribution of C. Texier for his endless comments, suggestions, and corrections which have certainly contributed to improve the quality of this book. Dov Levine accepted to help us in translating the book into English. This was a real challenge and we wish to thank him for his patience. We also wish to thank G. Bazalitsky for producing most of the figures with much dedication.

This venture was in many respects a roller-coaster ride and the caring support of Anne-Marie and Tirza was all the more precious.

Throughout this book, we use the (SI) international unit system, except in Chapter 13. The Planck constant $\hbar$ is generally taken equal to unity, in particular throughout Chapter 4. In the chapters where we think it is important to restore it, we have mentioned this at the beginning of the chapter. In order to simplify the notation, we have sometimes partially restored $\hbar$ in a given expression, especially when the correspondence between energy and frequency is straightforward.

To maintain homogeneous and consistent notation throughout a book which covers fields that are usually studied separately is a kind of challenge that, unfortunately, we have not always been able to overcome.

We have chosen not to give an exhaustive list of references, but instead to quote papers either for their obvious pedagogical value or because they discuss a particular point presented for instance as an exercise.

# How to use this book

This book is intended to provide self-contained material which will allow the reader to derive the main results. It does not require anything other than an elementary background in general physics and quantum mechanics.

We have chosen to treat in a parallel way similar concepts occurring in the propagation of electrons and light. The important background concepts are given in Chapter 4, where the notion of *probability of quantum diffusion* in random media is developed. This is a central quantity to which all physical quantities described in this book may be related.

This book is not intended to be read linearly. We have structured it into chapters which are supposed to present the main concepts, and appendices which focus on specific aspects or details of calculation. This choice may sometimes appear arbitrary. For example, the Landauer formalism is introduced in an appendix (A7.2), where it is developed for the diffusive regime, which to our knowledge has not been done in the textbook literature. The standard description of weak localization is presented within the Kubo formalism in the core of Chapter 7, while the Landauer picture of weak localization is developed in Appendix A7.2.

We suggest here a guide for lectures. Although we have tried to emphasize analogies between interference effects in the propagation of electrons and light, we propose two outlines, for two introductory courses respectively on the physics of electrons and the physics of light. We believe that, during the course of study, the interested reader will benefit from the analogies developed between the two fields, for example the relations between speckle fluctuations in optics and universal conductance fluctuations in electronics.

### ***Quantum transport in electronics***
### ***Main course***

1 *Introduction: mesoscopic physics* Provides a unified and general description of interference and multiple scattering effects in disordered systems. Introduces the physical problems and the main quantities of interest, the different length scales such as the mean free path and the phase coherence length, the notions of multiple scattering and disorder average. Relates the physical properties to the probability of returning to the origin in a random medium. Notion of quantum crossing. Analogies between electronics and optics.

Cambridge University Press
978-0-521-85512-9 - Mesoscopic Physics of Electrons and Photons
Eric Akkermans and Gilles Montambaux
Frontmatter
More information

2 *Wave equations and models of disorder* Schrödinger equation for electrons in solids and Helmholtz equation for electromagnetic waves. Gaussian, Edwards, Anderson models for disorder.

3 *Perturbation theory* Presents the minimal formalism of Green’s functions necessary for the notions developed further in the book. Multiple scattering and weak disorder expansion.

4.1–4.6 *Probability of quantum diffusion* Definition and description of essential concepts and tools used throughout the book. Iterative structure for the quantum probability, solution of a diffusion equation. Diffuson and Cooperon contributions. Formalism developed in real space. May also be useful to look at the reciprocal space formalism developed in Appendix A4.1.

6 *Dephasing* Proposes a general picture for dephasing and describes several mechanisms due to electron coupling to external parameters or degrees of freedom: magnetic field, Aharonov–Bohm flux, spin-orbit coupling and magnetic impurities. May be skipped at the introductory level, except for magnetic field and Aharonov–Bohm effects.

7 *Electronic transport* Deals with calculations of the average conductivity and of the weak localization correction. The latter is related to the probability of return to the origin for a diffusive particle. Applications to various geometries, plane, ring, cylinder, dimensionality effects. Section A7.2 is a comprehensive appendix on the Landauer formalism for diffusive systems.

10 *Spectral properties of disordered metals* Generalities on random matrix theory. Spectral correlation functions for disordered systems. The last part requires knowledge of correlation functions calculated in Appendix A4.4.

11 *Universal conductance fluctuations* Detailed calculation of the conductance fluctuations in the Kubo formalism, using the diagrammatics developed in Chapters 4 and 7. Many physical discussions on the role of various external parameters.

13 *Interactions and diffusion* Important chapter on the role of electron–electron interaction and its interplay with disorder. Density of states anomaly, correction to the conductivity. Important discussions about lifetime of quasiparticles and phase coherence time.

14 *Persistent currents* Can be considered optional. Thermodynamics and orbital magnetism of mesoscopic systems. Problematics of persistent currents, from the very simple one-dimensional description to the effect of disorder and interaction.

### *Optional*

5 *Properties of the diffusion equation* Provides a comprehensive and self-contained account of properties of the diffusion equation. Diffusion in finite systems, boundary conditions, diffusion on graphs.

*Miscellaneous* Various appendices are beyond an introductory level, or are not necessary in a first course on mesoscopic physics. They either develop technicalities such as Hikami boxes (A4.2), Cooperon in a time dependent magnetic field (A6.3), or important extensions

Cambridge University Press
978-0-521-85512-9 - Mesoscopic Physics of Electrons and Photons
Eric Akkermans and Gilles Montambaux
Frontmatter
More information

such as anisotropic collisions developed in A4.3 and their effect on weak localization (A7.4) and universal conductance fluctuations (A11.1). The Landauer formalism for diffusive systems is developed in A7.2 for the average conductance and the weak localization correction and in A11.2 for conductance fluctuations.

### *Propagation of light in random media*
### *Main course*

This course provides a comprehensive introduction to the propagation of light in random media. It describes coherent effects in multiple scattering: coherent backscattering, diffusing wave spectroscopy and angular and time correlations of speckle patterns. Compared to coherent electronic transport, this course emphasizes notions specific to electromagnetic waves such as angular correlations of transmission (or reflection) coefficients in open space geometry, correlation between channels in a wave guide geometry, as well as the effects of the dynamics of scatterers.

*1–4* These chapters are common to the two courses. In addition section 4.6 introduces the important formalism of radiative transfer which is developed in Appendix A5.2.

6 *Dephasing* Generalities on the mechanism of dephasing. Application to the polarization of electromagnetic waves, dynamics of the scatterers and dephasing associated with quantum internal degrees of freedom for the case of scattering of photons by cold atoms (the last topic is treated in Appendix A6.5).

8 *Coherent backscattering of light* Physics of the albedo, reflection coefficient of a diffusive medium. Coherent contribution (Cooperon) to the albedo, and its angular dependence. Uses the formalism developed in Chapter 4. Polarization and absorption effects (see also section 6.6). Extensive discussion of experimental results and coherent backscattering in various physical contexts. This chapter relies upon the results of section 5.6.

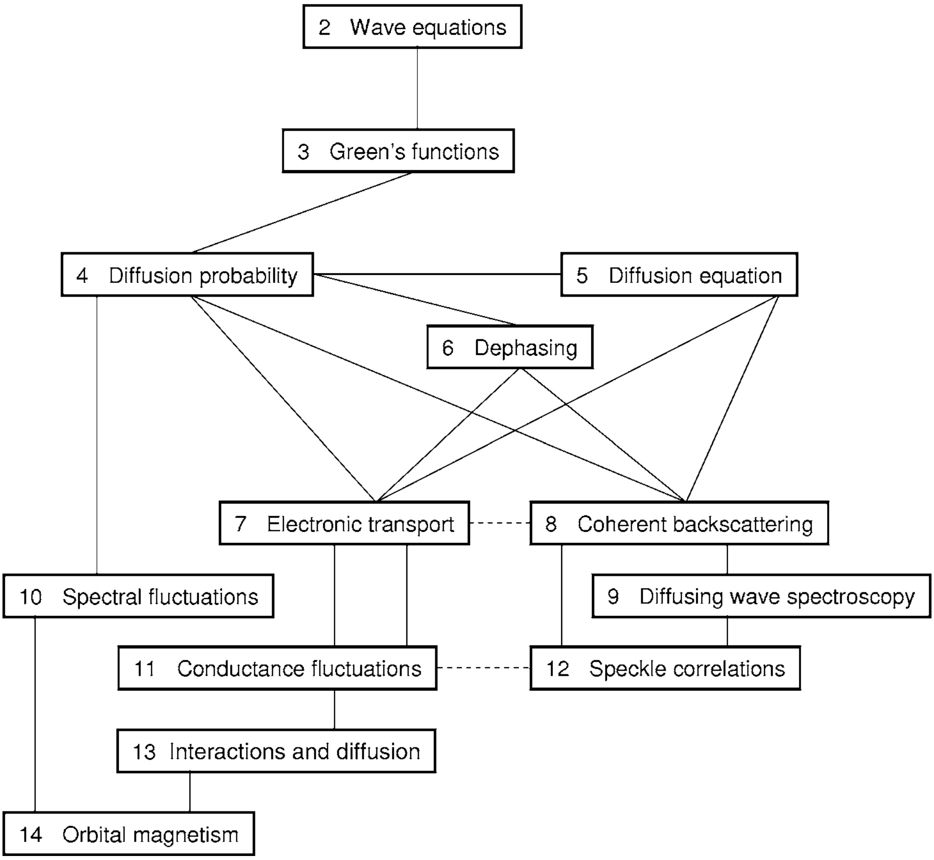
9 *Diffusing wave spectroscopy* An experimental technique routinely used to probe the dynamics of scatterers. Calculations result from a simple generalization of the formalism of Chapter 4. Interesting conceptually since diffusing wave spectroscopy exhibits the simplest example of decoherence introduced in a controlled way. Also interesting because this method probes the distribution of multiple scattering trajectories (reflection versus transmission experiments). Study of sections 5.6 and 5.7 is recommended.

12 *Correlations of speckle patterns* Analysis of a speckle pattern. Angular correlations of transmission coefficients. Classification and detailed calculation of the successive contributions $C_1$, $C_2$ and $C_3$. Simple description in terms of quantum crossings. Rayleigh law. Use of the Landauer formalism to relate speckle correlations to universal conductance fluctuations.

### *Optional*

5 *Properties of the diffusion equation* Solutions of the diffusion equation in quasi-one-dimensional geometries, useful for calculations developed in Chapters 8, 9 and 12. Important appendix A5.2 on radiative transfer.

*Miscellaneous* Various appendices are useful reminders for beginners, for example A2.1 on scattering theory and A2.3 on light scattering by individual scatterers (Rayleigh, Rayleigh–Gans, Mie, resonant). Other appendices go beyond a course at the introductory level, either because they develop additional technicalities such as Hikami boxes (A4.2), useful for the reader interested in detailed calculations of Chapter 12, or because they present additional aspects of multiple scattering of light by random media such as spatial correlations of light intensity (A12.1) or anisotropic collisions (A4.3) and their consequences. The Landauer formalism for diffusive systems is used extensively in Chapter 12 on speckle correlations. Appendix A6.5 gives an overview of the technical tools needed to study the specific problem of multiple scattering of photons by cold atoms.



Topics developed in this book. Lines represent logical links between chapters.