MESOSCOPIC PHYSICS OF ELECTRONS AND PHOTONS

Quantum mesoscopic physics covers a whole class of interference effects related to the propagation of waves in complex and random media. These effects are ubiquitous in physics, from the behavior of electrons in metals and semiconductors to the propagation of electromagnetic waves in suspensions such as colloids, and quantum systems like cold atomic gases. This book is a modern account of the problem of coherent wave propagation in random media.

As a solid introduction to quantum mesoscopic physics, this book provides a unified overview of the basic theoretical tools and methods. It highlights the common aspects of the various optical and electronic phenomena involved. With over 200 figures, and exercises throughout, the book is ideal for graduate students in physics, electrical engineering, optics, acoustics and astrophysics. It presents a large number of experimental results that cover a wide range of phenomena from semiconductors to optics, acoustics, and atomic physics. It will also be an important reference for researchers in this rapidly evolving field.

ERIC AKKERMANS is Professor of Physics in the Department of Physics at the Technion, Israel Institute of Technology, Israel. GILLES MONTAMBAUX is Directeur de Recherche at the CNRS, Laboratoire de Physique des Solides, Université Paris-Sud, France. Their research interests include the theory of condensed matter physics, mesoscopic quantum physics, and coherent effects in the propagation of waves in random media.
Mesoscopic Physics
of Electrons and Photons

Eric Akkermans
Technion, Israel Institute of Technology

Gilles Montambaux
CNRS, Université Paris-Sud
# Contents

Preface                                    page xiii

How to use this book                      xv

1 Introduction: mesoscopic physics         1

1.1 Interference and disorder             1

1.2 The Aharonov–Bohm effect              4

1.3 Phase coherence and the effect of disorder   7

1.4 Average coherence and multiple scattering 9

1.5 Phase coherence and self-averaging: universal fluctuations 12

1.6 Spectral correlations                  14

1.7 Classical probability and quantum crossings 15

1.7.1 Quantum crossings                    17

1.8 Objectives                             18

2 Wave equations in random media           31

2.1 Wave equations                         31

2.1.1 Electrons in a disordered metal       31

2.1.2 Electromagnetic wave equation – Helmholtz equation 32

2.1.3 Other examples of wave equations      33

2.2 Models of disorder                     36

2.2.1 The Gaussian model                   37

2.2.2 Localized impurities: the Edwards model 39

2.2.3 The Anderson model                   41

Appendix A2.1: Theory of elastic collisions and single scattering 43

A2.1.1 Asymptotic form of the solutions     44

A2.1.2 Scattering cross section and scattered flux 46

A2.1.3 Optical theorem                     47

A2.1.4 Born approximation                   51

Appendix A2.2: Reciprocity theorem         54

Appendix A2.3: Light scattering             56

A2.3.1 Classical Rayleigh scattering        56
### Contents

A2.3.2 Mie scattering 60
A2.3.3 Atom–photon scattering in the dipole approximation 61

3 Perturbation theory 70
  3.1 Green’s functions 71
    3.1.1 Green’s function for the Schrödinger equation 71
    3.1.2 Green’s function for the Helmholtz equation 77
  3.2 Multiple scattering expansion 79
    3.2.1 Dyson equation 79
    3.2.2 Self-energy 82
  3.3 Average Green’s function and average density of states 86
Appendix A3.1: Short range correlations 88

4 Probability of quantum diffusion 92
  4.1 Definition 92
  4.2 Free propagation 95
  4.3 Drude–Boltzmann approximation 96
  4.4 Diffuson or ladder approximation 97
  4.5 The Diffuson at the diffusion approximation 102
  4.6 Coherent propagation: the Cooperon 104
  4.7 Radiative transfer 110
Appendix A4.1: Diffuson and Cooperon in reciprocal space 113
  A4.1.1 Collisionless probability $P_0(q, \omega)$ 114
  A4.1.2 The Diffuson 115
  A4.1.3 The Cooperon 117
Appendix A4.2: Hikami boxes and Diffuson crossings 120
  A4.2.1 Hikami boxes 120
  A4.2.2 Normalization of the probability and renormalization of the diffusion coefficient 125
  A4.2.3 Crossing of two Diffusons 128
Appendix A4.3: Anisotropic collisions and transport mean free path 132
Appendix A4.4: Correlation of diagonal Green’s functions 138
Appendix A4.5: Other correlation functions 142
  A4.5.1 Correlations of Green’s functions 142
  A4.5.2 A Ward identity 145
  A4.5.3 Correlations of wave functions 145

5 Properties of the diffusion equation 148
  5.1 Introduction 148
  5.2 Heat kernel and recurrence time 149
    5.2.1 Heat kernel – probability of return to the origin 149
    5.2.2 Recurrence time 151
  5.3 Free diffusion 152
  5.4 Diffusion in a periodic box 155
## Contents

5.5 Diffusion in finite systems 156  
5.5.1 Diffusion time and Thouless energy 156  
5.5.2 Boundary conditions for the diffusion equation 156  
5.5.3 Finite volume and “zero mode” 157  
5.5.4 Diffusion in an anisotropic domain 158  
5.6 One-dimensional diffusion 159  
5.6.1 The ring: periodic boundary conditions 160  
5.6.2 Absorbing boundaries: connected wire 161  
5.6.3 Reflecting boundaries: isolated wire 162  
5.6.4 Semi-infinite wire 164  
5.7 The image method 165  
Appendix A5.1: Validity of the diffusion approximation in an infinite medium 166  
Appendix A5.2: Radiative transfer equation 168  
A5.2.1 Total intensity 168  
A5.2.2 Diffuse intensity 170  
A5.2.3 Boundary conditions 172  
A5.2.4 Slab illuminated by an extended source 175  
A5.2.5 Semi-infinite medium illuminated by a collimated beam 176  
Appendix A5.3: Multiple scattering in a finite medium 177  
A5.3.1 Multiple scattering in a half-space: the Milne problem 177  
A5.3.2 Diffusion in a finite medium 180  
Appendix A5.4: Spectral determinant 182  
Appendix A5.5: Diffusion in a domain of arbitrary shape – Weyl expansion 184  
Appendix A5.6: Diffusion on graphs 187  
A5.6.1 Spectral determinant on a graph 187  
A5.6.2 Examples 191  
A5.6.3 Thermodynamics, transport and spectral determinant 193  
6 Dephasing 195  
6.1 Dephasing and multiple scattering 195  
6.1.1 Generalities 195  
6.1.2 Mechanisms for dephasing: introduction 196  
6.1.3 The Goldstone mode 199  
6.2 Magnetic field and the Cooperon 199  
6.3 Probability of return to the origin in a uniform magnetic field 203  
6.4 Probability of return to the origin for an Aharonov–Bohm flux 205  
6.4.1 The ring 206  
6.4.2 The cylinder 208  
6.5 Spin-orbit coupling and magnetic impurities 210  
6.5.1 Transition amplitude and effective interaction potential 210  
6.5.2 Total scattering time 212  
6.5.3 Structure factor 214
Contents

6.5.4 The Diffuson 219
6.5.5 The Cooperon 221
6.5.6 The diffusion probability 223
6.5.7 The Cooperon $X_c$ 224

6.6 Polarization of electromagnetic waves 226
6.6.1 Elastic mean free path 227
6.6.2 Structure factor 228
6.6.3 Classical intensity 231
6.6.4 Coherent backscattering 233

6.7 Dephasing and motion of scatterers 234
6.7.1 General expression for the phase shift 234
6.7.2 Dephasing associated with the Brownian motion of the scatterers 237

6.8 Dephasing or decoherence? 238
Appendix A6.1: Aharonov–Bohm effect in an infinite plane 240
Appendix A6.2: Functional representation of the diffusion equation 242
   A6.2.1 Functional representation 242
   A6.2.2 Brownian motion and magnetic field 244
Appendix A6.3: The Cooperon in a time-dependent field 247
Appendix A6.4: Spin-orbit coupling and magnetic impurities, a heuristic point of view 251
   A6.4.1 Spin-orbit coupling 251
   A6.4.2 Magnetic impurities 254
Appendix A6.5: Decoherence in multiple scattering of light by cold atoms 256
   A6.5.1 Scattering amplitude and atomic collision time 256
   A6.5.2 Elementary atomic vertex 257
   A6.5.3 Structure factor 262

7 Electronic transport 270
7.1 Introduction 270
7.2 Incoherent contribution to conductivity 273
   7.2.1 Drude–Boltzmann approximation 273
   7.2.2 The multiple scattering regime: the Diffuson 276
   7.2.3 Transport time and vertex renormalization 278
7.3 Cooperon contribution 279
7.4 The weak localization regime 281
   7.4.1 Dimensionality effect 282
   7.4.2 Finite size conductors 284
   7.4.3 Temperature dependence 285
7.5 Weak localization in a magnetic field 286
   7.5.1 Negative magnetoresistance 286
   7.5.2 Spin-orbit coupling and magnetic impurities 290
7.6 Magnetoresistance and Aharonov–Bohm flux
   7.6.1 Ring 293
   7.6.2 Long cylinder: the Sharvin–Sharvin effect 294
   7.6.3 Remark on the Webb and Sharvin–Sharvin experiments: $\phi_0$ versus $\phi_0/2$ 295
   7.6.4 The Aharonov–Bohm effect in an infinite plane 296

Appendix A7.1: Kubo formulae 296
   A7.1.1 Conductivity and dissipation 296
   A7.1.2 Density-density response function 301

Appendix A7.2: Conductance and transmission 302
   A7.2.1 Introduction: Landauer formula 302
   A7.2.2 From Kubo to Landauer 305
   A7.2.3 Average conductance and transmission 307
   A7.2.4 Boundary conditions and impedance matching 311
   A7.2.5 Weak localization correction in the Landauer formalism 313
   A7.2.6 Landauer formalism for waves 314

Appendix A7.3: Real space description of conductivity 315
Appendix A7.4: Weak localization correction and anisotropic collisions 317

8 Coherent backscattering of light 320
   8.1 Introduction 320
   8.2 The geometry of the albedo 321
      8.2.1 Definition 321
      8.2.2 Albedo of a diffusive medium 322
   8.3 The average albedo 324
      8.3.1 Incoherent albedo: contribution of the Diffuson 324
      8.3.2 The coherent albedo: contribution of the Cooperon 326
   8.4 Time dependence of the albedo and study of the triangular cusp 330
   8.5 Effect of absorption 333
   8.6 Coherent albedo and anisotropic collisions 334
   8.7 The effect of polarization 336
      8.7.1 Depolarization coefficients 336
      8.7.2 Coherent albedo of a polarized wave 337
   8.8 Experimental results 339
      8.8.1 The triangular cusp 340
      8.8.2 Decrease of the height of the cone 341
      8.8.3 The role of absorption 343
   8.9 Coherent backscattering at large 347
      8.9.1 Coherent backscattering and the “glory” effect 347
      8.9.2 Coherent backscattering and opposition effect in astrophysics 348
      8.9.3 Coherent backscattering by cold atomic gases 349
      8.9.4 Coherent backscattering effect in acoustics 352
9 Diffusing wave spectroscopy 354
9.1 Introduction 354
9.2 Dynamic correlations of intensity 355
9.3 Single scattering: quasi-elastic light scattering 357
9.4 Multiple scattering: diffusing wave spectroscopy 358
9.5 Influence of the geometry on the time correlation function 359
  9.5.1 Reflection by a semi-infinite medium 359
  9.5.2 Comparison between $G_f(T)$ and $\alpha_c(\theta)$ 361
  9.5.3 Reflection from a finite slab 363
  9.5.4 Transmission 364
Appendix A9.1: Collective motion of scatterers 367

10 Spectral properties of disordered metals 370
10.1 Introduction 370
  10.1.1 Level repulsion and integrability 371
  10.1.2 Energy spectrum of a disordered metal 373
10.2 Characteristics of spectral correlations 374
10.3 Poisson distribution 376
10.4 Universality of spectral correlations: random matrix theory 377
  10.4.1 Level repulsion in $2 \times 2$ matrices 377
  10.4.2 Distribution of eigenvalues for $N \times N$ matrices 380
  10.4.3 Spectral properties of random matrices 382
10.5 Spectral correlations in the diffusive regime 385
  10.5.1 Two-point correlation function 386
  10.5.2 The ergodic limit 390
  10.5.3 Free diffusion limit 391
Appendix A10.1: The GOE–GUE transition 394

11 Universal conductance fluctuations 396
11.1 Introduction 396
11.2 Conductivity fluctuations 399
  11.2.1 Fluctuations of the density of states 402
  11.2.2 Fluctuations of the diffusion coefficient 405
11.3 Universal conductance fluctuations 406
11.4 Effect of external parameters 409
  11.4.1 Energy dependence 409
  11.4.2 Temperature dependence 409
  11.4.3 Phase coherence and the mesoscopic regime 411
  11.4.4 Magnetic field dependence 415
  11.4.5 Motion of scatterers 418
  11.4.6 Spin-orbit coupling and magnetic impurities 419
Appendix A11.1: Universal conductance fluctuations and anisotropic collisions 422
Appendix A11.2: Conductance fluctuations in the Landauer formalism 424
12 Correlations of speckle patterns 427
12.1 What is a speckle pattern? 427
12.2 How to analyze a speckle pattern 428
12.3 Average transmission coefficient 433
12.4 Angular correlations of the transmitted light 435
  12.4.1 Short range $C^{(1)}$ correlations 435
  12.4.2 Long range correlations $C^{(2)}$ 439
  12.4.3 Two-crossing contribution and $C^{(3)}$ correlation 441
  12.4.4 Relation with universal conductance fluctuations 445
12.5 Speckle correlations in the time domain 446
  12.5.1 Time dependent correlations $C^{(1)}(t)$ and $C^{(2)}(t)$ 447
  12.5.2 Time dependent correlation $C^{(3)}(t)$ 450
12.6 Spectral correlations of speckle patterns 452
12.7 Distribution function of the transmission coefficients 453
  12.7.1 Rayleigh distribution law 454
  12.7.2 Gaussian distribution of the transmission coefficient $T_a$ 455
  12.7.3 Gaussian distribution of the electrical conductance 456
Appendix A12.1: Spatial correlations of light intensity 458
  A12.1.1 Short range correlations 459
  A12.1.2 Long range correlations 461

13 Interactions and diffusion 465
13.1 Introduction 465
13.2 Screened Coulomb interaction 466
13.3 Hartree–Fock approximation 468
13.4 Density of states anomaly 470
  13.4.1 Static interaction 470
  13.4.2 Tunnel conductance and density of states anomaly 475
  13.4.3 Dynamically screened interaction 478
  13.4.4 Capacitive effects 481
13.5 Correction to the conductivity 483
13.6 Lifetime of a quasiparticle 487
  13.6.1 Introduction: Landau theory and disorder 487
  13.6.2 Lifetime at zero temperature 487
  13.6.3 Quasiparticle lifetime at finite temperature 494
  13.6.4 Quasiparticle lifetime at the Fermi level 495
13.7 Phase coherence 498
  13.7.1 Introduction 498
  13.7.2 Phase coherence in a fluctuating electric field 499
  13.7.3 Phase coherence time in dimension $d = 1$ 502
  13.7.4 Phase coherence and quasiparticle relaxation 506
  13.7.5 Phase coherence time in dimensions $d = 2$ and $d = 3$ 509
  13.7.6 Measurements of the phase coherence time $\tau_{ee}^{\phi}$ 510
Appendix A13.1: Screened Coulomb potential in confined geometry 512
Appendix A13.2: Lifetime in the absence of disorder 514

14 Orbital magnetism and persistent currents 516
14.1 Introduction 516
14.2 Free electron gas in a uniform field 518
14.2.1 A reminder: the case of no disorder 518
14.2.2 Average magnetization 521
14.2.3 Fluctuations of the magnetization 522
14.3 Effect of Coulomb interaction 524
14.3.1 Hartree–Fock approximation 525
14.3.2 Cooper renormalization 526
14.3.3 Finite temperature 528
14.4 Persistent current in a ring 528
14.4.1 Clean one-dimensional ring: periodicity and parity effects 529
14.4.2 Average current 534
14.5 Diffusive limit and persistent current 536
14.5.1 Typical current of a disordered ring 537
14.5.2 Effect of the Coulomb interaction on the average current 539
14.5.3 Persistent current and spin-orbit coupling 542
14.5.4 A brief overview of experiments 543
Appendix A14.1: Average persistent current in the canonical ensemble 545

15 Formulary 547
15.1 Density of states and conductance 547
15.2 Fourier transforms: definitions 548
15.3 Collisionless probability $P_0(r,r',t)$ 548
15.4 Probability $P(r,r',t)$ 548
15.5 Wigner–Eckart theorem and 3j-symbols 551
15.6 Miscellaneous 552
15.7 Poisson formula 558
15.8 Temperature dependences 558
15.9 Characteristic times introduced in this book 559

References 561
Index 582
Wave propagation in random media has been the subject of intense activity for more than two decades. This is now an important area of research, whose frontiers are still fuzzy, and which includes a variety of problems such as wave localization (weak and strong), mesoscopic physics, effects of electron–electron interactions in metals, etc. Moreover, since many disorder effects are not truly specific to a given kind of wave, various approaches have been developed independently in condensed matter physics, in optics, in atomic physics and in acoustics.

A large number of monographs or review articles already exist in the literature and they cover in detail various aspects of the field. Our aim is rather to present the basic common features of the effects of disorder on wave propagation and also to provide the non-specialist reader with the tools necessary to enter and practice this field of research.

Our first concern has been to give a description of the basic physical effects using a single formalism independent of the specific nature of the waves (electrons, electromagnetic waves, etc.). To this purpose, we have started with a detailed presentation of “single-particle” average quantities such as the density of states and elastic collision time using the framework of the so-called “Gaussian model” for the two most important examples of waves, namely Schrödinger and scalar Helmholtz wave equations. We have tried, as much as possible, to make precise the very basic notion of multiple scattering by an ensemble of independent effective scatterers whose scattering cross section may be obtained using standard one-particle scattering theory.

Nevertheless, the quantities of physical interest that are accessible experimentally and used to describe wave propagation in the multiple scattering regime depend essentially on the probability of quantum diffusion which describes the propagation of a wave packet. This probability thus plays a central role and Chapter 4 is devoted to its detailed study. We then see emerging notions such as classical (Diffuson) and coherent (Cooperon) contributions to the probability, which provide basic explanations of the observed physical phenomena such as weak localization corrections to electronic transport, negative magnetoresistance in a magnetic field, coherent backscattering of light, as well as universal conductance fluctuations, optical speckles and mesoscopic effects in orbital magnetism.

It thus happens that all these effects result from the behavior of a single quantity, namely the probability of quantum diffusion. However, in spite of the common background shared
by optics and electronics of random media, each of these domains has its own specificity which allows us to develop complementary approaches. For instance, the continuous change in the relative phases of electronic wave functions that can be achieved using a magnetic field or a vector potential has no obvious equivalent in optics. On the other hand, it is possible in optics to change directions of incident and outgoing beams and from this angular spectroscopy to trace back correlations between angular channels.

We have made a special effort to try to keep this book accessible to the largest audience, starting at a graduate level in physics with an elementary acquaintance of quantum mechanics as a prerequisite. We have also skipped a number of interesting but perhaps too specialized issues among which are the study of quantum dots, relations between spectral and transport quantities, strong localization and the Anderson metal–insulator transition, electronic ballistic billiards where “quantum complexity” does not result from disorder but instead from the boundary shape, and metal–superconductor interfaces. All these aspects reflect the richness of the field of “quantum mesoscopic physics” to which this book constitutes a first introduction.

A pleasant task in finishing the writing of a book is certainly the compilation of acknowledgments to all those who have helped us at various stages of the elaboration and writing, either through discussions, criticisms and especially encouragement and support: O. Assaf, H. Bouchiat, B. Huard, J. Cayssol, C. Cohen-Tannoudji, N. Dupuis, D. Estève, A. Georges, S. Guéron, M. Kouchnir, F. Maynard, F. Piéchon, H. Pothier, B. Reulet, B. Shapiro, B. van Tiggelen, D. Ullmo, J. Vidal, E. Wolf. We wish to single out the contribution of C. Texier for his endless comments, suggestions, and corrections which have certainly contributed to improve the quality of this book. Dov Levine accepted to help us in translating the book into English. This was a real challenge and we wish to thank him for his patience. We also wish to thank G. Bazalitsky for producing most of the figures with much dedication.

This venture was in many respects a roller-coaster ride and the caring support of Anne-Marie and Tirza was all the more precious.

Throughout this book, we use the (SI) international unit system, except in Chapter 13. The Planck constant $\hbar$ is generally taken equal to unity, in particular throughout Chapter 4. In the chapters where we think it is important to restore it, we have mentioned this at the beginning of the chapter. In order to simplify the notation, we have sometimes partially restored $\hbar$ in a given expression, especially when the correspondence between energy and frequency is straightforward.

To maintain homogeneous and consistent notation throughout a book which covers fields that are usually studied separately is a kind of challenge that, unfortunately, we have not always been able to overcome.

We have chosen not to give an exhaustive list of references, but instead to quote papers either for their obvious pedagogical value or because they discuss a particular point presented for instance as an exercise.
How to use this book

This book is intended to provide self-contained material which will allow the reader to derive the main results. It does not require anything other than an elementary background in general physics and quantum mechanics.

We have chosen to treat in a parallel way similar concepts occurring in the propagation of electrons and light. The important background concepts are given in Chapter 4, where the notion of probability of quantum diffusion in random media is developed. This is a central quantity to which all physical quantities described in this book may be related.

This book is not intended to be read linearly. We have structured it into chapters which are supposed to present the main concepts, and appendices which focus on specific aspects or details of calculation. This choice may sometimes appear arbitrary. For example, the Landauer formalism is introduced in an appendix (A7.2), where it is developed for the diffusive regime, which to our knowledge has not been done in the textbook literature. The standard description of weak localization is presented within the Kubo formalism in the core of Chapter 7, while the Landauer picture of weak localization is developed in Appendix A7.2.

We suggest here a guide for lectures. Although we have tried to emphasize analogies between interference effects in the propagation of electrons and light, we propose two outlines, for two introductory courses respectively on the physics of electrons and the physics of light. We believe that, during the course of study, the interested reader will benefit from the analogies developed between the two fields, for example the relations between speckle fluctuations in optics and universal conductance fluctuations in electronics.

Quantum transport in electronics

Main course

1 Introduction: mesoscopic physics Provides a unified and general description of interference and multiple scattering effects in disordered systems. Introduces the physical problems and the main quantities of interest, the different length scales such as the mean free path and the phase coherence length, the notions of multiple scattering and disorder average. Relates the physical properties to the probability of returning to the origin in a random medium. Notion of quantum crossing. Analogies between electronics and optics.
How to use this book

2 Wave equations and models of disorder  Schrödinger equation for electrons in solids and Helmholtz equation for electromagnetic waves. Gaussian, Edwards, Anderson models for disorder.

3 Perturbation theory  Presents the minimal formalism of Green’s functions necessary for the notions developed further in the book. Multiple scattering and weak disorder expansion.

4.1–4.6 Probability of quantum diffusion  Definition and description of essential concepts and tools used throughout the book. Iterative structure for the quantum probability, solution of a diffusion equation. Diffuson and Cooperon contributions. Formalism developed in real space. May also be useful to look at the reciprocal space formalism developed in Appendix A4.1.

6 Dephasing  Proposes a general picture for dephasing and describes several mechanisms due to electron coupling to external parameters or degrees of freedom: magnetic field, Aharonov–Bohm flux, spin-orbit coupling and magnetic impurities. May be skipped at the introductory level, except for magnetic field and Aharonov–Bohm effects.

7 Electronic transport  Deals with calculations of the average conductivity and of the weak localization correction. The latter is related to the probability of return to the origin for a diffusive particle. Applications to various geometries, plane, ring, cylinder, dimensionality effects. Section A7.2 is a comprehensive appendix on the Landauer formalism for diffusive systems.

10 Spectral properties of disordered metals  Generalities on random matrix theory. Spectral correlation functions for disordered systems. The last part requires knowledge of correlation functions calculated in Appendix A4.4.

11 Universal conductance fluctuations  Detailed calculation of the conductance fluctuations in the Kubo formalism, using the diagrammatics developed in Chapters 4 and 7. Many physical discussions on the role of various external parameters.

13 Interactions and diffusion  Important chapter on the role of electron–electron interaction and its interplay with disorder. Density of states anomaly, correction to the conductivity. Important discussions about lifetime of quasiparticles and phase coherence time.

14 Persistent currents  Can be considered optional. Thermodynamics and orbital magnetism of mesoscopic systems. Problematics of persistent currents, from the very simple one-dimensional description to the effect of disorder and interaction.

Optional

5 Properties of the diffusion equation  Provides a comprehensive and self-contained account of properties of the diffusion equation. Diffusion in finite systems, boundary conditions, diffusion on graphs.

Miscellaneous  Various appendices are beyond an introductory level, or are not necessary in a first course on mesoscopic physics. They either develop technicalities such as Hikami boxes (A4.2), Cooperon in a time dependent magnetic field (A6.3), or important extensions.
such as anisotropic collisions developed in A4.3 and their effect on weak localization (A7.4) and universal conductance fluctuations (A11.1). The Landauer formalism for diffusive systems is developed in A7.2 for the average conductance and the weak localization correction and in A11.2 for conductance fluctuations.

Propagation of light in random media

Main course

This course provides a comprehensive introduction to the propagation of light in random media. It describes coherent effects in multiple scattering: coherent backscattering, diffusing wave spectroscopy and angular and time correlations of speckle patterns. Compared to coherent electronic transport, this course emphasizes notions specific to electromagnetic waves such as angular correlations of transmission (or reflection) coefficients in open space geometry, correlation between channels in a wave guide geometry, as well as the effects of the dynamics of scatterers.

1–4 These chapters are common to the two courses. In addition section 4.6 introduces the important formalism of radiative transfer which is developed in Appendix A5.2.
6 Dephasing Generalities on the mechanism of dephasing. Application to the polarization of electromagnetic waves, dynamics of the scatterers and dephasing associated with quantum internal degrees of freedom for the case of scattering of photons by cold atoms (the last topic is treated in Appendix A6.5).
8 Coherent backscattering of light Physics of the albedo, reflection coefficient of a diffusive medium. Coherent contribution (Cooperon) to the albedo, and its angular dependence. Uses the formalism developed in Chapter 4. Polarization and absorption effects (see also section 6.6). Extensive discussion of experimental results and coherent backscattering in various physical contexts. This chapter relies upon the results of section 5.6.
9 Diffusing wave spectroscopy An experimental technique routinely used to probe the dynamics of scatterers. Calculations result from a simple generalization of the formalism of Chapter 4. Interesting conceptually since diffusing wave spectroscopy exhibits the simplest example of decoherence introduced in a controlled way. Also interesting because this method probes the distribution of multiple scattering trajectories (reflection versus transmission experiments). Study of sections 5.6 and 5.7 is recommended.
12 Correlations of speckle patterns Analysis of a speckle pattern. Angular correlations of transmission coefficients. Classification and detailed calculation of the successive contributions $C_1$, $C_2$ and $C_3$. Simple description in terms of quantum crossings. Rayleigh law. Use of the Landauer formalism to relate speckle correlations to universal conductance fluctuations.

Optional

5 Properties of the diffusion equation Solutions of the diffusion equation in quasi-one-dimensional geometries, useful for calculations developed in Chapters 8, 9 and 12. Important appendix A5.2 on radiative transfer.
Miscellaneous Various appendices are useful reminders for beginners, for example A2.1 on scattering theory and A2.3 on light scattering by individual scatterers (Rayleigh, Rayleigh–Gans, Mie, resonant). Other appendices go beyond a course at the introductory level, either because they develop additional technicalities such as Hikami boxes (A4.2), useful for the reader interested in detailed calculations of Chapter 12, or because they present additional aspects of multiple scattering of light by random media such as spatial correlations of light intensity (A12.1) or anisotropic collisions (A4.3) and their consequences. The Landauer formalism for diffusive systems is used extensively in Chapter 12 on speckle correlations. Appendix A6.5 gives an overview of the technical tools needed to study the specific problem of multiple scattering of photons by cold atoms.