

Magnetohydrodynamics of the Sun

Magnetohydrodynamics of the Sun describes the subtle and complex interaction between the Sun's plasma atmosphere and its magnetic field, which is responsible for many fascinating dynamic phenomena. Chapters cover the generation of the Sun's magnetic field by dynamo action, magnetoconvection and the nature of photospheric flux tubes such as sunspots, the heating of the outer atmosphere by waves or reconnection, the structure of prominences, the nature of eruptive instability and magnetic reconnection in solar flares and coronal mass ejections, and the acceleration of the solar wind by reconnection or wave-turbulence. Developed for a graduate course at St Andrews University, this advanced textbook provides a detailed account of our progress towards answering the key unsolved puzzles in solar physics. It is essential reading for graduate students and researchers in solar physics and related fields of astronomy, plasma physics and fluid dynamics. Problem sets and other resources are available at www.cambridge.org/9780521854719.

Eric Priest was elected Fellow of the Royal Society of Edinburgh in 1985, of the Norwegian Academy of Sciences and Letters in 1994, of the Royal Society in 2002 and of the European Academy of Sciences in 2005. He has delivered many named lectures, including the James Arthur Prize Lecture at Harvard and the Lindsay Memorial Lecture at the Goddard Space Flight Center. He was awarded the Hale Prize of the American Astronomical Society, only the second time it has been awarded to a British scientist, and the Gold Medal of the Royal Astronomical Society. Priest created and led an extremely active and successful group at St Andrews, served three times on UK Research Assessment Panels and, as Co-Chair of the PPARC Science Committee, he played an important role when the UK joined the European Southern Observatory.

Cambridge University Press
978-0-521-85471-9 - Magnetohydrodynamics of the Sun
Eric Priest
Frontmatter
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978-0-521-85471-9 - Magnetohydrodynamics of the Sun
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MAGNETOHYDRODYNAMICS OF THE SUN

Eric Priest
University of St Andrews



CAMBRIDGE
UNIVERSITY PRESS

32 Avenue of the Americas, New York, NY 10013-2473, USA

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It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9780521854719

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First published 2014

Printed in the United States of America

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloguing in Publication data

Priest, E. R. (Eric Ronald), 1943–

Magnetohydrodynamics of the Sun / Eric Priest.

pages cm

Includes bibliographical references and index.

ISBN 978-0-521-85471-9 (hardback)

1. Sun. 2. Solar activity. 3. Magnetohydrodynamics. 4. Solar magnetic fields.

5. Astrophysics. I. Title. II. Title: Magnetohydrodynamics of the Sun.

QB524.P75 2014

523.7'2-dc23 2013029372

ISBN 978-0-521-85471-9 Hardback

Additional resources for this publication at www.cambridge.org/9780521854719

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To Clare,

our wonderful children

(Andrew, Matthew, David and Naomi),

my ever-helpful brother (Gerry) and sister (June)

and amazing 97-year-old mum (Olive)

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Cambridge University Press
978-0-521-85471-9 - Magnetohydrodynamics of the Sun
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Preface

I finished writing a book entitled *Solar Magnetohydrodynamics* back in 1981, which grew out of a post-graduate lecture course at St Andrews. Since then, the whole field has been completely transformed by spectacular new observations from spacecraft and ground-based telescopes which have spawned entirely new theoretical concepts. Rather than tinker with that book, I have therefore completely rewritten it from scratch and have given it a new title, *Magnetohydrodynamics of the Sun*.

Magnetohydrodynamics (or MHD for short) is the study of the interaction between a plasma (or electrically conducting fluid) and a magnetic field. A magnetic field affects a plasma in several ways. It exerts a force which is able, for instance, to support material in a prominence against gravity or propel it away from the Sun at high speeds. It provides thermal insulation, and so allows cool plasma to exist alongside hotter material, as in prominences or spicules. It also stores energy, which may be released violently as a solar flare or sporadically to heat the corona.

Solar MHD has now blossomed to become a central part of solar physics, since the key role of the magnetic field in producing many dynamic processes on the Sun has been recognised, and in turn solar physics has become one of the most vibrant parts of astronomy. The Sun influences the Earth's climate and space weather and plays a crucial role as a key for unlocking the secrets of many cosmic plasma phenomena. It provides a natural route for learning at close hand about fundamental cosmic processes at work in the Universe, such as magnetic turbulence, dynamos, spots, cycles, coronae, winds, flares and particle acceleration. What riches would be in store for us if we could view more distant objects with as much precision!

Solar MHD has three strands with important complementary insights, namely, sophisticated analytical theory guided by physical principles, observation from space and ground, and state-of-the-art computational experiments. Often, the greatest progress comes when there is good communication between them and when they work closely together.

The Sun is an amazing object, which has continued to reveal completely unexpected features when observed in greater detail or at new wavelengths. Huge progress on the main questions in solar MHD has been made. The questions have been greatly refined and remain hot topics of investigation, but none has yet been conclusively answered:

- * How is the magnetic field generated by dynamo action to create the solar cycle?
- * What is the nature of flux emergence and how are sunspots created?
- * How is the corona heated and the solar wind accelerated?
- * How are prominences formed?
- * How do solar flares and coronal mass ejections occur?

This book follows a similar pattern to *Solar Magnetohydrodynamics*. It first describes in detail new observations of the Sun and the basic equations of MHD. Then follow chapters on the different aspects of MHD, namely, equilibria, waves, shocks, and instabilities, which have been greatly extended in the

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light of new research, such as on waves in non-uniform media. Next comes a substantial new chapter on the fundamental topic of magnetic reconnection, which is dear to my heart. The subsequent chapters on the applications of MHD to solar phenomena are completely different from the original book in view of the enormous changes in understanding. Chapter 10 deals with the theory of chromospheric and coronal loops and mechanisms for heating them by waves, reconnection or turbulence. Chapter 11 discusses flux-rope models for prominences and their fine-scale structure, namely, barbs and threads. Chapter 12 describes mechanisms for the eruption of a coronal flux rope to give an eruptive flare or coronal mass ejection, together with the latest models for energy release by reconnection. The final chapter summarises classical theories for the solar wind. It also describes wave turbulence models for the fast wind and reconnection models for the slow wind. Appendices describe units, useful values and expressions. Problem examples are referred to at various points in each chapter. They, together with their solutions and an appendix on ground- and space-based instruments, can be found on the Web page www.cambridge.org/9780521854719. The book ends with a comprehensive reference list and a detailed index.

The aim of mathematical modelling is not to account for all observational details of a particular phenomenon, or simply to produce an image that resembles an observation, but rather to focus on the key physical mechanisms at work. The full nonlinear equations of MHD are so complex that they often need to be approximated drastically by focusing on a particular mechanism. One begins with a simple model, and then extra effects are added in an attempt to make the model more realistic. Thus, simple analytical models and complex numerical computations can play complementary roles in enhancing understanding. Computational experiments are in many respects similar to laboratory experiments and can be so sophisticated that the underlying processes are far from transparent. When modelling is undertaken successfully, it is possible to predict behaviour with different parameter values and to give deep physical understanding of a phenomenon. However, in order to decide what are the important observations and physical processes, it is invariably crucial to listen carefully to insights from observers.

Single-fluid MHD is a remarkably successful model for describing solar plasma at large scales, even when it is collisionless, for reasons described in Chapter 2. However, it will be important to develop MHD in future to include multi-fluid effects, as well as kinetic models for the internal structure of non-ideal regions in reconnection, shock waves and wave dissipation structures.

Trying to summarise the advances of the past thirty years and the current state of solar MHD has been a formidable task. Although I have done my best, there are bound to be omissions and mistakes in understanding, for which I apologise.

The notations adopted for cylindrical and spherical polars are (R, ϕ, z) and (r, θ, ϕ) , respectively. All quantities are measured in rationalised mks units, with the magnetic field in tesla (T) in most formulae. In the text, however, magnetic field strengths are commonly quoted in gauss (G), such that $1 \text{ G} = 10^{-4} \text{ T}$, while energy is sometimes given in erg instead of joules (J), such that $1 \text{ erg} = 10^{-7} \text{ J}$, since these are much better known in the community. Lengths in formulae are usually measured in metres (m), although in the text they are often quoted in megametres (Mm) such that $1 \text{ Mm} = 10^6 \text{ m}$, since typical small observed structures in the photosphere and corona are about 1 Mm across.

Figures 1.8, 1.13, 1.18, 1.41, 3.10, 3.11, 3.14–3.16, 9.4, 9.12, 9.16, 9.17, 10.8b, 10.14–10.16, 10.23, 10.24, 10.27, 10.28, 10.30–10.32 and 13.8 appeared in the *Astrophysical Journal* and are reproduced by permission of the American Astronomical Society. Figure 9.11b appeared in *Monthly Notices of the Royal Astronomical Society* and is reproduced by permission of Oxford University Press on behalf of the Royal Astronomical Society.

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on SOHO, which is a project of international cooperation between ESA and NASA. Figures 1.13, 1.14a, 1.20ab, 1.28, 1.29b, 1.35a, 9.7 and 11.14c are from the Swedish 1-m Solar Telescope (SST) on La Palma (in the Spanish Observatorio del Roque de los Muchachos of the Instituto de Astrofísica de Canarias); this is operated by the Institute for Solar Physics, which was managed by the Royal Swedish Academy of Sciences until 1 January 2013 and since then by Stockholm University. Figures 1.22, 1.41 and 10.25 are from TRACE, which was a NASA Small Explorer (SMEX) mission. Figures 1.21c and 10.14a are from Yohkoh, which was a Japanese solar mission, developed and launched by ISAS/JAXA, Japan, with NASA and SERC/PPARC (UK) as international partners. Credits to the other figures appear in the captions.

It has been a privilege to be James Gregory Professor at St Andrews University. Many of James Gregory's discoveries are invaluable in modern solar physics. He was one of the inventors of calculus (with Newton and Leibniz), he invented the gregorian telescope and he discovered the diffraction grating by shining a light through a seagull's feather. He was appointed the first Regius Professor of Mathematics in the University in 1668 at the age of 30 and sadly died at the age of 37.

Many people have generously helped with advice and suggestions, for which I am extremely grateful, but they cannot be blamed for any inadequacy in treatment.

St Andrews and Dundee colleagues have given more support than they know and have provided a kindly and highly stimulating environment, including: Vasilis Archontis, Peter Cargill, Ineke De Moortel, Andrew Haynes, Alan Hood, Gunnar Hornig, Duncan Mackay, Julie McCormick, Karen Meyer, Thomas Neukirch, Paolo Pagano, Clare Parnell, David Pascoe, Sarah Platten, David Pontin, Laurel Rachmeler, Bernie Roberts, James Threlfall and Antonia Wilmot-Smith.

Many friends from over the world have also helped in countless ways, such as: Ernest Amouzou, Guillaume Aulanier, Tony Arber, Hubert Baty, Mitch Berger, Tom Berger, Véronique Bommier, Sean Brannon, Philippa Browning, Nic Brummell, Sacha Brun, Paul Cally, Robert Cameron, Dick Canfield, Mats Carlsson, Arnab Choudhuri, Steve Cranmer, Len Culhane, Pascal Démoulin, Bart De Pontieu, Mausumi Dikpati, George Doschek, Alec Engell, Oddbjørn Engvold, Robert Erdelyi, Lyndsay Fletcher, Terry Forbes, Laurent Gizon, Nat Gopalswamy, Mandy Hagenaar, Joanna Haigh, Louise Harra, Richard Harrison, Jack Harvey, David Hathaway, Jean Heyvaerts, Kiyoshi Ichimoto, Elena Khomenko, Bernhard Kliem, Sam Krucker, Jun Lin, Dana Longcope, Jean-Marie Malherbe, Eckart Marsch, Piet Martens, Marian Martinez Gonzalez, Valentin Martinez Pillet, Sarah Matthews, Scott McIntosh, David McKenzie, Nicole Meyer-Vernet, Fernando Moreno Insertis, Valera Nakariakov, Matthew Owens, Susi Parenti, Alex Pevtsov, Joseph Plowman, Jiong Qiu, Matthias Rempel, Alex Russell, Göran Scharmer, Rolf Schlichenmaier, Brigitte Schmieder, Karel Schrijver, Manfred Schüssler, Roger Scott, Kazunari Shibata, Lara Silvers, Sami Solanki, Henk Spruit, Bob Stein, Jack Thomas, Slava Titov, Steve Tobias, Juri Toomre, Javier Trujillo Bueno, Saku Tsuneta, Aad Van Ballegoijen, Luc Rouppe Van Der Voort, Lidia van Driel-Gesztelyi, Marco Velli, Harry Warren, Nigel Weiss, Thomas Wiegmann, Anthony Yeates and Takaaki Yokoyama.