

Continuous and Discrete Time Signals and Systems

Signals and systems is a core topic for electrical and computer engineers. This textbook presents an introduction to the fundamental concepts of continuous-time (CT) and discrete-time (DT) signals and systems, treating them separately in a pedagogical and self-contained manner. Emphasis is on the basic signal processing principles, with underlying concepts illustrated using practical examples from signal processing and multimedia communications. The text is divided into three parts. Part I presents two introductory chapters on signals and systems. Part II covers the theories, techniques, and applications of CT signals and systems and Part III discusses these topics for DT signals and systems, so that the two can be taught independently or together. The focus throughout is principally on linear time invariant systems. Accompanying the book is a CD-ROM containing `MATLAB` code for running illustrative simulations included in the text; data files containing audio clips, images and interactive programs used in the text, and two animations explaining the convolution operation. With over 300 illustrations, 287 worked examples and 409 homework problems, this textbook is an ideal introduction to the subject for undergraduates in electrical and computer engineering. Further resources, including solutions for instructors, are available online at www.cambridge.org/9780521854559.

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Preface

The book is primarily intended for instruction in an upper-level undergraduate or a first-year graduate course in the field of signal processing in electrical and computer engineering. Practising engineers would find the book useful for reference or for self study. Our main motivation in writing the book is to deal with continuous-time (CT) and discrete-time (DT) signals and systems separately. Many instructors have realized that covering CT and DT systems in parallel with each other often confuses students to the extent where they are not clear if a particular concept applies to a CT system, to a DT system, or to both. In this book, we treat DT and CT signals and systems separately. Following Part I, which provides an introduction to signals and systems, Part II focuses on CT signals and systems. Since many students are familiar with the theory of CT signals and systems from earlier courses, Part II can be taught to such students with relative ease. For students who are new to this area, we have supplemented the material covered in Part II with appendices, which are included at the end of the book. Appendices A–F cover background material on complex numbers, partial fraction expansion, differential equations, difference equations, and a review of the basic signal processing instructions available in MATLAB. Part III, which covers DT signals and systems, can either be covered independently or in conjunction with Part II.

The book focuses on linear time-invariant (LTI) systems and is organized as follows. Chapters 1 and 2 introduce signals and systems, including their mathematical and graphical interpretations. In Chapter 1, we cover the classification between CT and DT signals and we provide several practical examples in which CT and DT signals are observed. Chapter 2 defines systems as transformations that process the input signals and produce outputs in response to the applied inputs. Practical examples of CT and DT systems are included in Chapter 2. The remaining fifteen chapters of the book are divided into two parts. Part II constitutes Chapters 3–8 of the book and focuses primarily on the theories and applications of CT signals and systems. Part III comprises Chapters 9–17 and deals with the theories and applications of DT signals and systems. The organization of Parts II and III is described below.

Chapter 3 introduces the time-domain analysis of the linear time-invariant continuous-time (LTIC) systems, including the convolution integral used to evaluate the output in response to a given input signal. Chapter 4 defines the continuous-time Fourier series (CTFS) as a frequency domain representation for the CT periodic signals, and Chapter 5 generalizes the CTFS to aperiodic signals and develops an alternative representation, referred to as the continuous-time Fourier transform (CTFT). Not only do the CTFT and CTFS representations provide an alternative to the convolution integral for the evaluation of the output response, but also these frequency representations allow additional insights into the behavior of the LTIC systems that are exploited later in the book to design such systems. While the CTFT is useful for steady state analysis of the LTIC systems, the Laplace transform, introduced in Chapter 6, is used in control applications where transient and stability analyses are required. An important subset of LTIC systems are frequency-selective filters, whose characteristics are specified in the frequency domain. Chapter 7 presents design techniques for several CT frequency-selective filters including the Butterworth, Chebyshev, and elliptic filters. Finally, Chapter 8 concludes our treatment of LTIC signals and systems by reviewing important applications of CT signal processing.

The coverage of CT signals and systems concludes with Chapter 8 and a course emphasizing the CT domain can be completed at this stage. In Part III, Chapter 9 starts our consideration of DT signals and systems by providing several practical examples in which such signals are observed directly. Most DT sequences are, however, obtained by sampling CT signals. Chapter 9 shows how a band-limited CT signal can be accurately represented by a DT sequence such that no information is lost in the conversion from the CT to the DT domain. Chapter 10 provides the time-domain analysis of linear time-invariant discrete-time (LTID) systems, including the convolution sum used to calculate the output of a DT system. Chapter 11 introduces the frequency domain representations for DT sequences, namely the discrete-time Fourier series (DTFS) and the discrete-time Fourier transform (DTFT). The discrete Fourier transform (DFT) samples the DTFT representation in the frequency domain and is convenient for digital signal processing of finite-length sequences. Chapter 12 introduces the DFT, while Chapter 13 is devoted to a discussion of the z-transform. As for CT systems, DT systems are generally specified in the frequency domain. A particular class of DT systems, referred to as frequency-selective digital filters, is introduced in Chapter 14. Based on the length of the impulse response, digital filters can be further classified into finite impulse response (FIR) and infinite impulse response (IIR) filters. Chapter 15 covers the design techniques for the FIR filters, and Chapter 16 presents the design techniques for the IIR filters. Chapter 17 concludes the book by motivating the students with several applications of digital signal processing in audio and music, spectral analysis, and image and video processing.

Although the book has been designed to be as self-contained as possible, some basic prerequisites have been assumed. For example, an introductory

background in mathematics which includes trigonometry, differential calculus, integral calculus, and complex number theory, would be helpful. A course in electrical circuits, although not essential, would be highly useful as several examples of electrical circuits have been used as systems to motivate the students. For students who lack some of the required background information, a review of the core background materials such as complex numbers, partial fraction expansion, differential equations, and difference equations is provided in the appendices.

The normal use of this book should be as follows. For a first course in signal processing, at say sophomore or junior level, a reasonable goal is to teach Part II, covering continuous-time (CT) signals and systems. Part III provides the material for a more advanced course in discrete-time (DT) signal processing. We have also spent a great deal of time experimenting with different presentations for a single-semester signals and systems course. Typically, such a course should include Chapters 1, 2, 3, 10, 4, 5, 11, 6, and 13 in that order. Below, we provide course outlines for a few traditional signal processing courses. These course outlines should be useful to an instructor teaching this type of material or using the book for the first time.

- (1) Continuous-time signals and systems: Chapters 1–8.
- (2) Discrete-time signals and systems: Chapters 1, 2, 9–17.
- (3) Traditional signals and systems: Chapters 1, 2, (3, 10), (4, 5, 11), 6, 13.
- (4) Digital signal processing: Chapters 10–17.
- (5) Transform theory: Chapters (4, 5, 11), 6, 13.

Another useful feature of the book is that the chapters are self-contained so that they may be taught independent of each other. There is a significant difference between reading a book and being able to apply the material to solve actual problems of interest. An effective use of the book must include a fair coverage of the solved examples and problem solving by motivating the students to solve the problems included at the end of each chapter. As such, a major focus of the book is to illustrate the basic signal processing concepts with examples. We have included 287 worked examples, 409 supplementary problems at the ends of the chapters, and more than 300 figures to explain the important concepts. Wherever relevant, we have extensively used MATLAB to validate our analytical results and also to illustrate the design procedures for a variety of problems. In most cases, the MATLAB code is provided in the accompanying CD, so the students can readily run the code to satisfy their curiosity. To further enhance their understanding of the main signal processing concepts, students are encouraged to program extensively in MATLAB. Consequently, several MATLAB exercises have been included in the Problems sections.

Any suggestions or concerns regarding the book may be communicated to the authors; email addresses are listed at <http://www.cambridge.org/9780521854559>. Future updates on the book will also be available at the same website.

A number of people have contributed in different ways, and it is a pleasure to acknowledge them. Anna Littlewood, Irene Pizzie, and Emily Yossarian of Cambridge University Press contributed significantly during the production stage of the book. Professor Tyseer Aboulnasr reviewed the complete book and provided valuable feedback to enhance its quality. In addition, Mrinal Mandal would like to thank Gencheng Guo, Meghna Singh, Wen Chen, Saeed S. Tehrani, Sanjukta Mukhopadhyaya, and Professor Thomas Sikora for their help in the overall preparation of the book. On behalf of Amir Asif, special thanks are due to Professor José Moura, who introduced the fascinating field of signal processing to him for the first time and has served as his mentor for several years. Lastly, Mrinal Mandal thanks his parents, Iswar Chandra Mandal (late) and Mrs Kiran Bala Mandal, and his wife Rupa, and Amir Asif thanks his parents, Asif Mahmood (late) and Khalida Asif, his wife Sadia, and children Maaz and Sannah for their continuous support and love over the years.