Random Networks for Communication

When is a network (almost) connected? How much information can it carry? How can you find a particular destination within the network? And how do you approach these questions – and others – when the network is random?

The analysis of communication networks requires a fascinating synthesis of random graph theory, stochastic geometry and percolation theory to provide models for both structure and information flow. This book is the first comprehensive introduction for graduate students and scientists to techniques and problems in the field of spatial random networks. The selection of material is driven by applications arising in engineering, and the treatment is both readable and mathematically rigorous. Though mainly concerned with information-flow-related questions motivated by wireless data networks, the models developed are also of interest in a broader context, ranging from engineering to social networks, biology, and physics.

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From Statistical Physics to Information Systems

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and
Ronald Meester

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521854429

First published 2007

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-85442-9 hardback

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Preface

What is this book about, and who is it written for? To start with the first question, this book introduces a subject placed at the interface between mathematics, physics, and information theory of systems. In doing so, it is not intended to be a comprehensive monograph and collect all the mathematical results available in the literature, but rather pursues the more ambitious goal of laying the foundations. We have tried to give emphasis to the relevant mathematical techniques that are the essential ingredients for anybody interested in the field of random networks. Dynamic coupling, renormalisation, ergodicity and deviations from the mean, correlation inequalities, Poisson approximation, as well as some other tricks and constructions that often arise in the proofs are not only applied, but also discussed with the objective of clarifying the philosophy behind their arguments. We have also tried to make available to a larger community the main mathematical results on random networks, and to place them into a new communication theory framework, trying not to sacrifice mathematical rigour. As a result, the choice of the topics was influenced by personal taste, by the willingness to keep the flow consistent, and by the desire to present a modern, communication-theoretic view of a topic that originated some fifty years ago and that has had an incredible impact in mathematics and statistical physics since then. Sometimes this has come at the price of sacrificing the presentation of results that either did not fit well in what we thought was the ideal flow of the book, or that could be obtained using the same basic ideas, but at the expense of highly technical complications. One important topic that the reader will find missing, for example, is a complete treatment of the classic Erdös–Rényi model of random graphs and of its more recent extensions, including preferential attachment models used to describe properties of the Internet. Indeed, we felt that these models, lacking a geometric component, did not fit well in our framework and the reader is referred to the recent account of Durrett (2007) for a rigorous treatment of preferential attachment models. Other omissions are certainly present, and hopefully similarly justified. We also refer to the monographs by Bollobás (2001), Bollobás and Riordan (2006), Grimmett (1999), Meester and Roy (1996), and Penrose (2003), for a compendium of additional mathematical results.

Let us now turn to the second question: what is our intended readership? In the first place, we hope to inspire people in electrical engineering, computer science, and physics to learn more about very relevant mathematics. It is worthwhile to learn these mathematics, as it provides valuable intuition and structure. We have noticed that there is a tendency to re-invent the wheel when it comes to the use of mathematics, and we thought it would be very useful to have a standard reference text. But also, we want to inspire mathematicians to learn more about the communication setting. It raises specific questions that are mathematically interesting, and deep. Such questions would be hard to think about without the context of communication networks.

In summary: the mathematics is not too abstract for engineers, and the applications are certainly not too mechanical for mathematicians. The authors being from both communities – engineering and mathematics – have enjoyed over the years an interesting and fruitful collaboration, and we are convinced that both communities can profit from this book. In a way, our main concern is the interaction between people at either side of the interface, who desire to *break on through to the other side*.

A final word about the prerequisites. We assume that the reader is familiar with basic probability theory, with the basic notions of graph theory and with basic calculus. When we need concepts that go beyond these basics, we will introduce and explain them. We believe the book is suitable, and we have used it, for a first-year graduate course in mathematics or electrical engineering.

We thank Patrick Thiran and the School of Computer and Communication Sciences of the École Politechnique Fédérale de Lausanne for hosting us during the Summer of 2005, while working on this book. Massimo Franceschetti is also grateful to the Department of Mathematics of the Vrije Universiteit Amsterdam for hosting him several times. We thank Misja Nuyens who read the entire manuscript and provided many useful comments. We are also grateful to Nikhil Karamchandani, Young-Han Kim, and Olivier Lévêque, who have also provided useful feedback on different portions of the manuscript. Massimo Franceschetti also thanks Olivier Dousse, a close research collaborator of several years.

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978-0-521-85442-9 - Random Networks for Communication: From Statistical Physics to Information Systems
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List of notation

In the following, we collect some of the notation used throughout the book. Definitions are repeated within the text, in the specific context where they are used. Occasionally, in some local contexts, we introduce new notation and redefine terms to mean something different.

$\lvert\cdot\rvert$	Lebesgue measure
	Euclidean distance
	L_1 distance
	cardinality
$\lfloor\cdot\rfloor$	floor function, the argument is rounded down to the previous integer
$\lceil\cdot\rceil$	ceiling function, the argument is rounded up to the next integer
$\mathcal{A}$	an algorithm
	a region of the plane
a.a.s.	asymptotic almost surely
a.s.	almost surely
β	mean square constraint on the codeword symbols
B_n	box of side length $\sqrt{n}$
	box of side length n
$B_n^{\leftrightarrow}$	the event that there is a crossing path connecting
	the left side of B_n with its right side
$C(x)$	connected component containing the point x
C	connected component containing the origin
	channel capacity
$C(x, y)$	channel capacity between points x and y
	chemical distance between points x and y
C_n	sum of the information rates across a cut
$\partial(\cdot)$	inner boundary
$D(G)$	diameter of the graph G
$D(\mathcal{A})$	navigation length of the algorithm $\mathcal{A}$
d_{TV}	total variation distance
$E(\cdot)$	expectation
$g(x)$	connection function in a random connection model

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$\overline{g}(\lvert x\rvert)$	connection function depending only on the Euclidian distance, i.e., $\overline{g}:\mathbb{R}^+\to[0,1]$ such that $\overline{g}(\lvert x\rvert)=g(x)$
G	a graph
G_X	generating function of random variable X
γ	interference reduction factor in the SNIR model
$I(z)$	shot-noise process
$\tilde{I}(z)$	shifted shot-noise process
I	indicator random variable
i.i.d.	independent, identically distributed
k_c	critical value in nearest neighbour model
λ	density of a Poisson process, or parameter of a Poisson distribution
λ_c	critical density for boolean or random connection model
$\Lambda(x)$	density function of an inhomogeneous Poisson process
$\ell(x, y)$	attenuation function between points x and y
$l(\lvert x-y\rvert)$	attenuation function depending only on the Euclidian distance, i.e., $l:\mathbb{R}^+\to\mathbb{R}^+$ such that $l(\lvert x-y\rvert)=\ell(x, y)$
N	environmental noise
$N(A)$	number of pivotal edges for the event A
$N_\infty(B_n)$	number of Poisson points in the box B_n that are also part of the unbounded component on the whole plane
$N(n)$	number of paths of length n in the random grid starting at the origin
O	origin point on the plane
P	power of a signal, or just a probability measure
$Po(\lambda)$	Poisson random variable of parameter λ
p_c	critical probability for undirected percolation
$\vec{p}_c$	critical probability for directed percolation
p_c^{site}	critical probability for site percolation
p_c^{bond}	critical probability for bond percolation
p_α	critical probability for α-almost connectivity
$\psi(\cdot)$	probability that there exists an unbounded connected component
Q	the event that there exists at most one unbounded connected component
r_α	critical radius for α-almost connectivity in the boolean model
r_c	critical radius for the boolean model
R	rate of the information flow
$R(x, y)$	achievable information rate between x and y
$R(n)$	simultaneous achievable per-node rate in a box of area n
SNR	signal to noise ratio
SNIR	signal to noise plus interference ratio
T	a tree
	a threshold value
$\theta(\cdot)$	percolation function, i.e., the probability that there exists an unbounded connected component at the origin
U	the event that there exists an unbounded connected component

U_0	the event that there exists an unbounded connected component at the origin, when there is a Poisson point at the origin
W	channel bandwidth
	sum of indicator random variables
w.h.p.	with high probability
X	Poisson process
	a random variable
X_n	a sequence of random variables
X^m	a codeword of length m
$X(A)$	number of points of the Poisson process X falling in the set A
$X(e)$	uniform random variable in $[0, 1]$,
	where e is a random edge coupled with the outcome of X
$x \leftrightarrow y$	the event that there is a path connecting point x with point y
Z_n	nth generation in a branching process