

Contents

<i>Preface to the First Edition</i>	<i>page xvii</i>
<i>Preface to the Second Edition</i>	<i>xxi</i>
<i>Mathematical Prolegomenon</i>	<i>xxvii</i>
0.1 Set-theoretic Notation	xxvii
0.2 Proof by Induction	xxix
0.3 Equivalence Relations and Equivalence Classes	xxx
Part I Propositional Logic	1
1 Classical Logic and the Material Conditional	3
1.1 Introduction	3
1.2 The Syntax of the Object Language	4
1.3 Semantic Validity	5
1.4 Tableaux	6
1.5 Counter-models	10
1.6 Conditionals	11
1.7 The Material Conditional	12
1.8 Subjunctive and Counterfactual Conditionals	13
1.9 More Counter-examples	14
1.10 Arguments for $\supset$	15
1.11 *Proofs of Theorems	16
1.12 History	18
1.13 Further Reading	18
1.14 Problems	18
2 Basic Modal Logic	20
2.1 Introduction	20
2.2 Necessity and Possibility	20

2.3	Modal Semantics	21
2.4	Modal Tableaux	24
2.5	Possible Worlds: Representation	28
2.6	Modal Realism	28
2.7	Modal Actualism	29
2.8	Meinongianism	30
2.9	*Proofs of Theorems	31
2.10	History	33
2.11	Further Reading	34
2.12	Problems	34
3	Normal Modal Logics	36
3.1	Introduction	36
3.2	Semantics for Normal Modal Logics	36
3.3	Tableaux for Normal Modal Logics	38
3.4	Infinite Tableaux	42
3.5	S5	45
3.6	Which System Represents Necessity?	46
3.6a	The Tense Logic K^t	49
3.6b	Extensions of K^t	51
3.7	*Proofs of Theorems	56
3.8	History	60
3.9	Further Reading	60
3.10	Problems	60
4	Non-normal Modal Logics; Strict Conditionals	64
4.1	Introduction	64
4.2	Non-normal Worlds	64
4.3	Tableaux for Non-normal Modal Logics	65
4.4	The Properties of Non-normal Logics	67
4.4a	S0.5	69
4.5	Strict Conditionals	72
4.6	The Paradoxes of Strict Implication	72
4.7	... and their Problems	73
4.8	The Explosion of Contradictions	74
4.9	Lewis' Argument for Explosion	76
4.10	*Proofs of Theorems	77
4.11	History	79

	Contents	ix
4.12	Further Reading	80
4.13	Problems	80
5	Conditional Logics	82
5.1	Introduction	82
5.2	Some More Problematic Inferences	82
5.3	Conditional Semantics	84
5.4	Tableaux for C	86
5.5	Extensions of C	87
5.6	Similarity Spheres	90
5.7	C_1 and C_2	94
5.8	Further Philosophical Reflections	97
5.9	*Proofs of Theorems	98
5.10	History	100
5.11	Further Reading	101
5.12	Problems	101
6	Intuitionist Logic	103
6.1	Introduction	103
6.2	Intuitionism: The Rationale	103
6.3	Possible-world Semantics for Intuitionism	105
6.4	Tableaux for Intuitionist Logic	107
6.5	The Foundations of Intuitionism	112
6.6	The Intuitionist Conditional	113
6.7	*Proofs of Theorems	114
6.8	History	116
6.9	Further Reading	117
6.10	Problems	117
7	Many-valued Logics	120
7.1	Introduction	120
7.2	Many-valued Logic: The General Structure	120
7.3	The 3-valued Logics of Kleene and Łukasiewicz	122
7.4	LP and RM_3	124
7.5	Many-valued Logics and Conditionals	125
7.6	Truth-value Gluts: Inconsistent Laws	127
7.7	Truth-value Gluts: Paradoxes of Self-reference	129
7.8	Truth-value Gaps: Denotation Failure	130
7.9	Truth-value Gaps: Future Contingents	132
7.10	Supervaluations, Modality and Many-valued Logic	133

7.11	*Proofs of Theorems	137
7.12	History	139
7.13	Further Reading	140
7.14	Problems	140
8	First Degree Entailment	142
8.1	Introduction	142
8.2	The Semantics of <i>FDE</i>	142
8.3	Tableaux for <i>FDE</i>	144
8.4	<i>FDE</i> and Many-valued Logics	146
8.4a	Relational Semantics and Tableaux for L_3 and RM_3	149
8.5	The Routley Star	151
8.6	Paraconsistency and the Disjunctive Syllogism	154
8.7	*Proofs of Theorems	155
8.8	History	161
8.9	Further Reading	161
8.10	Problems	161
9	Logics with Gaps, Gluts and Worlds	163
9.1	Introduction	163
9.2	Adding $\rightarrow$	163
9.3	Tableaux for K_4	164
9.4	Non-normal Worlds Again	166
9.5	Tableaux for N_4	168
9.6	Star Again	169
9.7	Impossible Worlds and Relevant Logic	171
9.7a	Logics of Constructible Negation	175
9.8	*Proofs of Theorems	179
9.9	History	184
9.10	Further Reading	185
9.11	Problems	185
10	Relevant Logics	188
10.1	Introduction	188
10.2	The Logic B	188
10.3	Tableaux for B	190
10.4	Extensions of B	194
10.4a	Content Inclusion	197
10.5	The System R	203
10.6	The Ternary Relation	206

	Contents	xi
10.7	<i>Ceteris Paribus</i> Enthymemes	208
10.8	*Proofs of Theorems	211
10.9	History	216
10.10	Further Reading	217
10.11	Problems	218
11	Fuzzy Logics	221
11.1	Introduction	221
11.2	Sorites Paradoxes	221
11.3	... and Responses to Them	222
11.4	The Continuum-valued Logic L	224
11.5	Axioms for $L_{\aleph}$	227
11.6	Conditionals in L	230
11.7	Fuzzy Relevant Logic	231
11.7a	*Appendix: <i>t</i> -norm Logics	234
11.8	History	237
11.9	Further Reading	238
11.10	Problems	239
11a	Appendix: Many-valued Modal Logics	241
11a.1	Introduction	241
11a.2	General Structure	241
11a.3	Illustration: Modal Łukasiewicz Logic	243
11a.4	Modal <i>FDE</i>	244
11a.5	Tableaux	247
11a.6	Variations	250
11a.7	Future Contingents Revisited	251
11a.8	A Glimpse Beyond	254
11a.9	*Proofs of Theorems	255
	Postscript: An Historical Perspective on Conditionals	259
	Part II Quantification and Identity	261
12	Classical First-order Logic	263
12.1	Introduction	263
12.2	Syntax	263
12.3	Semantics	264
12.4	Tableaux	266
12.5	Identity	272

12.6	Some Philosophical Issues	275
12.7	Some Final Technical Comments	277
12.8	*Proofs of Theorems 1	278
12.9	*Proofs of Theorems 2	283
12.10	*Proofs of Theorems 3	285
12.11	History	287
12.12	Further Reading	287
12.13	Problems	288
13	Free Logics	290
13.1	Introduction	290
13.2	Syntax and Semantics	290
13.3	Tableaux	291
13.4	Free Logics: Positive, Negative and Neutral	293
13.5	Quantification and Existence	295
13.6	Identity in Free Logic	297
13.7	*Proofs of Theorems	300
13.8	History	304
13.9	Further Reading	305
13.10	Problems	305
14	Constant Domain Modal Logics	308
14.1	Introduction	308
14.2	Constant Domain <i>K</i>	308
14.3	Tableaux for <i>CK</i>	309
14.4	Other Normal Modal Logics	314
14.5	Modality <i>De Re</i> and <i>De Dicto</i>	315
14.6	Tense Logic	318
14.7	*Proofs of Theorems	320
14.8	History	325
14.9	Further Reading	326
14.10	Problems	327
15	Variable Domain Modal Logics	329
15.1	Introduction	329
15.2	Prolegomenon	329
15.3	Variable Domain <i>K</i> and its Normal Extensions	330
15.4	Tableaux for <i>VK</i> and its Normal Extensions	331
15.5	Variable Domain Tense Logic	335
15.6	Extensions	336

	Contents	xiii
15.7	Existence Across Worlds	339
15.8	Existence and Wide-Scope Quantifiers	341
15.9	*Proofs of Theorems	342
15.10	History	346
15.11	Further Reading	346
15.12	Problems	347
16	Necessary Identity in Modal Logic	349
16.1	Introduction	349
16.2	Necessary Identity	350
16.3	The Negativity Constraint	352
16.4	Rigid and Non-rigid Designators	354
16.5	Names and Descriptions	357
16.6	*Proofs of Theorems 1	358
16.7	*Proofs of Theorems 2	362
16.8	History	364
16.9	Further Reading	364
16.10	Problems	365
17	Contingent Identity in Modal Logic	367
17.1	Introduction	367
17.2	Contingent Identity	367
17.3	SI Again, and the Nature of Avatars	373
17.4	*Proofs of Theorems	376
17.5	History	382
17.6	Further Reading	382
17.7	Problems	382
18	Non-normal Modal Logics	384
18.1	Introduction	384
18.2	Non-normal Modal Logics and Matrices	384
18.3	Constant Domain Quantified L	385
18.4	Tableaux for Constant Domain L	386
18.5	Ringling the Changes	387
18.6	Identity	391
18.7	*Proofs of Theorems	393
18.8	History	397
18.9	Further Reading	397
18.10	Problems	397

19	Conditional Logics	399
19.1	Introduction	399
19.2	Constant and Variable Domain <i>C</i>	399
19.3	Extensions	403
19.4	Identity	408
19.5	Some Philosophical Issues	413
19.6	*Proofs of Theorems	415
19.7	History	419
19.8	Further Reading	419
19.9	Problems	419
20	Intuitionist Logic	421
20.1	Introduction	421
20.2	Existence and Construction	421
20.3	Quantified Intuitionist Logic	422
20.4	Tableaux for Intuitionist Logic 1	424
20.5	Tableaux for Intuitionist Logic 2	427
20.6	Mental Constructions	431
20.7	Necessary Identity	432
20.8	Intuitionist Identity	434
20.9	*Proofs of Theorems 1	437
20.10	*Proofs of Theorems 2	448
20.11	History	453
20.12	Further Reading	453
20.13	Problems	453
21	Many-valued Logics	456
21.1	Introduction	456
21.2	Quantified Many-valued Logics	456
21.3	$\forall$ and $\exists$	457
21.4	Some 3-valued Logics	459
21.5	Their Free Versions	461
21.6	Existence and Quantification	462
21.7	Neutral Free Logics	465
21.8	Identity	467
21.9	Non-classical Identity	468
21.10	Supervaluations and Subvaluations	469

	Contents	xv
21.11 *Proofs of Theorems	471	
21.12 History	473	
21.13 Further Reading	474	
21.14 Problems	474	
22 First Degree Entailment	476	
22.1 Introduction	476	
22.2 Relational and Many-valued Semantics	476	
22.3 Tableaux	479	
22.4 Free Logics with Relational Semantics	481	
22.5 Semantics with the Routley *	483	
22.6 Identity	486	
22.7 *Proofs of Theorems 1	489	
22.8 *Proofs of Theorems 2	493	
22.9 *Proofs of Theorems 3	499	
22.10 History	502	
22.11 Further Reading	502	
22.12 Problems	502	
23 Logics with Gaps, Gluts and Worlds	504	
23.1 Introduction	504	
23.2 Matrix Semantics Again	505	
23.3 N_4	505	
23.4 N_*	508	
23.5 K_4 and K_*	510	
23.6 Relevant Identity	512	
23.7 Relevant Predication	515	
23.8 Logics with Constructible Negation	517	
23.9 Identity for Logics with Constructible Negation	521	
23.10 *Proofs of Theorems 1	523	
23.11 *Proofs of Theorems 2	527	
23.12 *Proofs of Theorems 3	530	
23.13 History	532	
23.14 Further Reading	532	
23.15 Problems	533	
24 Relevant Logics	535	
24.1 Introduction	535	

24.2	Quantified B	535
24.3	Extensions of B	537
24.4	Restricted Quantification	541
24.5	Semantics vs Proof Theory	543
24.6	Identity	548
24.7	Properties of Identity	553
24.8	*Proofs of Theorems 1	555
24.9	*Proofs of Theorems 2	559
24.10	History	561
24.11	Further Reading	561
24.12	Problems	562
25	Fuzzy Logics	564
25.1	Introduction	564
25.2	Quantified Łukasiewicz Logic	564
25.3	Validity in $L_{\&}$	565
25.4	Deductions	570
25.5	The Sorites Again	572
25.6	Fuzzy Identity	573
25.7	Vague Objects	576
25.8	*Appendix: Quantification and Identity in t -norm Logics	578
25.9	History	581
25.10	Further Reading	582
25.11	Problems	582
	Postscript: A Methodological Coda	584
	<i>References</i>	587
	<i>Index of Names</i>	603
	<i>Index of Subjects</i>	607