Chapter 1
The celestial sphere and coordinate systems

In the night sky the stars appear as bright points on a dark spherical surface (Figure 1.1). No such surface really exists, of course, but the concept of a celestial sphere is a useful one that goes back thousands of years. Ptolemy described it and so did Pythagoras and many others. Today we no longer have to worry about the reality of that sphere, and so we eliminate the need for speculation on its composition, radius, thickness and so forth. On the other hand, even though the celestial sphere is not a physical entity, we have many practical uses for the concept. The observer is always at the center of it, and the direction from the observer to any star may be considered to be a radius of the celestial sphere.

The stars are so far very away that we can consider the celestial sphere to be very large and the Earth very small. From the perspective of an observer on the celestial sphere looking back, the entire Earth would appear as a single point. And on the surface of the Earth, when we point to objects in the sky, we don't need to know how far away they are for the purposes of positional astronomy. We need only be concerned with the angles between points on the celestial sphere. That's why a good planetarium fools us into thinking that we are looking at the real sky.

Coordinate systems

The most fundamental application of the concept of a celestial sphere is to determine the coordinates of objects that appear in the sky (or perhaps, on the sky). We can approach this problem of coordinates in a very general way and see first of all just what is involved in specifying the location of a point on the surface of any sphere. Assume, to begin

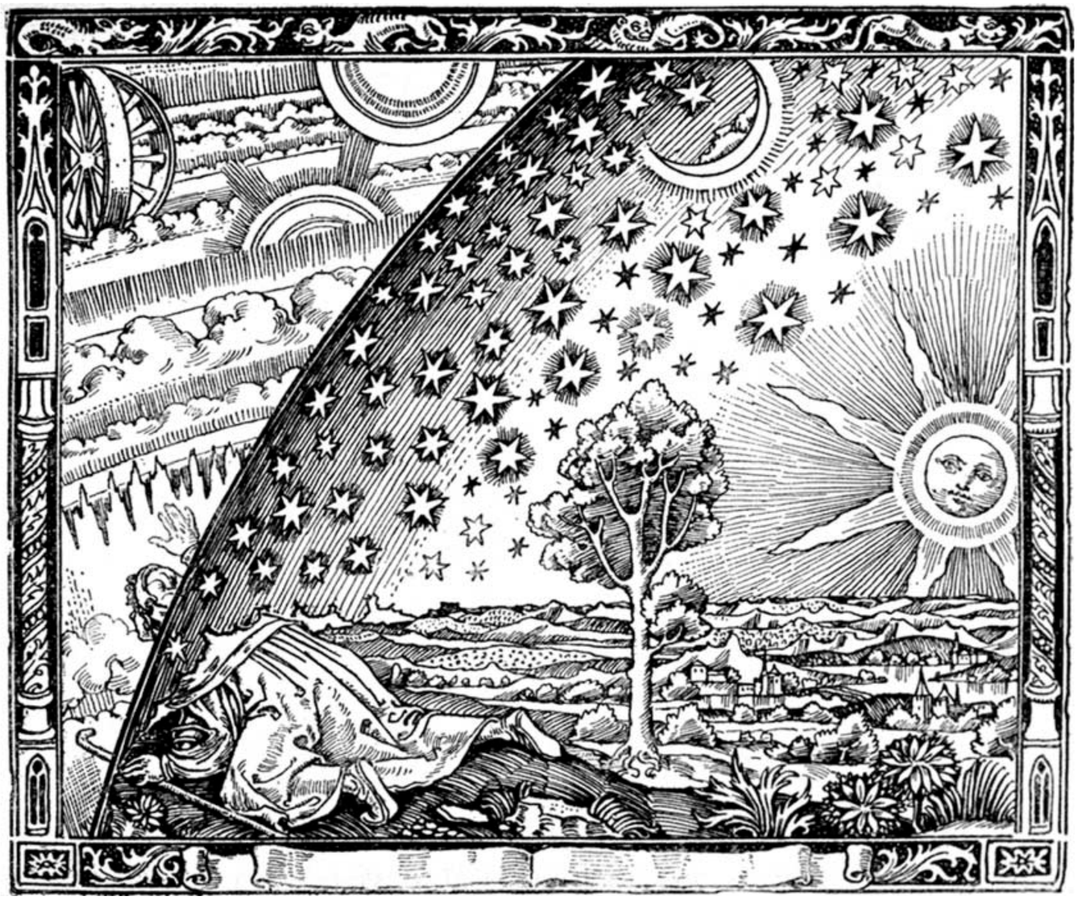

Figure 1.1. The celestial sphere depicted in a woodcut often mistakenly attributed to a medieval author. This powerful piece of visual rhetoric is often used to advance the incorrect claim that medievals believed the Earth was flat. Its earliest known appearance is in Camille Flammarion's *L'atmosphere: météorologie populaire* in 1888. (Courtesy History of Science Collections, University of Oklahoma Libraries.)

with, that the sphere is rotating. This requires the existence of an axis that passes through the center of the sphere as in Figure 1.2. The axis is thus defined by the rotation, and the axis defines, in turn, two points – the poles. Following the convention established for the Earth, we designate these as the **north pole** and the **south pole**. Now consider a plane passing through the sphere in such a way that it is perpendicular to the axis and includes the center of the sphere. In the case of the Earth this plane is called the **equator**. In the more general case this plane is referred to as the **fundamental plane**, taking on a more specific name depending on the system of coordinates.

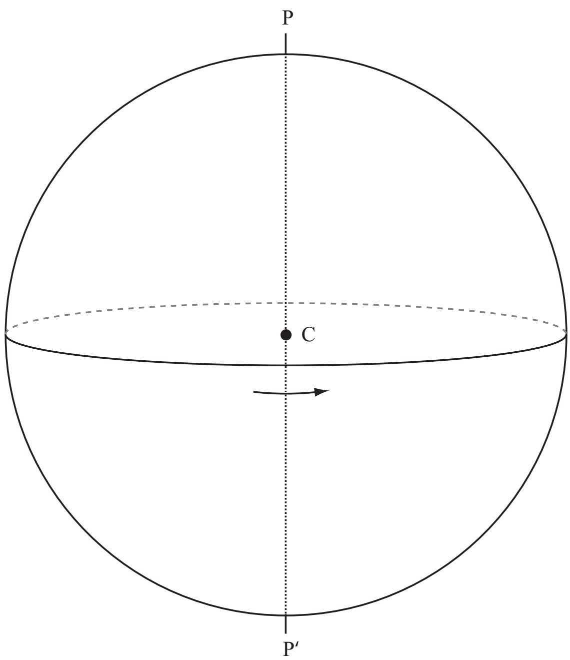

Figure 1.2. A rotating sphere. The poles and the equator are defined by the rotation.

Imagine now that we wish to define in a specific way the location of a point A on the sphere shown in Figure 1.3(a). Let us first pass a plane through the sphere so that the plane includes both the axis of the sphere and point A. In Figure 1.3(a) this plane has been indicated by the curve passing through the two poles and point A. Points B and C are also indicated. Point C is at the center of the sphere, and point B is the intersection of the circular arc PA with the equator. It should be obvious that ∠BCA defines the angular distance of A from the fundamental plane. On the Earth, an angle similar to ∠BCA is called the **latitude**. Of course, we do not go to the center of the Earth in order to determine the latitude of a place, but it is actually this angle that we are talking about when we use the term.

Now let us pass another plane through the sphere. Let this one be parallel to the fundamental plane, and let it pass through A. Notice that the radius of this circle is smaller than that of the fundamental circle. At this point we must introduce two terms. First, a **great circle** is the intersection of any plane with a sphere such that the plane passes through the center of the sphere. Thus, the fundamental plane forms a great circle, and this is called the **fundamental circle**, and the arc PAP′ is half of a great circle. Second, a **small circle** is the intersection of any plane with a sphere such that the plane *does not* contain the center of the sphere.

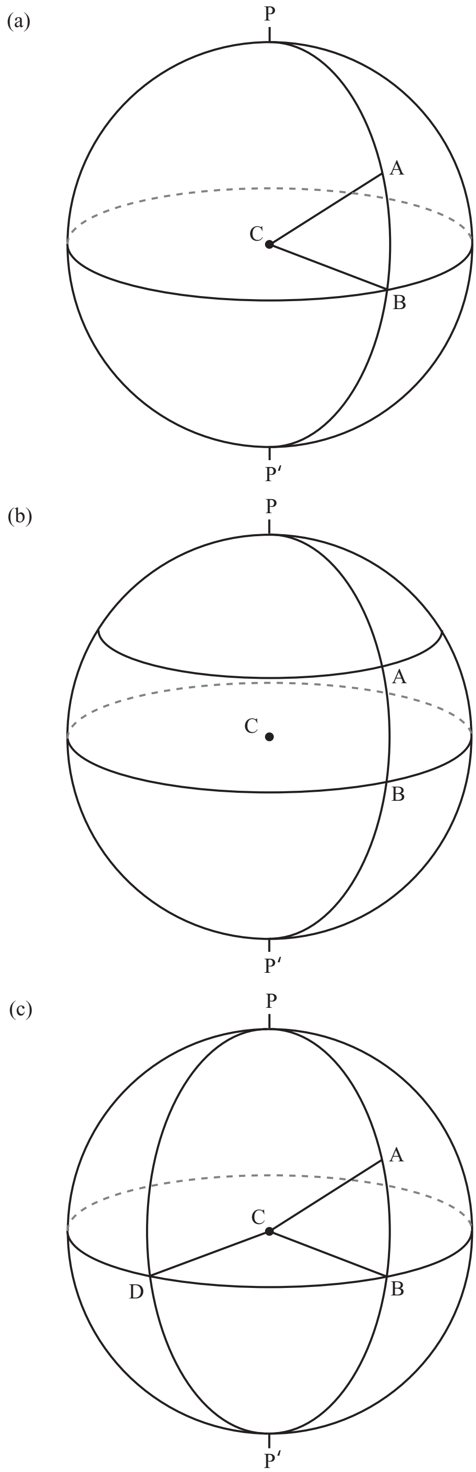

Figure 1.3. (a) The angle between two points on a sphere. (b) A small circle through point A. (c) Two angles define the location of point A with respect to the equator and an arbitrarily chosen point D.

In the case shown in Figure 1.3(b), the small circle happens to be parallel to the fundamental plane, but a small circle can have any orientation.

Returning to Figure 1.3(b), we can see that all points on the small circle are at the same angular distance from the fundamental circle. On the Earth we would say that all points on this small circle had the same latitude. In order to be precise about the location of A, we must specify in some way which of all of the possible great circles through the poles is the one that passes through A. We may do this by means of a second angle, measured this time in the fundamental plane. We must first select some arbitrary point as our zero point, and in Figure 1.3(c) this point has been indicated as D. Now ∠DCB quite specifically defines the great circle through A. On the earth the circle PDP′ represents the meridian of Greenwich, England, and ∠DCB represents the **longitude** of point A.

The discussion just presented is intended to show that the location of any point on a sphere can be specified by using two angles. One angle is measured perpendicular to the fundamental plane, and the other angle is measured in the fundamental plane. Just as we can uniquely identify any point on a Cartesian plane with x and y coordinates, any point on the surface of a sphere can be uniquely identified using two angles. Think of it as "wrapping" the sphere with a piece of graph paper.

Astronomers make use of several spherical coordinate systems. The principal difference between them is that the positions of the stars are referenced to a different fundamental plane.

Example 1.1 Great vs. small circles

At the equator, which is a great circle, the distance corresponding to 1° of longitude is simply 1/360 of the circumference of the Earth. The circumference of the Earth is given by $2\pi R$, where R is the radius of the Earth. So, 1° of longitude along the equator amounts to

$$\left(\frac{1}{360}\right)(2\pi)(6378\ \text{km}) = 111.3\ \text{km} \qquad \text{(at latitude } 0^\circ\text{)}$$

At latitude $\phi = 45^\circ$, however, 1° of longitude corresponds to a much shorter distance

$$(111.3\ \text{km})\cos\phi = (111.3\ \text{km})\cos 45 = 78.7\ \text{km} \qquad \text{(at latitude } 45^\circ\text{)}$$

An arc at constant latitude 45° that is 1° longitude in arc length is part of a small circle, and it has a physical length that is a factor of $\cos\phi$ smaller than the great circle distance corresponding to the same longitude arc length at the equator.

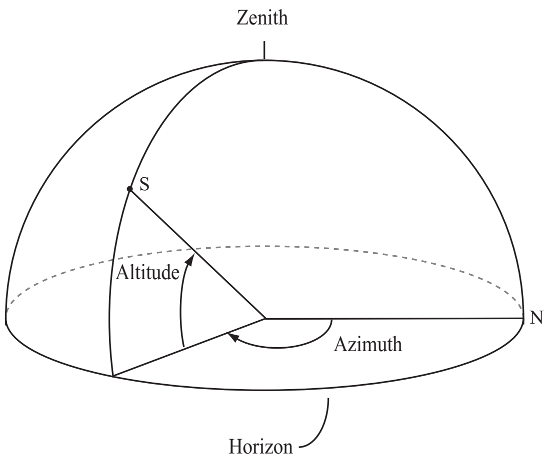

Figure 1.4. Altitude and azimuth in the horizon system. S is the position of the star.

Altitude–azimuth coordinates

The **horizon system** is the most intuitive when we first begin to discuss the positions of stars. Here, the fundamental plane is the plane of the observer's ideal horizon, and one pole is at the **zenith**, the point directly overhead. The unobstructed horizon is by definition 90° from the zenith. **Altitude** is the angle that the observer would measure from the horizon to an object in the sky, measured along a great-circle arc that passes through the zenith. Altitude varies from 0° for an object on the horizon to 90° for an object at the zenith. An object below the horizon would not be visible, but it may be considered as having a negative altitude. An example of this is the Sun and twilight. When the Sun is 18° below the horizon, we call this the moment of astronomical **twilight**. It marks the time when the last trace of evening twilight fades from the sky, or when morning twilight begins. The center of the Sun at the moment of astronomical twilight has an altitude of $-18°$.

In the horizon system, altitude is the first coordinate we need to uniquely identify any point on the sky. Determining the second coordinate requires selecting a zero point direction along the horizon, and for that we choose the north point. The **azimuth** is defined as the angle measured in the plane of the horizon from the north point to the point at which the arc from the zenith through the object crosses the horizon. By convention the azimuth increases in the direction toward the east. Figure 1.4 illustrates how altitude and azimuth are measured with respect to the horizon. Figure 1.5 shows a **theodolite** used to measure the altitudes and azimuths of celestial objects.

This coordinate system is variously referred to as the **horizon system**, the **altitude–azimuth system**, or as simply the **alt–az system**. Since

Figure 1.5. The theodolite, an instrument for measuring altitude and azimuth (Wellesley College photograph).

the Earth rotates continually towards the east, celestial objects appear to move from east to west across the sky. This means that in the horizon system the coordinates of objects are always changing. Such a system does have useful applications as we shall see, but it cannot be used to provide permanent descriptions of the locations of stars on the celestial sphere.

Equatorial coordinates

A coordinate system with more useful properties in astronomy is the **equatorial system** in which the fundamental plane is now the **celestial equator**. If one imagines a very small spherical Earth at the center of a very large spherical sky, then the extension of the Earth's axis defines the **celestial poles**, and the extension of the plane of the Earth's equator defines the celestial equator. In the equatorial system, then, coordinates of stars are defined with respect to the celestial equator. The angle

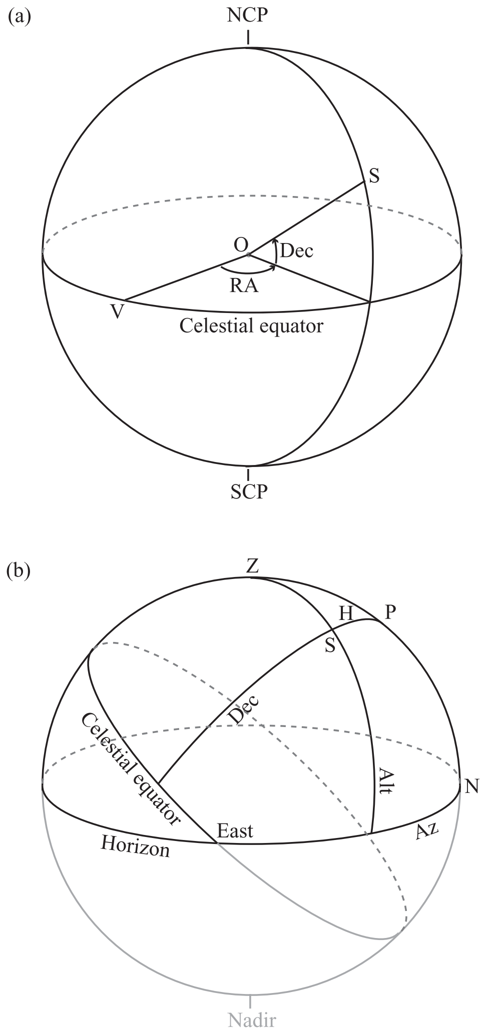

Figure 1.6. (a) The equatorial coordinate system. (b) The equatorial system superimposed on the horizon system. V is the vernal equinox.

measured northward or southward from the celestial equator is called the **declination**, and the angle measured in the plane of the celestial equator is called the **right ascension**. These two angles are indicated as Dec and RA respectively in Figure 1.6(a). A zero-point for the measurement of the RA angle must, of course, be selected, and for this we use the **vernal equinox** (♈), the point on the sky where the Sun crosses the celestial equator moving northward on or near March 21 each year. This point has been marked V in Figure 1.6(a). There is no obvious way in which one can identify the vernal equinox when looking at the sky, but later in Chapter 4 we shall show that this point can be located quite specifically from very simple observations. Later in this chapter we shall also describe the methods by which astronomers determine the right ascensions and declinations of stars.

By convention, declination is considered positive for objects in the northern hemisphere of the sky (north of the celestial equator) and negative for objects in the southern hemisphere of the sky (south of the celestial equator). Right ascension increases as one moves east along the celestial equator, and it is measured conveniently (we shall see why later) in units of time: hours, minutes, and seconds. Right ascension ranges from 0^h to 24^h, and because $24^h = 360^\circ$, there are $360^\circ/24^h = 15^\circ$ per hour of right ascension. Declination is measured in degrees. Astronomers often use the Greek letter alpha, α, to denote right ascension, and the Greek letter delta, δ, to denote declination.

The origin of the terms *right ascension* and *declination* is interesting. The apparent motion of the stars upon the celestial sphere as the Earth rotates appears quite different depending on where one is situated upon the surface of the Earth. At the Earth's equator, the stars all rise perpendicular to the eastern horizon, and the celestial equator passes through the zenith. This celestial sphere is called a *right sphere*. At either of Earth's poles, the stars appear to wheel around the zenith at constant altitude, and this celestial sphere is called a *parallel sphere*. At all other latitudes on the earth, we have a combination of rising (and setting) and wheeling motion (circumpolar stars), and this celestial sphere is called an *oblique sphere*. At the Earth's equator, all stars rising above the eastern horizon at a given moment have the same celestial longitude, that is, they have the same right ascension. The stars ascend vertically in the east in a right sphere, hence the term *right ascension*. An early meaning of the word *declination* refers to an inclination or leaning towards or away from something. This makes perfect sense when one considers that declination is the angular distance of a celestial object from the celestial equator. We will leave it as an exercise for the historically curious to figure out why the term *declination*, with all its negative nuances, was used instead of its more positive analog *inclination*.

In Figure 1.6(b) we have shown both the equatorial and the horizon systems superimposed on the same sketch. We remind the reader that as the sky appears to rotate counterclockwise around the north celestial pole, the altitude and azimuth of any celestial object will both be changing continuously, unless the object is fortuitously located precisely at the north celestial pole. The right ascension and declination will, of course, remain fixed. In the diagram, the great circle defined by the arc ZPN is called the observer's **celestial meridian** (or simply **meridian)**, and it always goes through the observer's zenith and the north and south points on the horizon (azimuth 0° and 180°, respectively), dividing the sky into an east half and west half. Here $\angle$ZPS has been marked H, and this angle is referred to as the **hour angle**. In other words, the hour angle indicates how far east or west of the meridian an object is, measured along the celestial equator. It is conventional to regard the hour angle as being negative when a star is east of the meridian and positive when a star is west of the meridian. Because the Earth is rotating, the hour angle is constantly changing, and as time passes the hour angle decreases progressively (that is, approaches zero) when a star is east of the meridian and becomes progressively larger after the star has crossed the meridian into the western sky. It is often useful to know the hour angle of a celestial object, because it tells you *when* the object crosses (or did cross) the meridian relative to the present time. The time of meridian crossing is the best time to look at a celestial object, because then it reaches its maximum altitude when the distorting effects of the Earth's atmosphere should be minimal. (**Circumpolar** stars cross the meridian above the horizon twice each day. We are usually interested in the meridian crossing that is above the north celestial pole, known as the **upper culmination**.) The hour angle of a star will depend upon the time at which the observation is being made and the right ascension of the star. We shall return to this subject in Chapter 4 where we shall see how the hour angle is determined.

Other coordinates

Another system of coordinates was of considerable use among astronomers several hundred years ago and is used today by those who are engaged in the study of the motions of the planets. In this system the fundamental plane is the **ecliptic**, the apparent path of the Sun around the sky (and the plane of the Earth's orbit projected upon the celestial sphere). The zero point on the ecliptic is again the vernal equinox, and the two coordinates are known as **ecliptic latitude**, β, and ecliptic longitude, λ. Figure 1.7 illustrates the ecliptic system of coordinates.

Finally, in studies of our Galaxy, astronomers make use of a system of coordinates in which the fundamental plane is the galactic equator, a great circle that closely approximates the "centerline" of the Milky Way on the sky. Astronomers have carefully chosen the right ascension and declination of the galactic north pole in such a way that the galactic equator is precisely defined. As illustrated in Figure 1.8, the two coordinates are known as **galactic latitude**, b, and **galactic longitude**, l, and the zero point on the galactic equator is a point in the constellation Sagittarius, which marks the direction to the center of our Galaxy. It is worthy of mention here that galactic coordinates cannot be determined by direct observation. They are calculated from the equatorial coordinates of the object of interest, the galactic pole, and the center of the Galaxy. Details of this computation will be presented in Chapter 4.