1
Introduction

'Faced with something unusual our thought should not be "What next?" but "Why?". By answering the second of these questions we can answer the first. And this, in brief, is the scientific method.'
Roger Scruton (c. 1990)

By definition, all exothermal reactors, including any fusion reactor one may envisage (tokamak, stellarator, etc.), produce both energy and spent reactants, or ash.[1] In order for the reactor to operate in steady-state, (i) fresh fuel must be added at the rate at which it is consumed, (ii) this fuel must be heated, ideally by the reactions themselves, (iii) fuel must be confined, by whatever means are available, for sufficiently long to allow the exothermic processes to continue, (iv) the energy and ash must be removed from the system at the rate at which they are created, (v) the impurities released from the reactor walls must likewise be removed at the rate at which they are produced, and (vi) the reactor itself, primarily its walls, must not be damaged by all the exhaust processes. Translating the above to a D–T burning tokamak, conditions (i)–(iii) may be labelled loosely as the *ignition* criteria, and conditions (iv)–(vi) as the *exhaust* criteria. Taken together they constitute the criteria of mutual *compatibility* between the burning plasma and first wall materials/components. Since the ignition criteria speak primarily to the central (core) plasma, while the exhaust criteria refer to the boundary (edge) plasma, and since the two regions are coupled by largely self-governing plasma transport processes, it is the exhaust criteria which determine the optimum reactor performance for a given reactor design. In the following, we introduce the basic concepts of fusion reactor operation, including the stability and exhaust limits on reactor performance.

[1] This applies both to chemical reactors, such as a candle or a steam engine, and nuclear reactors, such as a star or a fission power plant. It is equally true for all fusion reactors, irrespective of whether the reacting fuel is confined by *gravity*, as in the Sun, by *magnetic* fields and electric currents, as in a tokamak or stellarator, or by the *inertia* of the ions themselves, as in the violent implosion of a hydrogen ice pellet after it is heated by lasers or heavy ion beams.

1.1 Fusion reactor operating criteria

Let us consider the ignition and exhaust criteria for a *magnetically confined fusion* (MCF) reactor, operated either in steady-state or in successive pulse cycles; although we restrict the discussion to MCF, most of the following remarks apply equally well to *inertially confined fusion* (ICF). There are four fusion reactions of interest for energy production (Krane, 1988),

$$D + T \rightarrow He^4\,(3.52\,\text{MeV}) + n\,(14.06\,\text{MeV}), \tag{1.1}$$

$$D + D \rightarrow T\,(1.01\,\text{MeV}) + p\,(3.03\,\text{MeV}), \tag{1.2}$$

$$D + D \rightarrow He^3\,(0.82\,\text{MeV}) + n\,(2.45\,\text{MeV}), \tag{1.3}$$

$$D + He^3 \rightarrow He^4\,(3.67\,\text{MeV}) + p\,(14.67\,\text{MeV}), \tag{1.4}$$

where D and T represent the two isotopes of hydrogen: deuterium ($D \equiv H^2$) and tritium ($T \equiv H^3$). In all four cases, the strong Coulomb repulsion of the positively charged nuclei implies that the fusion cross-sections σ are only significant at ion energies above 10 keV, e.g. at 100 keV, $\sigma_{DT} \sim 5$ barn, $\sigma_{DD} \sim \sigma_{DHe3} \sim 0.01$ barn. In thermonuclear fusion, the supra-thermal particles in the tail of the Maxwellian distribution are responsible for most of the fusion reactions. Since the average reaction rate $\langle\sigma v\rangle$ is largest for (1.1), especially for $T_i < 100$ keV, e.g. at $T_i = 10$ keV, $\langle\sigma v\rangle_{DT} \sim 10^{-22}$ m^3/s, while $\langle\sigma v\rangle_{DD} \sim 10^{-24}$ m^3/s, a mixture of D and T is the preferred fuel for future fusion reactors, including ITER and DEMO. At keV temperatures, the atoms of hydrogen (for which the ionization potential is only 4 eV), as well as those of most low and medium Z elements, become fully ionized and the neutral gas mixture is transformed into an ion–electron *plasma*.

We now return to our six reactor criteria, the first two of which state that the D–T fuel burned in reaction (1.1) must be replenished, criterion (i), and heated to the operating reactor temperature, criterion (ii). In practice, (i) is achieved either by gas puffing or ice pellet injection, although neither of these methods is capable of delivering the fuel directly to the plasma core, i.e. the hot central region where the thermonuclear burn is active; instead the fresh fuel is deposited (ablated/ionized) in the edge plasma, and only reaches the core by a relatively slow diffusion process. In contrast, the steady flow of power required by (ii) is delivered directly to the core plasma either by external heating, e.g. by neutral beams or radio waves resonant with the gyration frequencies of ions and electrons, or by the charged fusion products, such as the 3.5 MeV alpha particle in (1.1) or the 14.7 MeV proton in (1.4), which are trapped by the magnetic fields.[2]

[2] Neutrons released in fusion reactions do not interact with, and thus cannot heat, the plasma. The same is true for photons released as bremsstrahlung and synchrotron radiation.

Particle and energy confinement of a thermonuclear plasma, criterion (iii), have been the central focus of MCF research over the past 50 years. To appreciate the difficulties posed by this task, recall that energy *break-even*,

$$Q_{DT} \equiv P_{DT}/P_{heat} > 1, \qquad Q_\alpha \equiv P_\alpha/P_{heat} = 0.2 Q_{DT}, \tag{1.5}$$

where Q_{DT} is the *energy multiplication factor*, defined as the ratio of fusion and auxiliary heating powers, was only approached recently. Since 80% of the energy released in (1.1) appears as the kinetic energy of neutrons and is thus promptly lost from the plasma, the fusion reactions can only be self-sustaining when $Q_\alpha \gg 1$. Note that P_{DT} and P_α may be evaluated as

$$P_{DT} = 5P_\alpha = \frac{\mathcal{E}_{DT}}{4} \int n^2 \langle \sigma v \rangle_{DT} \mathrm{d}\mathbf{x}, \qquad \mathcal{E}_{DT} = 5\mathcal{E}_\alpha = 17.58\,\mathrm{MeV} \tag{1.6}$$

where $\mathcal{E}_{DT}$ is the energy released per fusion reaction, $n = n_D + n_T$ is the particle density, $\langle \sigma v \rangle_{DT}$ is the fusion reaction cross-section and $\int \mathrm{d}\mathbf{x}$ is a volume integral over the plasma.

The slow progress towards $Q_\alpha > 1$ can be ultimately traced to one of the great unsolved problems of classical physics, namely fluid turbulence. Indeed, much of the success of MCF can be ascribed to the basic dimensional scaling: volume/area $\sim$ size, and thus to the building of ever bigger, and more expensive, devices, specifically the toroidal, axis-symmetric, inductively driven *tokamaks*, see Fig. 1.1. It is thus no coincidence that $Q_{DT} \sim 1$ was finally approached in the largest present day

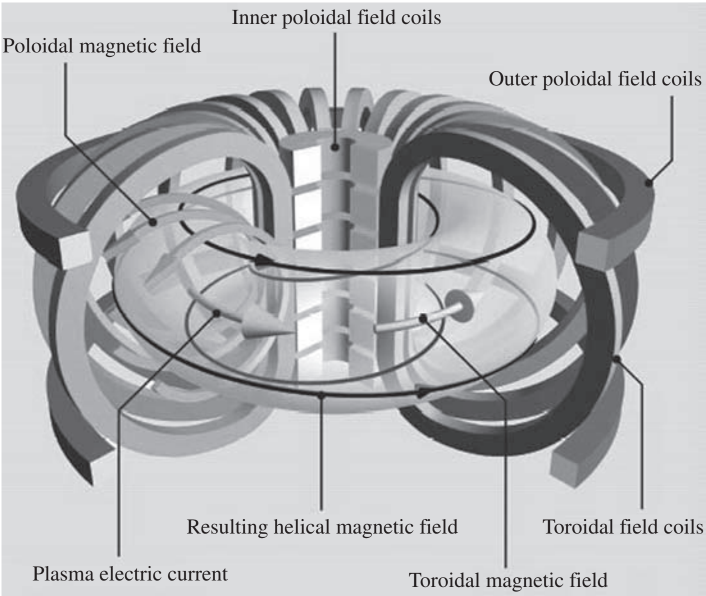

Fig. 1.1. Schematic representation of a *tokamak*. © EFDA-JET.

tokamak, namely the *Joint European Torus* (JET), with $Q_\alpha > 1$ expected in ITER. That future event may be viewed as the watershed beyond which issues related to energy confinement, criterion (iii), will be increasingly overshadowed by those related to power exhaust, criteria (iv)–(vi). This tendency, which first emerged in the technological challenges encountered during the conceptual, and later engineering, design of ITER, is also evident in a new generation of super-conducting, actively cooled machines.

Since both fusion power and ash (He^4) are generated in the plasma centre, their exhaust mechanisms are partly related. Thus, power is removed from an MCF plasma by three channels: (a) by neutrons released in the fusion reaction itself; (b) by photons emitted during bremsstrahlung, synchrotron and (hydrogenic or impurity) line radiation; and (c) by kinetic energy of the ions and electrons which are transported across the magnetic field largely by turbulent plasma motions; in contrast, fusion ash is removed from the core plasma only by turbulent advection. Assuming that bremsstrahlung is the dominant mode of radiation in the hot plasma core, we may approximate the steady-state power balance for a burning fusion plasma as

$$P_{heat} + P_\alpha = (1 + Q_\alpha)P_{heat} \approx P_{br} + P_{tr} = P_{loss}, \tag{1.7}$$

where the left-hand side represents the auxiliary (P_{heat}) and α particle (P_α) heating, and the right-hand side the total losses due to bremsstrahlung (P_{br}) and plasma transport by convection and conduction (P_{tr}),

$$P_{br} = \alpha_{br} \int n^2 T^{1/2} \mathrm{d}\mathbf{x}, \qquad W = 3 \int nT \,\mathrm{d}\mathbf{x} \equiv P_{tr}\tau_E \equiv P_{loss}\tau_E^*, \tag{1.8}$$

where n is the particle density, T is the plasma temperature and τ_E and τ_E^* are the thermal energy confinement times associated with plasma transport and transport + radiation, respectively. Hence Q_α may be estimated as

$$Q_\alpha \equiv \frac{P_\alpha}{P_{heat}} = \left[\frac{P_{loss}}{P_\alpha} - 1\right]^{-1} = \frac{P_\alpha}{P_{loss} - P_\alpha}. \tag{1.9}$$

The limit of vanishing heating power, which is equivalent to $Q_\alpha \to \infty$, can thus be expressed as $P_\alpha = P_{loss}$. Substituting from (1.6) and (1.8), we obtain the corresponding *ignition criterion*,

$$P_\alpha \geq P_{loss} \qquad \Rightarrow \qquad n\tau_E > \frac{12T}{\mathcal{E}_{DT}\langle\sigma v\rangle_{DT}} > 1.5 \times 10^{20}\,\mathrm{m}^{-3}\,\mathrm{s}, \tag{1.10}$$

where n and T represent volume-average values and the final expression represents a minimum value near $T \approx 30\,\mathrm{keV}$. In the keV temperature range, one finds that $\langle\sigma v\rangle_{DT} \propto T^2$ and the above result simplifies further to

$$nT\tau_E = p\tau_E > 3 \times 10^{21}\,\mathrm{m}^{-3}\,\mathrm{keV\,s} \approx 5\,\mathrm{bar} \cdot \mathrm{s}. \tag{1.11}$$

The numerical values in (1.10) and (1.11) assume flat radial profiles of n and T; for peaked profiles, these values are somewhat higher.

Let us next consider a corresponding condition for a *fusion reactor*, in which all power leaving the plasma is converted to electricity with an efficiency η_e and then used to heat the plasma with efficiency η_h. Defining $\eta = \eta_e\eta_h$, for which one expects values in the range 0.2–0.4, the requirement for net energy production may be written as,

$$\eta(P_{fus} + P_{loss}) > P_{loss}, \qquad P_{fus} = P_{DT} + P_{Li}, \tag{1.12}$$

where the additional power P_{Li} refers to the energy released in the breeder blanket by the reaction (1.14), see below. Substituting from (1.6) and (1.8) leads to the celebrated *Lawson's criterion* (Lawson, 1957),

$$n\tau_E > 3T\left(\frac{\eta}{1-\eta}\frac{\mathcal{E}_{DT}}{4}\langle\sigma v\rangle_{DT} - \alpha_{br}T^{1/2}\right)^{-1} \sim 3\times 10^{19}\,\mathrm{m}^{-3}\mathrm{s}, \tag{1.13}$$

where the final expression was evaluated near $T \approx 30$ keV and $\eta = 1/3$. The plasma ignition criterion (1.10) is equivalent to (1.13) with $\eta = 0.136$.[3]

Power exhaust channels (a)–(c) lead to three different types of heat loads on the first wall and require three different power removal systems: (a) the *neutron* energy is deposited volumetrically in a neutron-absorbing envelope surrounding the first wall, ideally a *breeder blanket*, employing the reactions,

$$\mathrm{Li}^6 + \mathrm{n} \rightarrow \mathrm{T} + \mathrm{He}^4 + 4.8\,\mathrm{MeV}, \tag{1.14}$$

$$\mathrm{Li}^7 + \mathrm{n} \rightarrow \mathrm{T} + \mathrm{He}^4 + \mathrm{n} - 2.5\,\mathrm{MeV}, \tag{1.15}$$

to breed tritium fuel from solid lithium, (b) the *photon* energy generates a fairly uniform surface heat load on first wall components, and (c) the *plasma* thermal energy is convected and conducted along the magnetic field lines to dedicated heat load bearing tiles. In each case, the power deposited on, or absorbed in, the vessel wall must be removed by an active coolant loop. Moreover, the effective heat load must not exceed some limit imposed by thermo-mechanical constraints. This in turn limits the energy flow crossing the outer boundary of the plasma in each of the three channels, i.e.

$$P_{\perp\sigma}/A_p \equiv q_{\perp\sigma} < q_{\perp\sigma}^{exh}, \qquad \sigma \in \{n, \gamma, tr\}, \tag{1.16}$$

where A_p is the plasma area. In practice, the last of these conditions imposes the most severe constraints on plasma operation, e.g. for ITER, the time-averaged power loads on plasma facing components (PFCs) are limited to $\sim$10 MW/m^2 and

[3] To demonstrate this, it suffices to insert $P_\alpha = P_{loss}$ in (1.12), which yields $\eta = P_\alpha/(P_\alpha + P_{DT} + P_{Li}) = 3.52/(3.52 + 17.58 + 4.8) = 0.136$.

transient energy loads to ~0.5 MJ/m^2 in ~250 μs.[4] Consequently, in the rest of the book we will focus on channel (c) above, i.e. the exhaust of fusion energy (and to a lesser extent, of fusion ash) by plasma transport processes. As motivation for this investigation, we first compare the limits on fusion reactor performance, which for simplicity we assume to be a tokamak, imposed by plasma stability and power exhaust.

1.2 Plasma stability limits on fusion reactor performance

Let us first assess the limits imposed by global (MHD, magneto-hydrodynamic) plasma stability requirements, which will be derived in Section 4.2 and summarized in Table 4.1.[5] Expressing the fusion power density in terms of the toroidal beta,

$$P_{DT}/\mathcal{V} = \tfrac{1}{4}\mathcal{E}_{DT}\langle n^2\langle\sigma v\rangle_{DT}\rangle_a \propto \langle p^2\rangle_a \propto \beta_T^2 B_0^4, \tag{1.17}$$

where β_T is given by (3.19) and $\langle\cdot\rangle_a$ is the average over the plasma volume, $\mathcal{V} = \int_a d\mathbf{x}$, and noting that the toroidal magnetic field (on axis) is limited by technological constraints to roughly $B_0^{max} \sim 5-10$ T, we find that the MHD pressure limits determine the maximum fusion power density and hence the reactor cost.[6] In order for the burning plasma equilibrium to be MHD stable, the MHD beta limit β_T^{mhd}, as given in Table 4.1, must exceed the minimum beta required for ignition β_T^{ign}, which may be inferred from (1.11).

$$\beta_T^{ign} \propto B_0^{-2}\tau_E^{-1}, \qquad \beta_T^{mhd} \propto \epsilon_a\kappa_a/q_0 q_a. \tag{1.18}$$

Here we defined the inverse aspect ratio, $\epsilon_a = a/R_0$, where a and R_0 are the minor and major radii of the torus, the elongation $\kappa_a = A_p/\pi a^2$ where A_p is the cross-sectional area of the plasma, and the safety factors on axis ($r = 0$) and at the edge of the plasma ($r = a$), q_0 and q_a, given by (3.18).

On the other hand, $\beta_T < \beta_T^{mhd}$ amounts to inefficient use of the 'expensive' toroidal magnetic field, which is optimally used only for $\beta_T \approx \beta_T^{mhd}$. Hence, the condition $\beta_T^{ign} \approx \beta_T^{mhd}$ determines the size a_{ign} of the smallest reactor able to ignite for given field B_0, inverse aspect ratio ϵ_a, elongation κ_a, etc.

To evaluate a_{ign}, we need to estimate the energy confinement time τ_E (1.8), e.g. we may assume that radial transport is purely diffusive, so that $\tau_E \approx a^2/\chi_\perp$, where

[4] This value should not be confused with $q_{\perp tr}^{exh}$, which refers to the power flux crossing the last closed flux surface, see Chapter 7.

[5] Here we will anticipate some of the definitions which will be made formally in Chapters 2 to 4.

[6] Since the reactor capital cost is roughly proportional to the plasma volume $\mathcal{V}$, being driven largely by the cost associated with super-conducting poloidal coils, one finds that the cost of electricity it generates scales inversely with $P_{DT}/\mathcal{V}$, i.e. an economical reactor should be as small as possible to generate a desired power output in MWe. Hence, the power density (1.17) may be interpreted as the financial figure of merit for a fusion reactor.

$\chi_\perp$ is the average radial heat diffusivity.[7] Anticipating the results of Chapter 6, we write down the generic expression,

$$B_0\tau_E \propto (q_a\rho_*)^{-x} \propto (q_a\rho_{ti}/a)^{-x} \propto (q_a\sqrt{T}/aB_0)^{-x}, \tag{1.19}$$

which states that $B_0\tau_E$ scales inversely with the product of the safety factor q_a, or q_* (4.110), and the normalized, toroidal gyro-radius $\rho^* \equiv \rho_{ti}/a$; here $x = 3$ corresponds to the *gyro-Bohm* scaling and $x = 2$ to the *Bohm* scaling.

Inserting (1.19) into (1.11), one finds a scaling of β_T^{ign} with reactor size,

$$\beta_T^{ign} \propto [q_a\sqrt{T}/aB_0]^x/B_0 \propto a^{-x}B_0^{-(x+1)}T^{x/2}q_a^x. \tag{1.20}$$

Since, the *minimum* beta needed for ignition, β_T^{ign}, decreases sharply with size, whereas the *maximum* beta imposed by MHD stability, β_T^{mhd}, is size independent, we find that ignition is always possible for large enough plasmas.[8] Equating β_T^{ign}, (1.20), and β_T^{mhd}, (1.18), gives the minimum ignition radius,

$$a_{ign} \propto (q_a/B_0)^{1+1/x}T^{1/2}(q_0/\varepsilon_a\kappa_a)^{1/x}, \tag{1.21}$$

which decreases with toroidal field as $B_0^{-3/2}$ ($x = 2$) and $B_0^{-4/3}$ ($x = 3$).

The plasma volume corresponding to (1.21) is found to scale as

$$\mathcal{V}_{ign} \propto a_{ign}^3\kappa_a/\varepsilon_a \propto (q_a/B_0)^{3(1+1/x)}T^{3/2}q_0^{3/x}\varepsilon_a^{-(1+3/x)}\kappa_a^{1-3/x}. \tag{1.22}$$

Since V_{ign} increases with q_a, q_0 and T, and decreases with B_0, ϵ_a and κ_a (although the κ_a dependence vanishes for $x = 3$), we would like to minimize (maximize) the former (latter) parameters. This can be done by (i) fixing $q_0 \approx q_0^{mhd} \approx 1$ and $q_a \approx q_a^{mhd} \approx 2-3$ at their MHD stability limits, (ii) choosing $T \sim 10$–30 keV, which, although below the maximum of the fusion cross-section, $\langle\sigma v\rangle_{DT}$, minimizes (1.10), and (iii) setting the axial field at $B_0^{max} \sim 5$ T. Moreover, (1.22) strongly favours small aspect ratios ($\epsilon_a \sim 1$) and weakly favours elongated plasma shapes ($\kappa_a > 1$), provided $x < 3$. The upper limit on ϵ_a and lower limit on R_0 are imposed by the requirement for a neutron heat shield on the inner solenoid;[9] the upper limit on κ_a is imposed by an axis-symmetric ($n = 0$) vertical displacement MHD instability, which becomes increasingly acute for elongated poloidal cross-sections. For instance, for ITER, whose aim is to achieve $Q_{DT} = 10$, or $Q_\alpha = 2$, and produce 500 MW of fusion power, the above

[7] This scaling expresses the easily verified fact that larger objects take longer too cool.
[8] However, as will be shown in Section 7.1, power exhaust considerations impose an upper limit on a cost effective reactor, i.e. one with $\beta_T \approx \beta_T^{mhd}$.
[9] Some designs dispense with this requirement by envisioning a replaceable central column, thus allowing a much smaller R_0 and thus a larger inverse aspect ratio ϵ_a.

parameters were carefully optimized with respect to cost and performance to yield

$$a = 2.0\,\text{m}, \quad B_0 = 5.3\,\text{T}, \quad q_a = 3.0, \quad \kappa_a = 1.7, \quad \epsilon_a = 0.33, \tag{1.23}$$

where $q_a = 3$ represents a plasma current of $I_T = 15\,\text{MA}$, see Fig. 8.1.

1.3 Power exhaust limits on fusion reactor performance

The limits on fusion reactor performance imposed by plasma stability, as derived above, should be compared with those imposed by power exhaust (1.16). Let us assume that the radial energy flow at the *last closed flux surface* (LCFS) is limited to some value $q_\perp^{exh}$, which can be written as

$$P_{tr}/A_p < q_\perp^{exh}, \qquad A_p = \int_{LCFS} \mathrm{d}S_\perp \approx 4\pi^2 a R_0 \kappa_a \approx 4\pi^2 a^2 \kappa_a/\epsilon_a. \tag{1.24}$$

Defining $0 < f_\alpha \equiv Q_\alpha/(1 + Q_\alpha) < 1$, it follows from (1.8) that

$$f_\alpha = \frac{P_\alpha}{P_{loss}} = \frac{\mathcal{E}_\alpha \langle n^2 \langle \sigma v \rangle_{DT} \rangle_a}{12 \langle p \rangle_a / \tau_E^*} \propto \frac{\langle p^2 \rangle_a \tau_E^*}{\langle p \rangle_a} \propto \langle p \rangle_a \tau_E^* \propto \beta_T B_0^2 \tau_E^*. \tag{1.25}$$

We next consider the low radiation limit ($P_{br} \ll P_{tr}$) for which $\tau_E^* \approx \tau_E$ and $P_{loss} \approx P_{tr}$. In this case, we can eliminate τ_E in P_{tr} using (1.25),

$$P_{tr}/\mathcal{V} \propto \beta_T B_0^2/\tau_E \propto \beta_T^2 B_0^4/f_\alpha, \qquad P_\alpha/\mathcal{V} \propto \beta_T^2 B_0^4. \tag{1.26}$$

Dividing (1.24) by (1.26) introduces the volume to surface ratio $\mathcal{V}/A_p \approx a/2$, which increases linearly with size. This imposes an upper, *power exhaust* limit a_{exh} on the reactor size for given values of $q_\perp^{exh}$, plasma pressure $p \propto \beta_T B_0^2$, energy confinement τ_E and/or level of ignition f_α, see Fig. 1.2,

$$a < a_{exh} \propto q_\perp^{exh} \tau_E / \beta_T B_0^2 \propto q_\perp^{exh} f_\alpha / \left(\beta_T B_0^2\right)^2. \tag{1.27}$$

For ignited ($f_\alpha = 1$) and marginally MHD stable $\left(\beta_T \approx \beta_T^{mhd}\right)$ plasmas,

$$a_{ign} \le a \le a_{exh}^{mhd} \propto q_\perp^{exh} (q_0 q_*/\epsilon_a \kappa_a)^2 B_0^{-4} \propto q_\perp^{exh} (q_a/\epsilon_a \kappa_a)^2 B_0^{-4}, \tag{1.28}$$

where a_{ign} is given by (1.21) and we set $q_0 = 1$ and $q_* = q_a$. When $q_\perp^{exh}$ is sufficiently large, i.e. when $q_\perp^{exh} > q_\perp^{ign} \propto a_{ign} \left(\beta_T^{mhd} B_0\right)^2$, the maximum β_T power exhaust radius exceeds the ignition radius, $a_{exh}^{mhd} > a_{ign}$, and (1.24) is satisfied automatically in the range of minor radii given by (1.28). This range of optimal a is reduced as $q_\perp^{exh}$ decreases, eventually prohibiting fusion burn at MHD marginal stability when $q_\perp^{exh} < q_\perp^{ign}$ $\left(a_{exh}^{mhd} < a_{ign}\right)$.

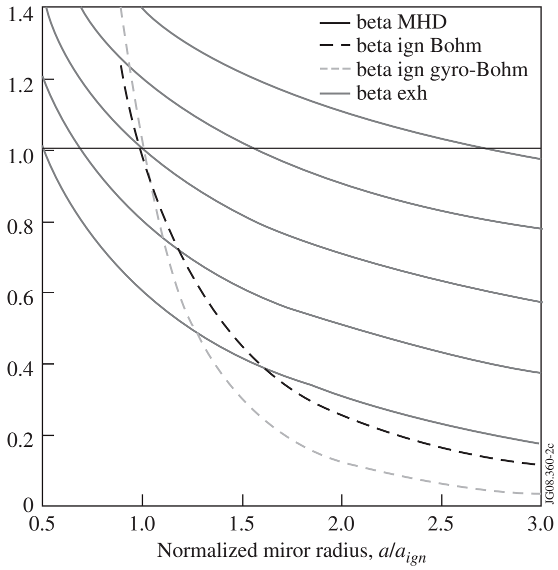

Fig. 1.2. Beta limits (ignition, stability and exhaust) vs. reactor size.

To estimate the minimum ignition radius in that case, we equate the minimum beta needed for ignition β_T^{ign} (1.20), with the maximum beta allowed by the power exhaust limit, which follows from (1.27) with $f_\alpha = 1$,

$$\beta_T^{exh} \propto \left(q_\perp^{exh}/aB_0^4\right)^{1/2}. \tag{1.29}$$

This yields an estimate of the power exhaust limited ignition radius,

$$a_{ign}^{exh} \propto \left(q_\perp^{exh}\right)^{-y/2} q_a^{xy} T^{xy/2} B_0^{(1-x)y}, \qquad y = 1/(x - 1/2). \tag{1.30}$$

Since $\beta_T^{ign} \propto a^{-x}$ decays faster than $\beta_T^{exh} \propto a^{-1/2}$, it is possible to achieve ignition for any value of $q_\perp^{exh}$, by increasing the size of the reactor; this is reflected in the weak, inverse scaling $a_{ign}^{exh} \propto \left(q_\perp^{exh}\right)^{-y/2}$, with the exponent being equal to $-1/3$ for $x = 2$ and $-1/5$ for $x = 3$. In short, power exhaust imposes the minimum reactor size only when $q_\perp^{exh} < q_\perp^{ign}$,

$$a_{min} = \max\left(a_{ign}, a_{ign}^{exh}\right). \tag{1.31}$$

The corresponding toroidal beta, $\beta_T^{min} \propto \left(q_\perp^{exh}/a_{min}B_0^2\right)^{1/2}$, is smaller than β_T^{mhd} thus reducing the fusion power density and the cost effectiveness of the reactor. The determination of $q_\perp^{exh}$ as a function of plasma and field quantities is the chief task of both experimental and numerical power exhaust studies. It is also one of the main incentives for investigating transport processes in the plasma boundary and the ultimate goal of the theoretical development, and the accompanying discussion, in the rest of the book.

Let us summarize the above findings. Since the exhaust limits provide the boundary conditions for the plasma thermodynamic quantities, they effectively determine the maximum achievable fusion gain, Q_α, for a given *reactor design* (RD), by which we mean a set of hardware including magnetic coils, heating, fuelling and current drive systems, vacuum vessel and mechanical support, cooling and pumping systems, and last, but not least, the plasma facing components (PFCs), i.e. the first wall armour against plasma fluxes. This relation may be expressed as

$$Q_\alpha = Q_\alpha(PS, RD), \qquad Q_\alpha^{max}(RD) = \max[Q_\alpha(PS, RD)|PS], \tag{1.32}$$

where *plasma scenario* (PS) refers to a combination of plasma shape, magnetic field, current profile, heating and fuelling methods, etc., i.e. to the way in which the given reactor design is utilized within each plasma discharge. Thus, the issue of compatibility or integration between the ignition and exhaust criteria, and specifically between plasma scenarios and PFCs, is really one of optimization of the PS for a given RD with respect to Q_α.[10]

Since the fusion power density is roughly proportional to the square of the central fuel ion plasma pressure, (1.6), while the plasma density is limited to roughly the *Greenwald density*, see (7.50), this optimization amounts to maximizing the ion temperature, T_i, and minimizing the effective charge, Z_{eff}, in the centre of the plasma column. In the absence of internal transport barriers, e.g. in the inductive or baseline tokamak plasma scenario, the ion temperature gradient (ITG) is set by the threshold for the ITG drift-wave turbulence (Garbet and Waltz, 1998). Hence, the central ion temperature is a linear function of the edge, or pedestal, temperature, T_{ped}, e.g. in ITER, it is predicted that $T_{ped} \sim 4\,\text{keV}$ is necessary to achieve the desired fusion gain factor, $Q_\alpha \sim 2$ (Doyle *et al.*, 2007).[11] The impact of any given PFC limit on the reactor performance can then be quantified as

$$\zeta(PFC) = 1 - Q_\alpha^{max}(PFC)/Q_\alpha^{max}(\infty), \tag{1.33}$$

where $Q_\alpha^{max}(PFC)$ is the maximum fusion gain factor for a specified PFC limit, i.e. (1.32) with PFC in place of RD, and $Q_\alpha^{max}(\infty)$ the same factor without any limit on PFC plasma loads, or some previously chosen reference limit value. One can recast (1.33) in terms of density and energy confinement degradation by estimating $Q_\alpha \propto p\tau_E \propto (f_{GW}H_{98})^z$, with $z \sim 2$–3,

$$\zeta(PFC) = 1 - [f_{GW}(PFC)/f_{GW}(\infty)]^z[H_{98}(PFC)/H_{98}(\infty)]^z, \tag{1.34}$$

[10] It is worth noting that the very terms ‘compatibility’ and ‘integration’ reflect the historical disconnection between the tasks of investigating, on the one hand, the plasma equilibrium, stability and transport, and, on the other, its particle and power exhaust properties. Such a disconnection is of course absent in a real plasma where the core and edge regions form an integrated whole.

[11] Whether such high edge plasma temperatures are compatible with the desired lifetime of the divertor and limiter PFCs remains a matter of active research.