COMMUNICATION IN MECHANISM DESIGN

Mechanism design is the field of economics that treats institutions and procedures as variables that can be selected in order to achieve desired objectives. An important aspect of a mechanism is the communication among its participants that it requires, which complements other design features such as incentives and complexity. A calculus-based theory of communication in mechanisms is developed in this book. The value of a calculus-based approach lies in its familiarity as well as the insight into mechanisms that it provides. Results are developed concerning (i) a first-order approach to the construction of mechanisms, (ii) the range of mechanisms that can be used to achieve a given objective, as well as (iii) lower bounds on the required communication.

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A Differential Approach

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9780521851312

First published 2008

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication Data

Williams, Steven R., 1954–
Communication in mechanism design : a differential approach / Steven R. Williams.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-521-85131-2 (hbk.)
1. Econometric models. 2. Information theory in economics.
3. Mathematical economics. I. Title.
HB141.W557 2008
330.01′5195–dc22 2007051003

ISBN 978-0-521-85131-2 hardback

To Christine

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Preface

This text develops a calculus-based, first-order approach to the construction of economic mechanisms. A mechanism here is *informationally decentralized* in the sense that it operates in an environment in which relevant information is dispersed among the participating agents. A mechanism thus requires a "language," or *message space*, that defines how the agents may communicate with one another. This text focuses on the task of constructing the alternative message spaces that a group of agents may use as languages for communicating with one another and thereby achieve a common objective. The relationship between the language that a group of agents may use and the ends that they may accomplish was identified in Hurwicz (1960); the model of a mechanism that is the main object of study in this text originated in this paper and in the long-term collaboration of Leonid Hurwicz with Stanley Reiter. Whereas constructing the message space is but one aspect of the design of a mechanism, it is fundamental in the sense that other aspects (such as dynamic stability and incentives) revolve around the choice of messages with which agents may communicate.

It is assumed here that the sets in the model of a mechanism are subsets of Euclidean space. Appropriate regularity assumptions are imposed on mappings and correspondences so that it is possible to identify necessary and sufficient differential conditions for the design of an economic mechanism. The technique of assuming that all sets in a model are Euclidean and all mappings and correspondences are differentiable is a standard method for making progress and gaining intuition into a scientific problem. Progress is facilitated because the techniques and concepts of a rich field of mathematics in this way become applicable to the problem. Intuition is gained because calculus is nearly universal in science. Although such a continuum model may not capture all aspects of the problem that may be of interest, and though it may in some cases seem to inadequately fit a particular instance of the problem, the successful development of a calculus-based approach is in general a significant step forward in the theoretical study of a problem.

This text complements Hurwicz and Reiter (2006), which develops a set theoretic approach to the construction of mechanisms. Because of the regularity assumptions imposed here, this text elaborates a branch of the theory of mechanism construction, with the set theoretic approach serving as the trunk. Insight and results are produced using the calculus approach; however, that may not be derived purely with set theory. It is worth noting that the calculus approach preceded and inspired much of the set theoretic approach of Hurwicz and Reiter (2006).

The target audience of this text is anyone interested in the field of economic theory known as mechanism design. Because some methods and concepts of differential geometry are not widely known among economic theorists, the second chapter presents the relevant mathematical theory in a style that is intended to be accessible to this community. The difficulty of a journey through differential geometry has deterred most economic theorists from learning about this approach to mechanism design; the second chapter thus provides a shortcut directly to the needed material. The third chapter then develops the first-order approach to the construction of an economic mechanism in a manner that parallels the mathematical theory of Chapter 2. This theory is then applied in the fourth chapter to explore the relationship between the ends that a group of agents can accomplish and the languages that they may use for communicating among themselves.

Acknowledgment

I was introduced to this topic when I was a graduate student at Northwestern University, working under the supervision of Donald Saari. In the summer of 1979, I was hired to proofread Hurwicz et al. (1978), which remains an incomplete manuscript that is full of promising ideas. The main purpose of this text is to assist in the completion of the research program that is outlined in Hurwicz and colleagues' manuscript. Over the years 1979–1982, Leonid Hurwicz and I discussed several results and proofs in the manuscript by phone, by mail, and with an occasional visit. As part of our dialogue and with Leo's encouragement, I wrote proofs of several results. The relevance of the Frobenius Theorem to the objective of developing a calculus-based approach to mechanism design had been identified in Hurwicz et al. (1978). I formulated and proved a version of the Frobenius Theorem (Theorem 6 in Chapter 3) that addresses aspects of the model of a mechanism. This theorem has proven useful in formalizing the Hurwicz, Reiter, and Saari research program. I profited from numerous conversations with Donald Saari while working on these results, and I also benefited from discussions with Stanley Reiter and Kenneth Mount concerning the broader research program. I therefore acknowledge the many contributions of Leo, Ken, Stan, and Don to this text.

A difference between my approach in this text and the approach of Hurwicz et al. (1978) is that I develop the theory of message processes as a separate topic from the theory of the relationship between an objective and a mechanism that realizes it. A message process is the component of a mechanism that captures the communication among the agents. Chapter 3 develops the calculus of message processes, and Chapter 4 applies this methodology to the relationship between objectives and mechanisms. Most of the results of Chapter 3 on message processes would only be complicated by including the objective to be realized; I thus separate the calculus of

message processes from the topic of realization of objectives for the sake of clarity. I mention this point at the beginning of this text to avoid confusing the reader who compares results of Chapter 3 of this text with cited results of Hurwicz et al. (1978), which mostly concern aspects of a mechanism that realize a given objective.

There is an alternative to the analytical approach followed in this text that is more algebraic in flavor. This approach formalizes the first-order conditions for the design of a mechanism using the theory of differential ideals. It was first presented in Part IV of Hurwicz et al. (1978). It has been extensively developed in a series of papers by Saari,[1] and so it is discussed in this text only in Section 3.7 and Subsections 4.1.2, 4.6.5, and 4.6.6. The differential ideal approach is mathematically equivalent to the approach described in this text; all results obtainable with one approach can be obtained with the other. The differential ideal approach is an alternative way of formulating and expressing ideas, however, which can provide intuition and facilitate proofs. Much of mathematics involves finding the right way to formally express complex ideas in order to facilitate further proof and understanding. This is especially true in the field of differential geometry, which underlies the first-order approach to the construction of mechanisms. This text therefore complements and does not perfectly substitute for the differential ideal approach to the construction of mechanisms.

Many of the results in this field of research have been presented at the Decentralization Conference, which is supported by the National Science Foundation. Attending this conference has greatly enhanced my understanding of the topic of decentralization and the field of mechanism design. I thank the many organizers and attendees of this conference over the more than twenty years that I have attended, especially Roy Radner, Ted Groves, and Matt Jackson, who managed the conference during this period. Among the attendees, I have particularly benefited from my conversations with Jim Jordan, John Ledyard, and Stefan Reichelstein. I also thank Tom Marschak for his encouragement and support of this project. The broad research program of which this text represents just one thread was furthered by a year of emphasis in 1983–1984 at the Institute for Mathematics and Its Applications at the University of Minnesota. This institute is also supported by the National Science Foundation.

I acknowledge my debt to Michael Spivak's excellent texts, *A Comprehensive Introduction to Differential Geometry*, vol. I (1979) and *Calculus on*

[1] Saari (1995) is perhaps the most accessible source on this approach (1984, 1985, 1988, 1990).

Manifolds (1965), from which most of the material in Chapter 2 is drawn. My debt goes beyond the results and the arguments from these texts that I have cited here: I learned most of what I know about the mathematics that I use here from reading these texts, and Spivak's distinctive style and geometric perspective have greatly influenced my presentation of this material. I encourage readers who wish to delve more deeply into the mathematics presented here to consult these texts.

Finally, I thank Naoko Miki for her assistance in preparing this manuscript for publication.