Dynamics of Multibody Systems

Third Edition

Dynamics of Multibody Systems, Third Edition, introduces multibody dynamics, with an emphasis on flexible body dynamics. Many common mechanisms such as automobiles, space structures, robots, and micromachines have mechanical and structural systems that consist of interconnected rigid and deformable components. The dynamics of these large-scale, multibody systems are highly nonlinear, presenting complex problems that in most cases can only be solved with computer-based techniques. The book begins with a review of the basic ideas of kinematics and the dynamics of rigid and deformable bodies before moving on to more advanced topics and computer implementation. This revised third edition now includes important new developments relating to the problem of large deformations and numerical algorithms as applied to flexible multibody systems. The book’s wealth of examples and practical applications will be useful to graduate students, researchers, and practicing engineers working on a wide variety of flexible multibody systems.

Ahmed A. Shabana is a Professor in the Department of Mechanical and Industrial Engineering at the University of Illinois, Chicago. Dr. Shabana received his Ph.D. in mechanical engineering from the University of Iowa. His active areas of research interest are in dynamics, vibration, and control of mechanical systems consisting of rigid and deformable interconnected bodies. He is also the author of three other works, Theory of Vibration: An Introduction, Vibration of Discrete and Continuous Systems, and Computational Dynamics.
DYNAMICS OF MULTIBODY SYSTEMS

Third Edition

Ahmed A. Shabana

University of Illinois at Chicago
To my father
and to
the memory of my mother
## Contents

### Preface

ix

### 1 INTRODUCTION

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Multibody Systems</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Reference Frames</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Particle Mechanics</td>
<td>6</td>
</tr>
<tr>
<td>1.4 Rigid Body Mechanics</td>
<td>11</td>
</tr>
<tr>
<td>1.5 Deformable Bodies</td>
<td>15</td>
</tr>
<tr>
<td>1.6 Constrained Motion</td>
<td>18</td>
</tr>
<tr>
<td>1.7 Computer Formulation and Coordinate Selection</td>
<td>22</td>
</tr>
<tr>
<td>1.8 Objectives and Scope of This Book</td>
<td>25</td>
</tr>
</tbody>
</table>

### 2 REFERENCE KINEMATICS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 Rotation Matrix</td>
<td>29</td>
</tr>
<tr>
<td>2.2 Properties of the Rotation Matrix</td>
<td>35</td>
</tr>
<tr>
<td>2.3 Successive Rotations</td>
<td>39</td>
</tr>
<tr>
<td>2.4 Velocity Equations</td>
<td>47</td>
</tr>
<tr>
<td>2.5 Accelerations and Important Identities</td>
<td>55</td>
</tr>
<tr>
<td>2.6 Rodriguez Parameters</td>
<td>59</td>
</tr>
<tr>
<td>2.7 Euler Angles</td>
<td>63</td>
</tr>
<tr>
<td>2.8 Direction Cosines</td>
<td>68</td>
</tr>
<tr>
<td>2.9 The 4 × 4 Transformation Matrix</td>
<td>72</td>
</tr>
<tr>
<td>2.10 Relationship between Different Orientation Coordinates Problems</td>
<td>80</td>
</tr>
</tbody>
</table>
CONTENTS

3 ANALYTICAL TECHNIQUES

3.1 Generalized Coordinates and Kinematic Constraints 86
3.2 Degrees of Freedom and Generalized Coordinate Partitioning 94
3.3 Virtual Work and Generalized Forces 102
3.4 Lagrangian Dynamics 115
3.5 Application to Rigid Body Dynamics 123
3.6 Calculus of Variations 129
3.7 Euler’s Equation in the Case of Several Variables 135
3.8 Equations of Motion of Rigid Body Systems 142
3.9 Newton–Euler Equations 150
3.10 Concluding Remarks 154
Problems 156

4 MECHANICS OF DEFORMABLE BODIES

4.1 Kinematics of Deformable Bodies 160
4.2 Strain Components 164
4.3 Physical Interpretation of Strains 168
4.4 Rigid Body Motion 169
4.5 Stress Components 172
4.6 Equations of Equilibrium 175
4.7 Constitutive Equations 178
4.8 Virtual Work of the Elastic Forces 183
Problems 186

5 FLOATING FRAME OF REFERENCE FORMULATION

5.1 Kinematic Description 189
5.2 Inertia of Deformable Bodies 200
5.3 Generalized Forces 213
5.4 Kinematic Constraints 219
5.5 Equations of Motion 223
5.6 Coupling between Reference and Elastic Displacements 228
5.7 Application to a Multibody System 231
5.8 Use of Independent Coordinates 241
5.9 Dynamic Equations with Multipliers 244
5.10 Generalized Coordinate Partitioning 248
5.11 Organization of Multibody Computer Programs 251
5.12 Numerical Algorithms 254
Problems 263

6 FINITE-ELEMENT FORMULATION

6.1 Element Shape Functions 268
6.2 Reference Conditions 276
CONTENTS

6.3 Kinetic Energy 278
6.4 Generalized Elastic Forces 287
6.5 Characterization of Planar Elastic Systems 288
6.6 Characterization of Spatial Elastic Systems 294
6.7 Coordinate Reduction 300
6.8 The Floating Frame of Reference and Large Deformation Problem 304
Problems 307

7 THE LARGE DEFORMATION PROBLEM 309
7.1 Background 310
7.2 Absolute Nodal Coordinate Formulation 314
7.3 Formulation of the Stiffness Matrix 318
7.4 Equations of Motion 322
7.5 Relationship to the Floating Frame of Reference Formulation 323
7.6 Coordinate Transformation 325
7.7 Consistent Mass Formulation 328
7.8 The Velocity Transformation Matrix 331
7.9 Lumped Mass Formulation 332
7.10 Extension of the Method 335
7.11 Comparison with Large Rotation Vector Formulation 339
Problems 342

APPENDIX: LINEAR ALGEBRA 345
A.1 Matrix Algebra 345
A.2 Eigenvalue Analysis 349
A.3 Vector Spaces 350
A.4 Chain Rule of Differentiation 353
A.5 Principle of Mathematical Induction 354
Problems 355

REFERENCES 357

INDEX 369
The methods for the nonlinear analysis of physical and mechanical systems developed for use on modern digital computers provide means for accurate analysis of large-scale systems under dynamic loading conditions. These methods are based on the concept of replacing the actual system by an equivalent model made up from discrete bodies having known elastic and inertia properties. The actual systems, in fact, form multibody systems consisting of interconnected rigid and deformable bodies, each of which may undergo large translational and rotational displacements. Examples of physical and mechanical systems that can be modeled as multibody systems are machines, mechanisms, vehicles, robotic manipulators, and space structures. Clearly, these systems consist of a set of interconnected bodies which may be rigid or deformable. Furthermore, the bodies may undergo large relative translational and rotational displacements. The dynamic equations that govern the motion of these systems are highly nonlinear which in most cases cannot be solved analytically in a closed form. One must resort to the numerical solution of the resulting dynamic equations.

The aim of this text, which is based on lectures that I have given during the past several years, is to provide an introduction to the subject of multibody mechanics in a form suitable for senior undergraduate and graduate students. The initial notes for the text were developed for two first-year graduate courses introduced and offered at the University of Illinois at Chicago. These courses were developed to emphasize both the general methodology of the nonlinear dynamic analysis of multibody systems and its actual implementation on the high-speed digital computer. This was prompted by the necessity to deal with complex problems arising in modern engineering and science. In this text, an attempt has been made to provide the rational development of the methods from their foundations and develop the techniques in clearly understandable stages. By understanding the basis of each step, readers can apply the method to their own problems.

The material covered in this text comprises an introductory chapter on the subjects of kinematics and dynamics of rigid and deformable bodies. In this chapter some
PREFACE

background materials and a few fundamental ideas are presented. In Chapter 2, the
kinematics of the body reference is discussed and the transformation matrices that
define the orientation of this body reference are developed. Alternate forms of the
transformation matrix are presented. The material presented in this chapter is es-
sential for understanding the dynamic motion of both rigid and deformable bodies.
Analytical techniques for deriving the system differential and algebraic equations of
motion of a multibody system consisting of rigid bodies are discussed in Chapter 3.
In Chapter 4, an introduction to the theory of elasticity is presented. The material
covered in this chapter is essential for understanding the dynamics of deformable
bodies that undergo large translational and rotational displacements. In Chapter 5,
the equations of motion of deformable multibody systems in which the reference mo-
tion and elastic deformation are coupled are derived using classical approximation
methods. In Chapters 6 and 7, two finite element formulations are presented. Both
formulations lead to exact modeling of the rigid body inertia and lead to zero strains
under an arbitrary rigid body motion. The first formulation discussed in Chapter 6,
which is based on the concept of the intermediate element coordinate system, uses
the definition of the coordinates used in the conventional finite element method. A
conceptually different finite element formulation that can be used in the large defor-
mation analysis of multibody systems is presented in Chapter 7. In this chapter, the
absolute nodal coordinate formulation in which no infinitesimal or finite rotations are
used as element coordinates is introduced.

I am grateful to many teachers, colleagues, and students who have contributed
to my education in this field. I owe a particular debt of gratitude to Dr. R.A. Wehage
and Dr. M.M. Nigm for their advice, encouragement, and assistance at various stages
of my educational career. Their work in the areas of computational mechanics and
vibration theory stimulated my early interest in the subject of nonlinear dynamics.
Several chapters of this book have been read, corrected, and improved by many of
my graduate students. I would like to acknowledge the collaboration with my stu-
dents Drs. Om Agrawal, E. Mohktar Bakr, Ipak Basdogan, Michael Brown, Bilin
Chang, Che-Wei Chang, Koroosh Changizi, Da-Chih Chen, Jui-Sheng Chen, Jin-
Hwan Choi, Hanaa El-Absy, Marian Gofron, Wei-Hsin Gau, Wei-Cheng Hsu, Kuo-
Hsing Hwang, Yunn-Lin Hwang, Yehia Khulief, John Kremer, Haichiang Lee, Jalil
Rismantab-Sany, Mohammad Sarwar, Marcello Berzeri, Marcello Campanelli,
Andrew Christensen, Hussien Husien, Refaat Yakoub, and Hiroyski Sugiyama. Their
work contributed significantly to the development of the material presented in this
book. Special thanks are due to Ms. Denise Burt for the excellent job in typing most
of the manuscript. Finally, I thank my family for their patience and encouragement
during the time of preparation of this text.

Chicago, Illinois

Ahmed Shabana

NOVEMBER 2004