

Cambridge University Press

978-0-521-84969-2 - Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-scale Circulation

Geoffrey K. Vallis

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Atmospheric and Oceanic Fluid Dynamics

Fundamentals and Large-Scale Circulation

Fluid dynamics is fundamental to our understanding of the atmosphere and oceans. Although many of the same principles of fluid dynamics apply to both the atmosphere and oceans, textbooks on the topic have tended to concentrate on either the atmosphere or the ocean, or on the theory of geophysical fluid dynamics (GFD). However, there is much to be said for a unified discussion, and this major new textbook provides a comprehensive, coherent treatment of all these topics. It is based on course notes that the author has developed over a number of years at Princeton and the University of California.

The first part of the book provides an introduction to the fundamentals of geophysical fluid dynamics, including discussions of rotation and stratification, the role of vorticity and potential vorticity, and scaling and approximations. The second part of the book discusses baroclinic and barotropic instabilities, wave–mean flow interactions and turbulence. The third and fourth parts discuss the general circulation of the atmosphere and ocean. Student problems and exercises, as well as bibliographic and historical notes, are included at the end of each chapter.

Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-Scale Circulation will prove to be an invaluable graduate textbook on advanced courses in GFD, meteorology, atmospheric science, and oceanography, and will also be an excellent review volume for researchers. Additional resources are available at www.cambridge.org/9780521849692

GEOFFREY K. VALLIS is a senior scientist and professor in the Program in Atmospheric and Oceanic Sciences and NOAA's Geophysical Fluid Dynamics Laboratory at Princeton University. He is also an associate faculty member at the Program in Applied and Computational Mathematics, and a former professor at the University of California, Santa Cruz. Until recently he was editor of the *Journal of the Atmospheric Sciences*. His research interests include the general circulation of the ocean and atmosphere, turbulence theory, and climate dynamics. He has taught a wide range of topics at Princeton and the University of California, and he has published extensively in both the oceanographic and meteorological literature.

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Pre-publication praise for *Atmospheric and Oceanic Fluid Dynamics*

“Geoff Vallis’ *Atmosphere and Ocean Dynamics* will become the standard text on modern large-scale atmosphere and ocean dynamics. It covers the field from the equations of motion to modern developments such as wave–mean flow interaction theory and theories for the global-scale circulations of atmospheres and oceans. There is no book of comparable comprehensiveness, spanning the needs of beginning graduate students and researchers alike.”

Tapio Schneider, California Institute of Technology

“This clearly written, self-contained new book is a modern treatment of atmospheric and oceanic dynamics. The book starts from classical concepts in fluid dynamics and thermodynamics and takes the reader to the frontier of current research. This is an accessible textbook for beginning students in meteorology, oceanography and climate sciences. Mature researchers will welcome this work as a stimulating resource. This is also the only textbook on geophysical fluid dynamics with a comprehensive collection of problems; these cement the material and expand it to a more advanced level. Highly recommended!”

Paola Cessi, Scripps Institution of Oceanography, University of California, San Diego

“Vallis provides a cohesive view of GFD that smoothly blends classic results with modern interpretations. The book strikes an ideal balance between mathematical rigor and physical intuition, and between atmosphere- and ocean-relevant applications. The use of a hierarchy of models is particularly welcome. Each physical phenomenon is modeled with the right degree of complexity, and the reader is introduced to the value of the hierarchy at an early stage. Well-designed homework problems spanning a broad range of difficulty make the book very appropriate for use in introductory courses in GFD.”

Adam Sobel, Lamont-Doherty Earth Observatory, Columbia University

“I have adopted this text for my course in Atmosphere–Ocean Dynamics because the ideas are clearly presented and up-to-date. The text provides the flexibility for the instructor to choose among a variety of paths that take the student from the foundations of the subject to current research topics. For me as a researcher, the text is satisfying because it presents a unified view of the ideas that underlie the modern theory of large scale atmospheric and oceanic circulations.”

Paul J. Kushner, University of Toronto

“The large-scale circulation in the atmosphere–ocean system is maintained by small-scale turbulent motions that interact with large-scale radiative processes. The first half of the book introduces the basic theories of large-scale atmosphere–ocean flows and of small-scale turbulent motions. In the second half, the two theories are brought together to explain how the interactions of motions on different scales maintain the global-scale climate. The emphasis on turbulent motions and their effect on larger scales makes this book a gem in the GFD literature. Finally, we have a textbook that is up to date with our current understanding of the climate system.”

Raffaele Ferrari, Massachusetts Institute of Technology

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ATMOSPHERIC AND OCEANIC FLUID DYNAMICS

Fundamentals and Large-scale Circulation

GEOFFREY K. VALLIS

Princeton University, New Jersey



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To my parents, Jim and Doreen Vallis.

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Contents

An asterisk indicates more advanced material that may be omitted on a first reading. A dagger indicates material that is still a topic of research or that is not settled.

Preface	<i>page</i> xix
Notation	xxiv
Part I FUNDAMENTALS OF GEOPHYSICAL FLUID DYNAMICS	1
1 Equations of Motion	3
1.1 Time Derivatives for Fluids	3
1.1.1 Field and material viewpoints	3
1.1.2 The material derivative of a fluid property	4
1.1.3 Material derivative of a volume	6
1.2 The Mass Continuity Equation	8
1.2.1 An Eulerian derivation	8
1.2.2 Mass continuity via the material derivative	10
1.2.3 A general continuity equation	10
1.3 The Momentum Equation	11
1.3.1 Advection	11
1.3.2 The pressure force	12
1.3.3 Viscosity and diffusion	12
1.3.4 Hydrostatic balance	13
1.4 The Equation of State	14
1.5 Thermodynamic Relations	15
1.5.1 A few fundamentals	16
1.5.2 Various thermodynamic relations	18
1.6 Thermodynamic Equations for Fluids	22

viii	Contents
1.6.1	Thermodynamic equation for an ideal gas 23
1.6.2	* Thermodynamic equation for liquids 26
1.7	* More Thermodynamics of Liquids 31
1.7.1	Potential temperature, potential density and entropy 31
1.7.2	* Thermodynamic properties of seawater 33
1.8	Sound Waves 37
1.9	Compressible and Incompressible Flow 38
1.9.1	Constant density fluids 38
1.9.2	Incompressible flows 39
1.10	The Energy Budget 40
1.10.1	Constant density fluid 40
1.10.2	Variable density fluids 42
1.10.3	Viscous effects 43
1.11	An Introduction to Non-Dimensionalization and Scaling 43
1.11.1	The Reynolds number 44
2	Effects of Rotation and Stratification 51
2.1	Equations in a Rotating Frame 51
2.1.1	Rate of change of a vector 52
2.1.2	Velocity and acceleration in a rotating frame 53
2.1.3	Momentum equation in a rotating frame 54
2.1.4	Mass and tracer conservation in a rotating frame 54
2.2	Equations of Motion in Spherical Coordinates 55
2.2.1	* The centrifugal force and spherical coordinates 55
2.2.2	Some identities in spherical coordinates 57
2.2.3	Equations of motion 60
2.2.4	The primitive equations 61
2.2.5	Primitive equations in vector form 62
2.2.6	The vector invariant form of the momentum equation 63
2.2.7	Angular momentum 64
2.3	Cartesian Approximations: The Tangent Plane 66
2.3.1	The f-plane 66
2.3.2	The beta-plane approximation 67
2.4	The Boussinesq Approximation 67
2.4.1	Variation of density in the ocean 68
2.4.2	The Boussinesq equations 68
2.4.3	Energetics of the Boussinesq system 72
2.5	The Anelastic Approximation 73
2.5.1	Preliminaries 73
2.5.2	The momentum equation 74
2.5.3	Mass conservation 75
2.5.4	Thermodynamic equation 76
2.5.5	* Energetics of the anelastic equations 76
2.6	Changing Vertical Coordinate 77
2.6.1	General relations 77
2.6.2	Pressure coordinates 78

Contents	ix
2.6.3 Log-pressure coordinates	80
2.7 Scaling for Hydrostatic Balance	80
2.7.1 Preliminaries	80
2.7.2 Scaling and the aspect ratio	81
2.7.3 * Effects of stratification on hydrostatic balance	82
2.7.4 Hydrostasy in the ocean and atmosphere	84
2.8 Geostrophic and Thermal Wind Balance	85
2.8.1 The Rossby number	85
2.8.2 Geostrophic balance	86
2.8.3 Taylor–Proudman effect	88
2.8.4 Thermal wind balance	89
2.8.5 * Effects of rotation on hydrostatic balance	91
2.9 Static Instability and the Parcel Method	91
2.9.1 A simple special case: a density-conserving fluid	92
2.9.2 The general case: using potential density	93
2.9.3 Lapse rates in dry and moist atmospheres	95
2.10 Gravity Waves	98
2.10.1 Gravity waves and convection in a Boussinesq fluid	98
2.11 * Acoustic-Gravity Waves in an Ideal Gas	100
2.11.1 Interpretation	101
2.12 The Ekman Layer	104
2.12.1 Equations of motion and scaling	105
2.12.2 Integral properties of the Ekman layer	107
2.12.3 Explicit solutions. I: a bottom boundary layer	109
2.12.4 Explicit solutions. II: the upper ocean	112
2.12.5 Observations of the Ekman layer	113
2.12.6 * Frictional parameterization of the Ekman layer	114
3 Shallow Water Systems and Isentropic Coordinates	123
3.1 Dynamics of a Single, Shallow Layer	123
3.1.1 Momentum equations	124
3.1.2 Mass continuity equation	125
3.1.3 A rigid lid	127
3.1.4 Stretching and the vertical velocity	128
3.1.5 Analogy with compressible flow	129
3.2 Reduced Gravity Equations	129
3.2.1 Pressure gradient in the active layer	130
3.3 Multi-Layer Shallow Water Equations	131
3.3.1 Reduced-gravity multi-layer equation	133
3.4 Geostrophic Balance and Thermal wind	134
3.5 Form Drag	135
3.6 Conservation Properties of Shallow Water Systems	136
3.6.1 Potential vorticity: a material invariant	136
3.6.2 Energy conservation: an integral invariant	139
3.7 Shallow Water Waves	140
3.7.1 Non-rotating shallow water waves	140

x	Contents
3.7.2	Rotating shallow water (Poincaré) waves 141
3.7.3	Kelvin waves 143
3.8	Geostrophic Adjustment 144
3.8.1	Non-rotating flow 145
3.8.2	Rotating flow 146
3.8.3	* Energetics of adjustment 148
3.8.4	* General initial conditions 149
3.8.5	A variational perspective 151
3.9	Isentropic Coordinates 152
3.9.1	A hydrostatic Boussinesq fluid 152
3.9.2	A hydrostatic ideal gas 153
3.9.3	* Analogy to shallow water equations 154
3.10	Available Potential Energy 155
3.10.1	A Boussinesq fluid 156
3.10.2	An ideal gas 158
3.10.3	Use, interpretation, and the atmosphere and ocean 159
4	Vorticity and Potential Vorticity 163
4.1	Vorticity and Circulation 163
4.1.1	Preliminaries 163
4.1.2	Simple axisymmetric examples 164
4.2	The Vorticity Equation 165
4.2.1	Two-dimensional flow 167
4.3	Vorticity and Circulation Theorems 168
4.3.1	The 'frozen-in' property of vorticity 168
4.3.2	Kelvin's circulation theorem 171
4.3.3	Baroclinic flow and the solenoidal term 173
4.3.4	Circulation in a rotating frame 173
4.3.5	The circulation theorem for hydrostatic flow 174
4.4	Vorticity Equation in a Rotating Frame 175
4.4.1	The circulation theorem and the beta effect 175
4.4.2	The vertical component of the vorticity equation 176
4.5	Potential Vorticity Conservation 178
4.5.1	PV conservation from the circulation theorem 178
4.5.2	PV conservation from the frozen-in property 180
4.5.3	PV conservation: an algebraic derivation 182
4.5.4	Effects of salinity and moisture 183
4.5.5	Effects of rotation, and summary remarks 183
4.6	* Potential Vorticity in the Shallow Water System 184
4.6.1	Using Kelvin's theorem 184
4.6.2	Using an appropriate scalar field 185
4.7	Potential Vorticity in Approximate, Stratified Models 186
4.7.1	The Boussinesq equations 186
4.7.2	The hydrostatic equations 187
4.7.3	Potential vorticity on isentropic surfaces 187
4.8	* The Impermeability of Isentropes to Potential Vorticity 188

Contents	xi
4.8.1 Interpretation and application	190
5 Simplified Equations for Ocean and Atmosphere	197
5.1 Geostrophic Scaling	198
5.1.1 Scaling in the shallow water equations	198
5.1.2 Geostrophic scaling in the stratified equations	200
5.2 The Planetary-Geostrophic Equations	203
5.2.1 Using the shallow water equations	203
5.2.2 Planetary-geostrophic equations for stratified flow	205
5.3 The Shallow Water Quasi-Geostrophic Equations	207
5.3.1 Single-layer shallow water quasi-geostrophic equations	207
5.3.2 Two-layer and multi-layer quasi-geostrophic systems	211
5.3.3 † Non-asymptotic and intermediate models	214
5.4 The Continuously Stratified Quasi-Geostrophic System	215
5.4.1 Scaling and assumptions	215
5.4.2 Asymptotics	216
5.4.3 Buoyancy advection at the surface	219
5.4.4 Quasi-geostrophy in pressure coordinates	220
5.4.5 The two-level quasi-geostrophic system	221
5.5 * Quasi-geostrophy and Ertel Potential Vorticity	224
5.5.1 * Using height coordinates	224
5.5.2 Using isentropic coordinates	225
5.6 * Energetics of Quasi-Geostrophy	226
5.6.1 Conversion between APE and KE	227
5.6.2 Energetics of two-layer flows	228
5.6.3 Enstrophy conservation	229
5.7 Rossby Waves	229
5.7.1 Waves in a single layer	229
5.7.2 Rossby waves in two layers	232
5.8 * Rossby Waves in Stratified Quasi-Geostrophic Flow	234
5.8.1 Setting up the problem	234
5.8.2 Wave motion	235
Appendix: Wave Kinematics, Group Velocity and Phase Speed	236
5.A.1 Kinematics and definitions	236
5.A.2 Wave propagation	237
5.A.3 Meaning of group velocity	239
Part II INSTABILITIES, WAVE-MEAN FLOW INTERACTION AND TURBULENCE	245
6 Barotropic and Baroclinic Instability	247
6.1 Kelvin-Helmholtz Instability	248
6.2 Instability of Parallel Shear Flow	250
6.2.1 Piecewise linear flows	251

6.2.2	Kelvin–Helmholtz instability, revisited	253
6.2.3	Edge waves	254
6.2.4	Interacting edge waves producing instability	254
6.3	Necessary Conditions for Instability	258
6.3.1	Rayleigh’s criterion	258
6.3.2	Fjørtoft’s criterion	260
6.4	Baroclinic Instability	261
6.4.1	A physical picture	261
6.4.2	Linearized quasi-geostrophic equations	263
6.4.3	Necessary conditions for baroclinic instability	264
6.5	The Eady Problem	265
6.5.1	The linearized problem	266
6.5.2	Atmospheric and oceanic parameters	268
6.6	Two-Layer Baroclinic Instability	271
6.6.1	Posing the problem	271
6.6.2	The solution	272
6.7	An Informal View of the Mechanism of Baroclinic Instability	277
6.7.1	The two-layer model	278
6.7.2	Interacting edge waves in the Eady problem	280
6.8	* The Energetics of Linear Baroclinic Instability	282
6.9	* Beta, Shear and Stratification in a Continuous Model	284
6.9.1	Scaling arguments for growth rates, scales and depth	285
6.9.2	Some numerical calculations	287
7	Wave–Mean Flow Interaction	295
7.1	Quasi-geostrophic Preliminaries	296
7.1.1	Potential vorticity flux in the linear equations	297
7.2	The Eliassen–Palm Flux	298
7.2.1	The Eliassen–Palm relation	299
7.2.2	The group velocity property	300
7.2.3	* The orthogonality of modes	302
7.3	The Transformed Eulerian Mean	304
7.3.1	Quasi-geostrophic form	304
7.3.2	The TEM in isentropic coordinates	306
7.3.3	Residual and thickness-weighted circulation	307
7.3.4	* The TEM in the primitive equations	309
7.4	The Non-acceleration Result	314
7.4.1	A derivation from the potential vorticity equation	314
7.4.2	Using TEM to give the non-acceleration result	316
7.4.3	The EP flux and form drag	317
7.5	Influence of Eddies on the Mean Flow in the Eady Problem	319
7.5.1	Formulation	319
7.5.2	Solution	321
7.5.3	The two-level problem	324
7.6	* Necessary Conditions for Instability	324
7.6.1	Stability conditions from pseudomomentum conservation	325

Contents	xiii
7.6.2 Inclusion of boundary terms	325
7.7 * Necessary Conditions for Instability: Use of Pseudoenergy	327
7.7.1 Two-dimensional flow	327
7.7.2 * Stratified quasi-geostrophic flow	330
7.7.3 * Applications to baroclinic instability	331
8 Basic Theory of Incompressible Turbulence	337
8.1 The Fundamental Problem of Turbulence	338
8.1.1 The closure problem	338
8.1.2 Triad interactions in turbulence	339
8.2 The Kolmogorov Theory	341
8.2.1 The physical picture	341
8.2.2 Inertial-range theory	342
8.2.3 * Another expression of the inertial-range scaling argument	348
8.2.4 A final note on our assumptions	349
8.3 Two-Dimensional Turbulence	349
8.3.1 Energy and enstrophy transfer	351
8.3.2 Inertial ranges in two-dimensional turbulence	354
8.3.3 † More about the phenomenology	357
8.3.4 Numerical illustrations	360
8.4 Predictability of Turbulence	361
8.4.1 Low-dimensional chaos and unpredictability	361
8.4.2 * Predictability of a turbulent flow	363
8.4.3 Implications and weather predictability	365
8.5 * Spectra of Passive Tracers	366
8.5.1 Examples of tracer spectra	367
9 Geostrophic Turbulence and Baroclinic Eddies	377
9.1 Effects of Differential Rotation	377
9.1.1 The wave-turbulence cross-over	378
9.1.2 Generation of zonal flows and jets	380
9.1.3 † Joint effect of β and friction	382
9.2 Stratified Geostrophic Turbulence	384
9.2.1 An analogue to two-dimensional flow	384
9.2.2 Two-layer geostrophic turbulence	385
9.2.3 Phenomenology of two-layer turbulence	388
9.3 † A Scaling Theory for Geostrophic Turbulence	391
9.3.1 Preliminaries	392
9.3.2 Scaling properties	393
9.3.3 The halting scale and the β -effect	395
9.4 † Phenomenology of Baroclinic Eddies in the Atmosphere and Ocean	395
9.4.1 The magnitude and scale of baroclinic eddies	396
9.4.2 Baroclinic eddies and their lifecycle in the atmosphere	397
9.4.3 Baroclinic eddies and their lifecycle in the ocean	400

10	Turbulent Diffusion and Eddy Transport	407
10.1	Diffusive Transport	408
10.1.1	An explicit example	409
10.2	Turbulent Diffusion	409
10.2.1	Simple theory	409
10.2.2	* An anisotropic generalization	413
10.2.3	Discussion	415
10.3	Two-Particle Diffusivity	415
10.3.1	Large particle separation	416
10.3.2	Separation within the inertial range	417
10.4	Mixing Length Theory	419
10.4.1	Requirements for turbulent diffusion	421
10.4.2	A macroscopic perspective	422
10.5	Homogenization of a Scalar that is Advected and Diffused	423
10.5.1	Non-existence of extrema	423
10.5.2	Homogenization in two-dimensional flow	424
10.6	† Transport by Baroclinic Eddies	425
10.6.1	Symmetric and antisymmetric diffusivity tensors	426
10.6.2	* Diffusion with the symmetric tensor	426
10.6.3	* The skew flux	427
10.6.4	The story so far	429
10.7	† Eddy Diffusion in the Atmosphere and Ocean	430
10.7.1	Preliminaries	430
10.7.2	Magnitude of the eddy diffusivity	430
10.7.3	* Structure: the symmetric transport tensor	432
10.7.4	* Structure: the antisymmetric transport tensor	435
10.7.5	Examples	437
10.8	† Thickness Diffusion	440
10.8.1	Equations of motion	440
10.8.2	Diffusive thickness transport	442
10.9	† Eddy Transport and the Transformed Eulerian Mean	443
10.9.1	Potential vorticity diffusion	443
Part III	LARGE-SCALE ATMOSPHERIC CIRCULATION	449
11	The Overturning Circulation: Hadley and Ferrel Cells	451
11.1	Basic Features of the Atmosphere	452
11.1.1	The radiative equilibrium distribution	452
11.1.2	Observed wind and temperature fields	453
11.1.3	Meridional overturning circulation	456
11.1.4	Summary	457
11.2	A Steady Model of the Hadley Cell	457
11.2.1	Assumptions	457
11.2.2	Dynamics	458

Contents	xv
11.2.3 Thermodynamics	460
11.2.4 Zonal wind	462
11.2.5 Properties of solution	463
11.2.6 Strength of the circulation	464
11.2.7 † Effects of moisture	465
11.2.8 The radiative equilibrium solution	466
11.3 A Shallow Water Model of the Hadley Cell	468
11.3.1 Momentum balance	468
11.3.2 Thermodynamic balance	468
11.4 † Asymmetry Around the Equator	469
11.5 Eddies, Viscosity and the Hadley Cell	473
11.5.1 Qualitative considerations	473
11.5.2 An idealized eddy-driven model	474
11.6 The Hadley Cell: Summary and Numerical Solutions	477
11.7 The Ferrel Cell	480
12 Zonally Averaged Mid-Latitude Atmospheric Circulation	485
12.1 Surface Westerlies and the Maintenance of a Barotropic Jet	486
12.1.1 Observations and motivation	486
12.1.2 The mechanism of jet production	487
12.1.3 A numerical example	495
12.2 Layered Models of the Mid-latitude Circulation	496
12.2.1 A single-layer model	497
12.2.2 A two-layer model	503
12.2.3 Dynamics of the two-layer model	507
12.3 † Eddy Fluxes and an Example of a Closed Model	513
12.3.1 Equations for a closed model	513
12.3.2 * Eddy fluxes and necessary conditions for instability	514
12.4 A Stratified Model and the Real Atmosphere	516
12.4.1 Potential vorticity and its fluxes	516
12.4.2 Overturning circulation	522
12.5 † The Tropopause and the Stratification of the Atmosphere	522
12.5.1 A radiative–convective model	526
12.5.2 Radiative and dynamical constraints	528
12.6 † Baroclinic eddies and Potential Vorticity Transport	529
12.6.1 A linear argument	530
12.6.2 Mixing potential vorticity and baroclinic adjustment	530
12.6.3 Diffusive transport of potential vorticity	532
12.7 † Extratropical Convection and the Ventilated Troposphere	533
Appendix: TEM for the Primitive Equations in Spherical Coordinates	536
13 Planetary Waves and the Stratosphere	541
13.1 Forced and Stationary Rossby Waves	542
13.1.1 A simple one-layer case	542
13.1.2 Application to Earth's atmosphere	543

xvi	Contents
13.1.3 * One-dimensional Rossby wave trains	545
13.1.4 The adequacy of linear theory	548
13.2 * Meridional Propagation and Dispersion	549
13.2.1 Ray tracing	549
13.2.2 Rossby waves and Rossby rays	550
13.2.3 Application to an idealized atmosphere	553
13.3 * Vertical Propagation of Rossby Waves in a Stratified Medium	554
13.3.1 Model formulation	554
13.3.2 Model solution	555
13.3.3 Properties of the solution	559
13.4 * Effects of Thermal Forcing	560
13.4.1 Thermodynamic balances	561
13.4.2 Properties of the solution	562
13.4.3 Numerical solutions	563
13.5 Stratospheric Dynamics	566
13.5.1 A descriptive overview	566
13.5.2 † Dynamics of the overturning circulation	569
13.5.3 † The polar vortex and the quasi-horizontal circulation	575
Part IV LARGE-SCALE OCEANIC CIRCULATION	581
14 Wind-Driven Gyres	583
14.1 The Depth Integrated Wind-Driven Circulation	585
14.1.1 The Stommel model	586
14.1.2 Alternative formulations	587
14.1.3 Approximate solution of Stommel model	589
14.2 Using Viscosity Instead of Drag	593
14.3 Zonal Boundary Layers	597
14.4 * The Nonlinear Problem	599
14.4.1 A perturbative approach	599
14.4.2 A numerical approach	600
14.5 * Inertial Solutions	601
14.5.1 Roles of friction and inertia	601
14.5.2 Attempting an inertial western boundary solution	603
14.5.3 A fully inertial approach: the Fofonoff model	606
14.6 Topographic Effects on Western Boundary Currents	608
14.6.1 Homogeneous model	608
14.6.2 Advective dynamics	609
14.6.3 Bottom pressure stress and form drag	610
14.7 * Vertical Structure of the Wind-Driven Circulation	613
14.7.1 A two-layer quasi-geostrophic Model	613
14.7.2 The functional relationship between ψ and q	616
14.8 * A Model with Continuous Stratification	618
14.8.1 Depth of the wind's influence	618
14.8.2 The complete solution	620

15	The Buoyancy-Driven Ocean Circulation	627
15.1	Sideways Convection	629
15.1.1	Two-dimensional convection	630
15.1.2	† Phenomenology of the overturning circulation	633
15.2	The Maintenance of Sideways Convection	634
15.2.1	The energy budget	635
15.2.2	Conditions for maintaining a thermally-driven circulation	635
15.2.3	Surface fluxes and non-turbulent flow at small diffusivities	637
15.2.4	The importance of mechanical forcing	639
15.3	Simple Box Models	640
15.3.1	A two-box model	640
15.3.2	* More boxes	644
15.4	A Laboratory Model of the Abyssal Circulation	646
15.4.1	Set-up of the laboratory model	646
15.4.2	Dynamics of flow in the tank	647
15.5	A Model for Oceanic Abyssal Flow	650
15.5.1	Completing the solution	652
15.5.2	Application to the ocean	653
15.5.3	A two-hemisphere model	655
15.6	* A Shallow Water Model of the Abyssal Flow	656
15.6.1	Potential vorticity and poleward interior flow	657
15.6.2	The solution	658
15.7	Scaling for the Buoyancy-Driven Circulation	659
15.7.1	Summary remarks on the Stommel-Arons model	661
16	The Wind- and Buoyancy-Driven Ocean Circulation	667
16.1	The Main Thermocline: an Introduction	667
16.1.1	A simple kinematic model	668
16.2	Scaling and Simple Dynamics of the Main Thermocline	670
16.2.1	An advective scale	671
16.2.2	A diffusive scale	672
16.2.3	Summary of the physical picture	673
16.3	The Internal Thermocline	674
16.3.1	The M equation	674
16.3.2	* Boundary-layer analysis	676
16.4	The Ventilated Thermocline	681
16.4.1	A reduced gravity, single-layer model	682
16.4.2	A two-layer model	683
16.4.3	The shadow zone	686
16.4.4	† The western pool	688
16.5	† A Model of Deep Wind-Driven Overturning	691
16.5.1	A single-hemisphere model	693
16.5.2	A cross-equatorial wind-driven deep circulation	697
16.6	† Flow in a Channel and the Antarctic Circumpolar Current	700
16.6.1	Steady and eddying flow	701

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Geoffrey K. Vallis

Frontmatter

[More information](#)

xviii	Contents
16.6.2 Vertically integrated momentum balance	702
16.6.3 Form drag and baroclinic eddies	703
16.6.4 † An idealized adiabatic model	708
16.6.5 Form stress and Ekman stress at the ocean bottom	709
16.6.6 Differences between gyres and channels	710
Appendix: Miscellaneous Relationships in a Layered Model	710
16.A.1 Hydrostatic balance	711
16.A.2 Geostrophic and thermal wind balance	711
16.A.3 Explicit cases	712
References	717

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[More information](#)

We must be ignorant of much, if we would know anything.

Cardinal John Henry Newman (1801–1890).

Preface

THIS IS A BOOK on the fluid dynamics of the atmosphere and ocean, with an emphasis on the fundamentals and on the large-scale circulation, the latter meaning flows from the scale of the first deformation radius (a few tens of kilometres in the ocean, several hundred kilometres in the atmosphere) to the global scale. The book is primarily a textbook; it is designed to be accessible to students and could be used as a text for graduate courses. It may be also useful as an introduction to the field for scientists in other areas and as a reference for researchers in the field, and some aspects of the book have the flavour of a research monograph.

Atmospheric and oceanic fluid dynamics (AOFD) is a fascinating field, and simultaneously both pure and applied. It is a pure field because it is intimately tied to some of the most fundamental and unsolved problems in fluid dynamics — problems in turbulence and wave-mean flow interaction, problems in chaos and predictability, and problems in the general circulation itself. Yet it is applied because the climate and weather so profoundly affect the human condition, and so a great deal of effort goes into making predictions — indeed the practice of weather forecasting is a remarkable example of a successful applied science, in spite of the natural limitations to predictability that are now reasonably well understood. The field is plainly important, for we live in the atmosphere and the ocean covers about two-thirds of the Earth. It is also very broad, encompassing such diverse topics as the general circulation, gyres, boundary layers, waves, convection and turbulence. My goal in this book is to present a coherent selection of these topics, concentrating on the foundations but without shying away from the boundaries of active areas of research — for a book that limits itself to what is absolutely settled would, I think, be rather dry, a quality best reserved for martinis and humour.

AOFD is closely related to the field of *geophysical fluid dynamics* (GFD). The latter can be, depending on one's point of view, both a larger and a smaller field than the former. It is larger because GFD, in its broadest meaning, includes not just the fluid dynamics of the Earth's atmosphere and ocean, but also the fluid dynamics of such things as the Earth's interior, volcanoes, lava flows and planetary atmospheres; it is the fluid mechanics of all

things geophysical. But at the same time the appellation ‘GFD’ implies a certain austerity, and the subject is often seen as the one that provides the fundamental principles and language for understanding geophysical flows without being suffocated by the overwhelming detail of the real world. In this book we are guided by the ascetic spirit of GFD, and my hope is that the reader will gain a solid grounding in the fundamentals, motivated by and with an appreciation for the problems of the real world.

The book is an outgrowth of various courses that I have taught over the years, mainly at Princeton University but also at the University of California and at summer schools or similar in Boulder and Kyoto. There are four parts to the book: fundamentals of geophysical fluid dynamics; instabilities, wave–mean flow interaction and turbulence; atmospheric circulation; and ocean circulation. Each corresponds, very roughly, to a one-term graduate course, although parts could also be used for undergraduates. Limitations enforced both by the need to keep the book coherent and focused, and my own expertise or (especially) lack thereof, naturally limit the choice of topics. In particular the chapters on the circulation focus on the steady and statistically steady large-scale circulation and perforce a number of important topics are omitted — tropical and equatorial dynamics, many of the effects of moisture on atmospheric circulation, the spin-up of the ocean circulation, atmospheric and oceanic tides, the quasi-biennial oscillation, and so on. I have however — and at no extra charge, mind you — discussed the large-scale circulation of both atmosphere and ocean. The similarities and differences between the two systems are, I believe, so instructive that even if one’s interest is solely in one, there is much to be gained by studying the other. The references at the end of the book are representative and not exhaustive, and almost certainly disproportionately represent articles written in English and those with which I happen to be familiar. For the benefit of the reader interested in exploring the development of the subject I have included references to a number of historical articles, even when the presentation given does not draw from them. If there are other references that are particularly relevant I trust the reader will inform me.

I have tried to keep the overall treatment of topics as straightforward and as clear as I know how. In particular, I have tried to be as explicit as possible in my explanations, even at the risk of descending from pedagogy into pedantry. Relatedly, there is a certain amount of repetition between sections, and this serves both to emphasize the important things and to keep chapters reasonably self-contained. The chapters are of course intellectually linked, for example, heat transport in the atmosphere depends on baroclinic instability, but hopefully the reader already familiar with the latter will be able to read about the former without too much cross-referencing. The treatment generally is fairly physical and phenomenological, and rigour in the mathematical sense is absent; I treat the derivatives of integrals and of infinitesimal quantities rather informally, for example.

The figures (many in colour) may all be downloaded from the CUP web site associated with this book. An asterisk, *, next to a section heading means that the section may be omitted on first reading; although normally uncontroversial, it may contain advanced material that is not essential for subsequent sections. A dagger, †, next to a section heading means that the section discusses topics of research. Very roughly speaking, one might interpret as asterisk as indicating there is advanced manipulation of the equations, whereas a dagger might indicate there is approximation of the equations, or an interpretation that is not universally regarded as settled; *caveat emptor*. There is some arbitrariness in such markings, especially where the section deals with a well understood model of a poorly understood

reality. Sections so-marked may be regarded as providing an introduction to the literature, rather than a complete or finished treatment, and the section may also require knowledge of material that appears later in the book. If the asterisk or dagger is applied to a section it applies to all the subsections within, and if a dagger or asterisk appears within a section that is already marked, the warning is even more emphatic. Reading the endnotes at the end of each chapter is not needed in order to follow the arguments in the main text. Problems marked with black diamonds may, like similarly marked ski-slopes, be difficult, and I do not know the solutions to all of them. Good answers to some of them may be publishable and I would appreciate hearing about any such work. I would also appreciate any comments on the material presented in the text. *Qui docet discit*.

Finally, I should say that this book owes its existence in part to my own hubris and selfishness: hubris to think that others might wish to read what I have written, and selfishness because the enjoyable task of writing such a book masquerades as work.

Summary of Contents

The chapters within each part, and the sections within each chapter, form *logical* units, and do not necessarily directly correspond to a single lecture or set number of lectures.

Part I. Fundamentals of geophysical fluid dynamics

Chapter 1 is a brief introduction to fluid dynamics and the basic equations of motion, assuming no prior knowledge of the field. Readers with prior knowledge of fluid dynamics might skim it lightly, concentrating on those aspects unique to the atmosphere or ocean.

Chapter 2 introduces the effects of stratification and rotation, these being the two main effects that most differentiate AOFD from other branches of fluid dynamics. Fundamental topics such as the primitive equations, the Boussinesq equations, and Ekman layers are introduced here, and these form the foundation for the rest of the book.

Chapter 3 focuses on the shallow water equations. Many of the principles of geophysical fluid dynamics have their simplest expression in the shallow water equations, because the effects of stratification are either eliminated or much simplified. The equations thus provide a relatively gentle introduction to the field.

Chapter 4 discusses vorticity and potential vorticity. Potential vorticity plays an especially important role in large-scale, rotating and stratified flows, and its conservation provides the basis for the equation sets of chapter 5.

Chapter 5 derives simplified equation sets for large-scale flows, in particular the quasi-geostrophic and planetary-geostrophic equation sets, and introduces a simple application, Rossby waves. Much of our theoretical understanding of the large-scale circulation has arisen through the use of these equations.

Part II. Instabilities, wave-mean flow interaction and turbulence

Chapter 6 covers barotropic and baroclinic instability, the latter being the instability that gives rise to weather — and therefore being, perhaps, the form of hydrodynamic instability that most affects the human condition.

Chapter 7 provides an introduction to the important topic, albeit one that is regarded as difficult, of wave-mean flow interaction. That is, how do the waves and instabilities affect the mean flow in which they propagate?

Chapter 8 and 9 are on the statistical theory of turbulence. Chapter 8 introduces the basic concepts of two- and three-dimensional turbulence, and chapter 9 applies similar ideas to geostrophic turbulence.

Chapter 10 discusses turbulent diffusion — a hoary subject, both used and abused, yet one that plays a central role in our thinking about the transport properties of eddies in the atmosphere and ocean.

Part III. Large-scale atmospheric circulation

Chapter 11 is mostly concerned with the dynamics of the Hadley Cell, and, rather descriptively, with the Ferrel Cell.

Chapter 12 addresses the mid-latitude circulation. The goal of this chapter is to provide the basis of an understanding of such topics as the surface westerly winds, the dynamics of the Ferrel Cell, the stratification of the atmosphere and the height of the tropopause. These are still active topics of research, so we cannot always be definitive, although many of the underlying principles are now established, and our discussion emphasizes the fundamentals that have, I think, permanent value.

Chapter 13 discusses stationary waves in the atmosphere, mainly produced by the interaction of the zonal wind with mountains and land–sea temperature contrasts. It also discusses the vertical propagation of Rossby waves, and how these induce a stratospheric circulation.

Part IV. Large-scale oceanic circulation

Chapter 14 discusses the wind-driven circulation, in particular the ocean gyres, either ignoring buoyancy effects or assuming that buoyancy forcing acts primarily to set up a stratification that can be taken as a given.

Chapter 15 discusses the buoyancy-driven circulation, largely neglecting the effects of wind forcing.

Chapter 16 addresses the combined effects of wind and buoyancy forcing in setting up the stratification and in driving both the predominantly horizontal gyral circulation and the overturning circulation. As with many sections of Part III, many of these topics are still being actively researched, although, again, many of the underlying principles have, I think, now been established and have permanent value.

Acknowledgements

This book would not have been possible without the input, criticism, encouragement and advice of a large number of people, from students to senior scientists. Students at Princeton University, New York University, Columbia University, California Institute of Technology, MIT and the University of Toronto have used earlier versions of the text in various courses, and I am grateful for the feedback received from the instructors and students about what works and what doesn't, as well as for numerous detailed comments. Parts of the first few chapters and many of the problems draw on notes prepared over the years for a graduate class at Princeton University taught by Steve Garner, Isaac Held, Yoshio Kurihara, Paul Kushner and me. Steve Garner has been notably generous with his time and ideas with respect to this material.

I would like to thank following individuals for providing comments on the text or for

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My first exposure to the field came as a graduate student in the fecund atmosphere of Atmospheric Physics Group at Imperial College in the late 1970s and early 1980s, and I’d like to thank everyone who was there at that time. You know who you are. I would also like to thank everyone at the Geophysical Fluid Dynamics Laboratory (GFDL) and in Princeton University’s Atmospheric and Oceanic Science program for creating the pleasant and stimulating work environment that made it possible to write this book. I am grateful to the staff and scientists at the Plymouth Marine Laboratory and the U. K. Meteorological Office for their hospitality during a sabbatical visit. Finally, I would like to thank the National Science Foundation and the National Oceanic and Atmospheric Administration for providing financial support over the years, and Bess for support of a much greater kind.

Note on later printings

I have taken advantage the opportunity afforded by later printings of this book to correct a number of typographic errors and make a number of small corrections to the text throughout. The pagination and equation numbering are unaltered, although some microtypographic improvements have changed some of the line breaks. I am grateful to many readers who have sent in comments and corrections, and I would particularly like to acknowledge Roger Berland for his exceptionally detailed and perceptive comments on the entire book. Additional resources, including downloadable figures and solutions to many of the end-of-chapter exercises, are available at www.vallisbook.org and www.cambridge.org/9780521849692.

NOTATION

The list below contains only the more important variables, or instances of non-obvious notation. Distinct meanings are separated with a semi-colon. Variables are normally set in italics, constants (e.g. π) in roman (i.e., upright), differential operators in roman, vectors in bold, and tensors in bold sans serif. Thus, vector variables are in bold italics, vector constants (e.g., unit vectors) in bold roman, and tensor variables are in bold slanting sans serif. Physical units are set in roman. A subscript denotes a derivative only if the subscript is a coordinate, such as x , y , z or t ; a subscript 0 generally denotes a constant reference value (e.g., ρ_0). The components of a vector are denoted by superscripts.

Variable	Description
a	Radius of Earth.
b	Buoyancy, $-g\delta\rho/\rho_0$ or $g\delta\theta/\tilde{\theta}$.
\mathbf{c}_g	Group velocity, (c_g^x, c_g^y, c_g^z) .
c_p	Phase speed; heat capacity at constant pressure.
c_v	Heat capacity constant volume.
c_s	Sound speed.
f, f_0	Coriolis parameter, and its reference value.
\mathbf{g}, g	Vector acceleration due to gravity, magnitude of \mathbf{g} .
h	Layer thickness (in shallow water equations).
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	Unit vectors in (x, y, z) directions.
i	An integer index.
i	Square root of -1 .
\mathbf{k}	Wave vector, with components (k, l, m) or (k^x, k^y, k^z) .
k_d	Wave number corresponding to deformation radius.
L_d	Deformation radius.
L, H	Horizontal length scale, vertical (height) scale.
m	Angular momentum about the Earth's axis of rotation.
M	Montgomery function, $M = c_p T + \Phi$.
N	Buoyancy, or Brunt-Väisälä, frequency.
p	Pressure.
Pr	Prandtl ratio, f_0/N .
q	Quasi-geostrophic potential vorticity.
Q	Potential vorticity (in particular Ertel PV).
\dot{Q}	Rate of heating.
Ra	Rayleigh number.
Re	Real part of expression.
Re	Reynolds number, UL/ν .
Ro	Rossby number, U/fL .
S	Salinity; source term on right-hand side of an evolution equation.
S_o, \mathbf{S}_o	Solenoidal term, solenoidal vector.
T	Temperature.
t	Time.
\mathbf{u}	Two-dimensional (horizontal) velocity, (u, v) .
\mathbf{v}	Three-dimensional velocity, (u, v, z) .
x, y, z	Cartesian coordinates, usually in zonal, meridional and vertical directions.
Z	Log-pressure, $-H \log p/p_R$. Usually, $H = 7.5$ km and $p_R = 10^5$ Pa.

Variable	Description
\mathcal{A}	Wave activity.
α	Inverse density, or specific volume; aspect ratio.
β	Rate of change of f with latitude, $\partial f / \partial y$.
β_T, β_S	Coefficient of expansion with respect to temperature, salinity.
ϵ	Generic small parameter (epsilon).
ε	Cascade or dissipation rate of energy (varepsilon).
η	Specific entropy; perturbation height; enstrophy cascade or dissipation rate.
\mathcal{F}	Eliassen Palm flux, $(\mathcal{F}^y, \mathcal{F}^z)$.
γ	Vorticity gradient, $\beta - u_{yy}$; the ratio c_p / c_v .
Γ	Lapse rate.
κ	Diffusivity; the ratio R / c_p .
\mathcal{K}	Kolmogorov or Kolmogorov-like constant.
Λ	Shear, e.g., $\partial U / \partial z$.
μ	Viscosity.
ν	Kinematic viscosity, μ / ρ .
v	Meridional component of velocity.
ϕ	Pressure divided by density, p / ρ ; passive tracer.
Φ	Geopotential, usually gz .
Π	Exner function, $\Pi = c_p T / \theta = c_p (p / p_R)^{R / c_p}$.
ω	Vorticity.
$\Omega, \boldsymbol{\Omega}$	Rotation rate of Earth and associated vector.
ψ	Streamfunction.
ρ	Density.
ρ_θ	Potential density.
σ	Layer thickness, $\partial z / \partial \theta$; Prandtl number ν / κ ; measure of density, $\rho - 1000$.
$\boldsymbol{\tau}$	Stress vector, often wind stress.
$\tilde{\boldsymbol{\tau}}$	Kinematic stress, $\tilde{\boldsymbol{\tau}} / \rho$.
τ	Zonal component or magnitude of wind stress; eddy turnover time.
θ	Potential temperature.
ϑ, λ	Latitude, longitude.
ζ	Vertical component of vorticity.
$\left(\frac{\partial a}{\partial b}\right)_c$	Derivative of a with respect to b at constant c .
$\left.\frac{\partial a}{\partial b}\right _{a=c}$	Derivative of a with respect to b evaluated at $a = c$.
∇_a	Gradient operator at constant value of coordinate a , e.g., $\nabla_z = \mathbf{i} \partial_x + \mathbf{j} \partial_y$.
$\nabla_{a \cdot}$	Divergence operator at constant value of coordinate a , e.g., $\nabla_{z \cdot} = (\mathbf{i} \partial_x + \mathbf{j} \partial_y) \cdot$.
∇^\perp	Perpendicular gradient, $\nabla^\perp \phi \equiv \mathbf{k} \times \nabla \phi$.
curl_z	Vertical component of $\nabla \times$ operator, $\text{curl}_z \mathbf{A} = \mathbf{k} \cdot \nabla \times \mathbf{A} = \partial_x A^y - \partial_y A^x$.
$\frac{D}{Dt}$	Material derivative (generic).
$\frac{D_g}{Dt}$	Material derivative using geostrophic velocity, for example $\partial / \partial t + \mathbf{u}_g \cdot \nabla$.
$\frac{D_3}{Dt}, \frac{D_2}{Dt}$	Material derivative in three dimensions and in two dimensions, for example $\partial / \partial t + \mathbf{v} \cdot \nabla$ and $\partial / \partial t + \mathbf{u} \cdot \nabla$ respectively.