CHAPTER

Introduction to signals and systems

Welcome to *Introduction to Signals and Systems*. This text will focus on the properties of signals and systems, and the relationship between the inputs and outputs of physical systems. We will develop tools and techniques to help us analyze, design, and simulate signals and systems.

1.1 Signals and systems

A signal is a pattern of variation of a physical quantity: a definition which covers a wide territory. You are processing signals as you read this text. A lecturer creates a signal as he or she talks and your ear processes these signals. Signals are all around us. Examples include acoustical, electrical, and mechanical signals. Signals may depend on one or more independent variables. As the name implies, one-dimensional signals depend on one independent variable. An example is the location of a particle moving in a rectilinear motion, in which case the independent variable is time, t. Two-dimensional signals depend on two independent variables. An example is a picture that varies spatially, in which case the independent variables are the spatial coordinates, x and y. Many of the signals and systems that you have routinely dealt with have interesting properties that this text will explore.

A system processes signals. For example, a compact disc (CD) player is a system that reads a digital signal from a CD and transforms it into an electrical signal. The electrical signal goes to the speaker, which is another system that transforms electrical signals into acoustical signals. Many signals contain information. Other signals are used only to transport energy. For example, the signal from a wall socket is boring in terms of information content, but very useful for carrying energy.

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Signals can be categorized as analog, discrete or digital. They are summarized as follows:

- Analog signals: signals that vary continuously in amplitude and time. The independent variable is not necessarily time, it could also be a spatial coordinate.
- **Discrete-time signals:** signals that have continuous amplitude but only exist at discrete times. These signals are represented as sequences of numbers.
- **Digital signals:** signals that have discrete amplitude and time. These signals are represented by sequences of numbers with finite precision. They are used when processing information by computer.

We will often be interested in converting signals from one type to another. For example, in a chemistry laboratory, a continuous-time transducer measures the analog value of a physical quantity. We will often use a continuousto-discrete (analog-to-digital) converter to capture the signal into a computer for processing.

Systems can be categorized as linear or nonlinear. Systems are said to be linear when scaling and superposition hold. For linear systems, if the input to the system is scaled by some constant a, the output of the system will be scaled by the same amount. Thus, if the input to the system is doubled, so will be the output of the system. Linear systems also obey the superposition principle. Thus, for a linear system, the response of the system to a combination of N inputs is simply the sum of the responses to each input considered individually.

Systems can also be classified as time-variant or time-invariant. When the parameters of a system remain constant during operation, the system is said to be time-invariant. When the parameters can vary as a function of time, the system is said to be time-variant. For time-invariant systems, the system responds the same yesterday, today, and tomorrow. Although time-varying systems without nonlinearity are still considered linear, such systems are considerably more difficult to analyze and design.

In this text, we will primarily be interested in linear and time-invariant systems, or LTI systems for short. They are very useful for signal processing and system modeling. While most physical signals and systems are not LTI, surprisingly many can be approximated as LTI over a specified time domain of interest. Nonlinear systems have some very interesting and surprising

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1.1 Signals and systems

properties but are usually much more difficult to handle mathematically, and there are limited methods available for solving nonlinear systems.

A familiar example of signals and systems is the recording and playback of audio signals such as music and voice. Over the years there have been major changes in the technology used resulting in dramatic improvements in quality. Milestones in audio technology include:

- Phonograph: invented by Thomas Edison in 1877.
- Gramophone: developed by Emile Berliner in 1887 (70 revolution per minute or rpm).
- 78 rpm record: 1930s.
- AM (amplitude modulation) radio: 1920s
- 33 1/3 LP (long-playing) record: introduced by CBS in 1948 (held about 20 minutes on each side).
- 45 EP (extended-playing) record: introduced by RCA in 1948 (more portable, held 5–6 minutes on each side).
- Stereo: 1956.
- Stereo FM (frequency modulation): 1960s.
- CD (compact disc): developed by Sony and Philips in 1982 (holds about 70 minutes).

The information density has improved over time. On a conventional LP record, each track is about 100 micrometers wide. On a CD, the tracks are 1.6 micrometers wide, so tracks are packed 60 times more densely. On a CD, the information is stored by a series of pits burned with a powerful laser. The information is read by measuring the reflection from another laser to determine where the pits are located. The locations encode a series of binary numbers, so a CD is really just a physical encoding of a long sequence of 1s and 0s.

Signals may be represented as a graph with time on the horizontal axis and amplitude of the signals on the vertical axis. An *oscilloscope* is a system that converts an electrical signal into an optical signal showing such a graph, allowing us to examine how the signal varies as a function of time. A *spectrum analyzer* is a system that converts an electrical signal into an optical signal showing a graph of what frequencies are in the input, called the *spectrum* of a signal. We will have much more to say about the spectrum of a signal. Knowing the frequency content of a signal allows us to characterize the signal.

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1.2 Examples of signals

Let us now consider some simple examples of signals. Sinusoidal signals will prove to be one of the most useful signals we will encounter. Mathematically, a sinusoidal signal may be represent as

$$x(t) = A\cos(2\pi f t + \phi), \tag{1.1}$$

where A denotes the *amplitude* or magnitude of the signal, f its corresponding *frequency*, and ϕ its *phase*. Figure 1.1 shows a sine wave whose amplitude is 1 and phase is 0, and which has a frequency of 200 Hz (0.2 kHz). Figure 1.2 shows the spectrum of the same sine wave. The spectrum of a signal consists of a graph that shows what frequencies are present in the signal as well as the magnitudes of the frequency components. Thus, as expected, the spectrum of a 200 Hz sine wave (Figure 1.2) shows a single peak centered at 200 Hz with amplitude 1.

Figure 1.3 shows a periodic square wave at 200 Hz, and Figure 1.4 shows its spectrum. The square wave alternates between 1 and -1 with a period of 0.005 s. The spectrum consists of spikes at 200 Hz, 600 Hz, 1000 Hz, 1400 Hz, etc. The largest peak is at the fundamental frequency f_0 of 200 Hz. The next peaks are at the third harmonic $3f_0$, fifth harmonic

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1.2 Examples of signals



 $5f_0$, seventh harmonic $7f_0$, and so forth. The amplitudes drop off for higher harmonics. Later we will show how these amplitudes and frequencies can be determined analytically using Fourier series and numerically using the MATLAB command fft.

Figure 1.5 shows a triangle wave at 200 Hz and Figure 1.6 shows its spectrum. Note that the spectrum also contains the odd harmonics, but the amplitudes drop off quickly compared to the amplitudes of the square wave of Figure 1.4.

The previous examples suggest that we could construct any signal by summing sinusoids of different amplitudes and frequencies. This underlying principle forms the basis of *Fourier series*. The spectrum shows the am-

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plitude of the coefficients of the Fourier series. We will be interested in both synthesizing signals from sums of other signals (for example sinusoids) and analyzing signals by determining the amplitudes of the frequencies within the signal.

As a more elaborate example, Figure 1.7 shows a digital representation of a two-channel audio signal sampled at 44 kHz (CD quality). Clearly, the eye is not trained to interpret the signal, even when zooming in on a short segment. Transforming the signal into an acoustical signal, however, makes it clearly recognizable as the beginning of Beethoven's Fifth Symphony. The above examples serve to remind us that there are many different ways to represent a signal. We can identify a signal by plotting its variation with time, analyzing its spectrum, or listening to how the signal changes as a function of time. These approaches can all be used to characterize the same signal. Depending on the application, one approach may be more useful than the others.

Another example of a system that uses signals is the telephone. Dialing a touch-tone telephone generates a series of tones. These tones are the *superposition* or sum of a pair of sine waves, as shown in Table 1.1. You could build your own dialer by producing these tones with another system such as your computer. Similarly, you could determine which phone number 7

1.3 Mathematical foundations

Frequencies	1209 Hz	1336 Hz	1477 Hz	
697 Hz	1	2	3	
770 Hz	4	5	6	
852 Hz	7	8	9	
941 Hz	*	0	#	

Table 1.1. Touch-tone telephone tones

was dialed by looking at the spectrum of the tones and observing what frequencies are present.

Having seen various examples of signals and systems, we are now ready to lay the mathematical foundations to understand them in detail.

1.3 Mathematical foundations

Euler's formula (or identity) was introduced in calculus. It states that a complex exponential¹ can be expressed as the sum of a cosine function and a sine function:

$$e^{j\theta} = \cos\theta + j\sin\theta, \qquad (1.2)$$

where $j = \sqrt{-1}$.² Euler's identity can be easily proved by expanding e^x in Taylor series and replacing x by $j\theta$. Figure 1.8 shows Eq. (1.2) plotted as a vector on the complex plane, where the horizontal axis corresponds to the real axis, and the vertical axis corresponds to the imaginary axis. Observe that the length or magnitude of the vector is 1 and the angle is θ , where θ is measured in radians, and is positive in the counterclockwise sense from the horizontal or real axis. Similarly, we can show that

$$e^{-j\theta} = \cos\theta - j\sin\theta. \tag{1.3}$$

Therefore, in terms of complex exponentials, cosine and sine can be expressed as

$$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$
 and $\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta}).$ (1.4)

¹ Because complex arithmetic will be used extensively throughout the text, the reader is encouraged to review Appendix A for a detailed discussion.

² Engineers often use j to represent complex unity. The variable i is reserved to denote the current of electrical systems.



Fig. 1.8. Complex exponential plotted on a complex plane.

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These sums are represented graphically in Figure 1.9. Recall that the complex conjugate of a complex number a + jb is simply a - jb, where a and b are real constants. Thus, $e^{j\theta}$ and $e^{-j\theta}$ are complex conjugates, because their real parts are identical and their imaginary parts are the negative of one another.

In terms of complex exponentials, we can now write a simple sine wave as follows:

$$\mathbf{x}(t) = A\cos(\omega t + \phi) = \frac{A}{2} \left[e^{\mathbf{j}(\omega t + \phi)} + e^{-\mathbf{j}(\omega t + \phi)} \right],\tag{1.5}$$

where A and ϕ are real constants. The units of ω are radians per second. This is shown graphically in Figure 1.10, where both vectors are now functions of time, t. Note that as t increases, the upper vector rotates counterclockwise at a rate determined by the angular frequency ω and the lower vector rotates clockwise at an angular frequency of $-\omega$. Thus, we define a vector with a negative frequency $(-\omega)$ as one that rotates in the clockwise direction.

There are many different ways of representing a sinusoid. Another representation utilizes complex exponentials and the definition of the real part of a complex number as follows:

$$x(t) = A\cos(\omega t + \phi) = \operatorname{Re}\left\{Ae^{j(\omega t + \phi)}\right\},\tag{1.6}$$

where A and ϕ are real constants, and Re{ } indicates the real part of an expression. Using Euler's identity, we have:

$$Ae^{j(\omega t + \phi)} = A\cos(\omega t + \phi) + jA\sin(\omega t + \phi).$$
(1.7)

By inspection, note that x(t) of Eq. (1.5) is simply the real part of Eq. (1.7).



Fig. 1.10. Graphical representation of sinusoid on a complex plane.

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1.4 Phasors

Finally, using Eq. (1.6), we can also represent a sinusoid as:

$$x(t) = \operatorname{Re}\{Ae^{j\phi}e^{j\omega t}\} = \operatorname{Re}\{Xe^{j\omega t}\}.$$
(1.8)

This is called the *phasor* notation. The phasor $X = Ae^{j\phi}$ contains the amplitude (A) and phase (ϕ) information of the sinusoid, corresponding to the length or magnitude and angle of the vector in the complex plane.

We have introduced various ways of representing signals. They will be used extensively throughout the text, and the reader should become very familiar with these different representations and how they are related.

1.4 Phasors



waves of various frequencies, amplitudes, and phases. We can write any sine wave as

We have seen that sine waves appear to be important building blocks. Thus, we expect that we can construct more elaborate signals as the sums of sine

$$x(t) = A\cos(\omega t + \theta). \tag{1.9}$$

Fig. 1.11. The real part of a rotating phasor.

This can also be represented with a phasor as follows:

$$x(t) = \operatorname{Re}\{Ae^{j\theta}e^{j\omega t}\},\tag{1.10}$$

which can be viewed as a vector on the complex plane, as shown in Figure 1.11. The vector rotates at a rate (called the *angular frequency*) of ω in units of radians per second. The real part of the vector corresponds to the value of the sinusoid of Eq. (1.8) at time *t*. The wave is periodic with a period *T* between repetitions, where the period is given by $T = 2\pi/\omega$. We often wish to refer to the rate at which the wave repeats. Let us define this rate as $f = \omega/(2\pi) = 1/T$, where *f* is called the *frequency*, measured in units of hertz (Hz) or cycles/second. Therefore, in terms of frequency, we often rewrite Eq. (1.9) as

$$x(t) = A\cos\left(2\pi f t + \theta\right). \tag{1.11}$$

Also observe that the peaks occur at times

$$t = \frac{2\pi n - \theta}{2\pi f}, \quad n = 0, \pm 1, \pm 2, \dots$$
 (1.12)

Signals generally represent physical quantities and should be dimensioned appropriately. For example, the *x*-axis may be in dimensions of seconds,

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minutes, or feet. The *y*-axis may be in volts, meters, degrees Celsius, etc. For example, a facetious set of dimensions for calibrating speedometers is furlongs/fortnight. In this text we will typically use the SI (Système International) metric units.

In analyzing signals, we often will be interested in adding multiple sine waves of the same frequency. Consider two signals $x_1(t)$ and $x_2(t)$ of the same angular frequency, ω :

$$x_1(t) = A_1 \cos(\omega t + \theta_1)$$

$$x_2(t) = A_2 \cos(\omega t + \theta_2),$$
(1.13)

whose sum is given by

$$x(t) = x_1(t) + x_2(t) = A\cos(\omega t + \theta).$$
(1.14)

Not surprisingly, the amplitude (A) and phase (θ) of the new signal x(t) are related to the amplitudes and phases of $x_1(t)$ and $x_2(t)$, and they can be obtained through simple algebraic manipulations. Let us first expand $x_1(t)$ and $x_2(t)$ using the trigonometric identity for the cosine of the sum of two angles,

$$x_1(t) = A_1 \cos(\omega t) \cos \theta_1 - A_1 \sin(\omega t) \sin \theta_1$$

$$x_2(t) = A_2 \cos(\omega t) \cos \theta_2 - A_2 \sin(\omega t) \sin \theta_2.$$
(1.15)

Summing the coefficients of the cosine and sine terms, we get

$$x(t) = \left(\underbrace{A_1 \cos \theta_1 + A_2 \cos \theta_2}_{A_c}\right) \cos(\omega t) - \left(\underbrace{A_1 \sin \theta_1 + A_2 \sin \theta_2}_{A_s}\right) \sin(\omega t),$$
(1.16)

which can be rewritten as

$$x(t) = A_{c} \cos(\omega t) - A_{s} \sin(\omega t)$$

= $A \cos \theta \cos(\omega t) - A \sin \theta \sin(\omega t)$ (1.17)
= $A \cos(\omega t + \theta)$.

By matching the coefficients of $cos(\omega t)$ and $sin(\omega t)$, we note immediately that

$$A\cos\theta = A_{\rm c}$$

$$A\sin\theta = A_{\rm s},$$
(1.18)