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0521849608 - Gravitational Radiation, Luminous Black Holes, and Gamma-Ray Burst Supernovae

Maurice H. P. M. van Putten

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Gravitational Radiation, Luminous Black Holes, and Gamma-Ray Burst Supernovae

Black holes and gravitational radiation are two of the most dramatic predictions of general relativity. The quest for rotating black holes – discovered by Roy P. Kerr as exact solutions to the Einstein equations – is one of the most exciting challenges currently facing physicists and astronomers.

Gravitational Radiation, Luminous Black Holes and Gamma-ray Burst Supernovae takes the reader through the theory of gravitational radiation and rotating black holes, and the phenomenology of GRB supernovae. Topics covered include Kerr black holes and the frame-dragging of spacetime, luminous black holes, compact tori around black holes, and black hole–spin interactions. It concludes with a discussion of prospects for gravitational-wave detections of a long-duration burst in gravitational waves as a method of choice for identifying Kerr black holes in the universe.

This book is ideal for a special topics graduate course on gravitational-wave astronomy and as an introduction to those interested in this contemporary development in physics.

MAURICE H. P. M. VAN PUTTEN studied at Delft University of Technology, The Netherlands and received his Ph.D. from the California Institute of Technology. He has held postdoctoral positions at the Institute of Theoretical Physics at the University of California at Santa Barbara, and the Center for Radiophysics and Space Research at Cornell University. He then joined the faculty of the Massachusetts Institute of Technology and became a member of the new Laser Interferometric Gravitational-wave Observatory (MIT-LIGO), where he teaches a special-topic graduate course based on his research.

Professor van Putten's research in theoretical astrophysics has spanned a broad range of topics in relativistic magnetohydrodynamics, hyperbolic formulations of general relativity, and radiation processes around rotating black holes. He has led global collaborations on the theory of gamma-ray burst supernovae from rotating black holes as burst sources of gravitational radiation. His theory describes a unique link between gravitational waves and Kerr black holes, two of the most dramatic predictions of general relativity. Discovery of *triplets* – gamma-ray burst supernovae accompanied by a long-duration gravitational-wave burst – provides a method for calorimetric identification of Kerr black holes in the universe.

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To my parents Anton and Maria,
and
Michael, Pascal, and Antoinette

Contents

<i>Foreword</i>	<i>page</i> xi
<i>Acknowledgments</i>	xii
<i>Introduction</i>	xiii
<i>Notation</i>	xvii
1 Superluminal motion in the quasar 3C273	1
1.1 Lorentz transformations	1
1.2 Kinematic effects	5
1.3 Quasar redshifts	6
1.4 Superluminal motion in 3C273	7
1.5 Doppler shift	9
1.6 Relativistic equations of motion	9
2 Curved spacetime and SgrA*	13
2.1 The accelerated letter “L”	14
2.2 The length of timelike trajectories	15
2.3 Gravitational redshift	16
2.4 Spacetime around a star	18
2.5 Mercury’s perihelion precession	20
2.6 A supermassive black hole in SgrA*	22
3 Parallel transport and isometry of tangent bundles	26
3.1 Covariant and contravariant tensors	27
3.2 The metric g_{ab}	29
3.3 The volume element	30
3.4 Geodesic trajectories	31
3.5 The equation of parallel transport	32
3.6 Parallel transport on the sphere	34
3.7 Fermi–Walker transport	34
3.8 Nongeodesic observers	35
3.9 The Lie derivative	39

viii	<i>Contents</i>	
4	Maxwell's equations	43
4.1	p -forms and duality	43
4.2	Geometrical interpretation of F_{ab}	44
4.3	Two representations of F_{ab}	46
4.4	Exterior derivatives	47
4.5	Stokes' theorem	48
4.6	Some specific expressions	49
4.7	The limit of ideal MHD	50
5	Riemannian curvature	55
5.1	Derivations of the Riemann tensor	55
5.2	Symmetries of the Riemann tensor	57
5.3	Foliation in spacelike hypersurfaces	58
5.4	Curvature coupling to spin	59
5.5	The Riemann tensor in connection form	62
5.6	The Weyl tensor	64
5.7	The Hilbert action	64
6	Gravitational radiation	67
6.1	Nonlinear wave equations	69
6.2	Linear gravitational waves in h_{ij}	72
6.3	Quadrupole emissions	75
6.4	Summary of equations	79
7	Cosmological event rates	81
7.1	The cosmological principle	82
7.2	Our flat and open universe	83
7.3	The cosmological star-formation rate	85
7.4	Background radiation from transients	85
7.5	Observed versus true event rates	86
8	Compressible fluid dynamics	89
8.1	Shocks in 1D conservation laws	91
8.2	Compressible gas dynamics	94
8.3	Shock jump conditions	95
8.4	Entropy creation in a shock	98
8.5	Relations for strong shocks	98
8.6	The Mach number of a shock	100
8.7	Polytropic equation of state	101
8.8	Relativistic perfect fluids	103
9	Waves in relativistic magnetohydrodynamics	110
9.1	Ideal magnetohydrodynamics	112
9.2	A covariant hyperbolic formulation	113

	<i>Contents</i>	ix
	9.3 Characteristic determinant	115
	9.4 Small amplitude waves	117
	9.5 Right nullvectors	118
	9.6 Well-posedness	122
	9.7 Shock capturing in relativistic MHD	125
	9.8 Morphology of a relativistic magnetized jet	132
10	Nonaxisymmetric waves in a torus	138
	10.1 The Kelvin–Helmholtz instability	139
	10.2 Multipole mass-moments in a torus	141
	10.3 Rayleigh’s stability criterion	142
	10.4 Derivation of linearized equations	142
	10.5 Free boundary conditions	144
	10.6 Stability diagram	145
	10.7 Numerical results	146
	10.8 Gravitational radiation-reaction force	148
11	Phenomenology of GRB supernovae	152
	11.1 True GRB energies	162
	11.2 A redshift sample of 33 GRBs	164
	11.3 True GRB supernova event rate	165
	11.4 Supernovae: the endpoint of massive stars	168
	11.5 Supernova event rates	174
	11.6 Remnants of GRB supernovae	175
	11.7 X-ray flashes	176
	11.8 Candidate inner engines of GRB/XRF supernovae	177
12	Kerr black holes	179
	12.1 Kerr metric	180
	12.2 Mach’s principle	183
	12.3 Rotational energy	183
	12.4 Gravitational spin–orbit energy $E = \omega J$	185
	12.5 Orbits around Kerr black holes	187
	12.6 Event horizons have no hair	189
	12.7 Penrose process in the ergosphere	192
13	Luminous black holes	197
	13.1 Black holes surrounded by a torus	197
	13.2 Horizon flux of a Kerr black hole	199
	13.3 Active black holes	202
14	A luminous torus in gravitational radiation	215
	14.1 Suspended accretion	216
	14.2 Magnetic stability of the torus	217
	14.3 Lifetime and luminosity of black holes	222

x	<i>Contents</i>	
	14.4 Radiation channels by the torus	222
	14.5 Equations of suspended accretion	224
	14.6 Energies emitted by the torus	226
	14.7 A compactness measure	228
15	GRB supernovae from rotating black holes	231
	15.1 Centered nucleation at low kick velocities	233
	15.2 Branching ratio by kick velocities	237
	15.3 Single and double bursters	237
	15.4 Radiatively driven supernovae	238
	15.5 SN1998bw and SN2002dh	240
	15.6 True GRB afterglow energies	241
16	Observational opportunities for LIGO and Virgo	245
	16.1 Signal-to-noise ratios	249
	16.2 Dimensionless strain amplitudes	250
	16.3 Background radiation from GRB-SNe	251
	16.4 LIGO and Virgo detectors	253
	16.5 Signal-to-noise ratios for GRB-SNe	256
	16.6 A time-frequency detection algorithm	260
	16.7 Conclusions	262
17	Epilogue: GRB/XRF singlets, doublets? Triplets!	266
	Appendix A. Landau's derivation of a maximal mass	269
	Appendix B. Thermodynamics of luminous black holes	271
	Appendix C. Spin-orbit coupling in the ergotube	273
	Appendix D. Pair creation in a Wald field	275
	Appendix E. Black hole spacetimes in the complex plane	280
	Appendix F. Some units, constants and numbers	283
	<i>References</i>	285
	<i>Index</i>	300

Foreword

General relativity is one of the most elegant and fundamental theories of physics, describing the gravitational force with a most awesome precision. When it was first discovered, by Einstein in 1915, the theory appeared to do little more than provide for minute corrections to the older formalism: Newton's law of gravity. Today, more and more stellar systems are discovered, in the far outreaches of the universe, where extreme conditions are suspected to exist that lead to incredibly strong gravitational forces, and where relativistic effects are no longer a tiny perturbation, but they dominate, yielding totally new phenomena. One of these phenomena is gravitational radiation – gravity then acts in a way very similar to what happens with electric and magnetic fields when they oscillate: they form waves that transmit information and energy.

Only the most violent sources emit gravitational waves that can perhaps be detected from the Earth, and this makes investigating such sources interesting. The physics and mathematics of these sources is highly complex.

Maurice van Putten has great expertise in setting up the required physical models and in solving the complicated equations emerging from them. This book explains his methods in dealing with these equations. Not much time is wasted on philosophical questions or fundamental motivations or justifications. The really relevant physical questions are confronted with direct attacks. Of course, we encounter all sorts of difficulties on our way. Here, we ask for practical ways out, rather than indulging on formalities. Different fields of physics are seen to merge: relativity, quantum mechanics, plasma physics, elementary particle physics, numerical analysis and, of course, astrophysics. A book for those who want to get their hands dirty.

Gerard 't Hooft

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Frontmatter

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Epigraph to Chapter 16, reprinted with permission from *Gravitation and Cosmology*, by Stephen Weinberg.

Introduction

Observations of gravitational radiation from black holes and neutron stars promise to dramatically transform our view of the universe. This new topic of gravitational-wave astronomy will be initiated with detections by recently commissioned gravitational-wave detectors. These are notably the Laser Interferometric Gravitational wave Observatory LIGO (US), Virgo (Europe), TAMA (Japan) and GEO (Germany), and various bar detectors in the US and Europe.

This book is intended for graduate students and postdoctoral researchers who are interested in this emerging opportunity. The audience is expected to be familiar with electromagnetism, thermodynamics, classical and quantum mechanics. Given the rapid development in gravitational wave experiments and our understanding of sources of gravitational waves, it is recommended that this book is used in combination with current review articles.

This book developed as a graduate text on general relativity and gravitational radiation in a one-semester special topics graduate course at MIT. It started with an invitation of Gerald E. Brown for a *Physics Reports* on gamma-ray bursts. *Why study gamma-ray bursters?* Because they are there, representing the most energetic and relativistic transients in the sky? Or perhaps because they hold further promise as burst sources of gravitational radiation?

Our focus is on gravitational radiation powered with rotating black holes – the two most fundamental predictions of general relativity for astronomy (other than cosmology). General relativity is a classical field theory, and we believe it applies to all macroscopic bodies. We do not know whether general relativity is valid down to the Planck scale without modifications at intermediate scales, without any extra dimensions or additional internal symmetries.

Observations of neutron star binaries PSR 1913+16 and, more recently, PSR 0737-3039, tell us that gravitational waves exist and carry energy. This discovery is a considerable advance beyond the earlier phenomenology of quasi-static

spacetimes in general relativity, such as the deflection of light by the Sun and the orbital precession of Mercury.

Observational evidence of black holes is presently limited to compact stellar mass objects as black hole candidates in soft X-ray transients and their supermassive counterparts at centers of galaxies. Particularly striking is the discovery of compact stellar trajectories in SgrA* in our own galaxy, which reveals a supermassive black hole of a few million solar masses.

Rotating black holes are believed to nucleate in core collapse of massive stars. The exact solution of rotating black holes was discovered by Roy P. Kerr[293]. It shows frame-dragging to be the explicit manifestation of curvature induced by angular momentum. It further predicts a large energy reservoir in rotation in the black hole: its energy content may exceed that in a rapidly rotating neutron star by at least an order of magnitude. While in isolation stellar black holes are stable and essentially nonradiating, in interaction with their environment black holes can become luminous upon emitting angular momentum in various radiation channels.

Essential to the interaction of Kerr black holes with the environment is the Rayleigh criterion. Rotating black holes tend to lower their energy by radiating high specific angular momentum to infinity. In isolation, these radiative processes are suppressed by canonical angular momentum barriers, rendering macroscopic black holes stable. Penrose recognized that, in principle, the rotational energy of a black hole can be liberated by splitting surrounding matter into high and low angular momentum particles[416, 417]. Absorption of low-angular momentum and ejection of high-angular momentum with positive energy to infinity is consistent with the Rayleigh criterion and conservation of mass and angular momentum. These processes are restricted to the so-called *ergosphere*. Black hole spin-induced curvature and curvature coupling to spin combined further give rise to spin-orbit coupling – an effective interaction of black hole spin with angular momentum in an *ergotube* along the axis of rotation. Calorimetry on the ensuing radiation energies promises first-principle evidence for Kerr black holes and, consequently, evidence for general relativity in the nonlinear regime.

While currently observed neutron star binary systems provide us with laboratories to study linearized general relativity, could gamma ray burst supernovae serve a similar role for fully nonlinear general relativity?

Cosmological gamma-ray bursts were accidentally discovered by Vela and Konus satellites in the late 1960s. Their association with supernovae, in its earliest form proposed by Stirling Colgate, has been confirmed by GRB 980425/SN1988bw[224, 536] and GRB030329/SN2003dh[506, 265]. Thus, Type Ib/c supernovae are probably the parent population of long GRBs. It has been appreciated that the observed GRB afterglow emissions represent the dissipation of ultrarelativistic baryon-poor outflows[451, 452], while

the associated supernova is strongly aspherical[268] and bright in X-ray line-emissions[17, 432, 613, 434, 454]. These observations further show the time of onset of the gamma-ray burst and the supernova to be the same within observational uncertainties.

This phenomenology reveals a baryon-poor active nucleus as the powerhouse of GRB supernovae in core collapse in massive stars. The only known baryon-free energy source is a rotating black hole. This presents an *energy paradox*: *the rotational energy of a rapidly rotating black hole is orders of magnitude larger than the energy requirements set by the observed radiation energies in GRB supernovae*. A rapidly rotating nucleus formed in core-collapse is relativistically compact and radiative primarily in “unseen” gravitational radiation and MeV-neutrino emissions. These channels provide a new opportunity for probing the inner engine of cosmological GRB supernovae.

The promise of a link between gravitational radiation and black holes in GRB supernovae provides a method for the gravitational wave-detectors LIGO and Virgo to provide first-principle evidence for Kerr black holes in association with a currently known observational phenomenon.

This book consists of three parts: gravitational radiation, waves in astrophysical fluids, and a theory of GRB supernovae from rotating black holes. Chapters 1–7 introduce general relativity and gravitational radiation. Chapters 8–10 discuss fluid dynamical waves in jets and tori around black holes. Gamma-ray burst supernovae are introduced in Chapter 11. A theory of gravitational waves created by GRB supernovae from rotating black holes is discussed in Chapters 12–15. Chapter 16 discusses GRB supernovae as observational opportunities for gravitational wave experiments LIGO and Virgo.

The author is greatly indebted to his collaborators and many colleagues for constructive discussions over many years, which made possible this venture into gravitational-wave astronomy: Amir Levinson, Eve C. Ostriker, Gerald E. Brown, Roy P. Kerr, Garry Tee, Gerard 't Hooft, H. Cheng, S.-T. Yau, Félix Mirabel, Dale A. Frail, Kevin Hurley, Douglas M. Eardley, John Heise, Stirling Colgate, Andy Fabian, Alain Brillet, Rainer Weiss, David Shoemaker, Barry Barish, Kip S. Thorne, Roger D. Blandford, Robert V. Wagoner, E. Sterl Phinney, Jacob Bekenstein, Gary Gibbons, Shrinivastas Kulkarni, Giora Shaviv, Tsvi Piran, Gennadii S. Bisnovatyi-Kogan, Ramesh Narayan, Bohdan Paczyński, Peter Mészáros, Saul Teukolsky, Stuart Shapiro, Edward E. Salpeter, Ira Wasserman, David Chernoff, Yvonne Choquet-Bruhat, Tim de Zeeuw, John F. Hawley, David Coward, Ron Burman, David Blair, Sungeun Kim, Hyun Kyu Lee, Tania Regimbau, Gregory M. Harry, Michele Punturo, Linqing Wen, Stephen Eikenberry, Mark Abramowicz, Michael L. Norman, Valeri Frolov, Donald

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The reader is referred to other texts for more general discussions on stellar structure, compact objects and general relativity, notably: *Gravitation* by C. W. Misner, K. S. Thorne and J. A. Wheeler[382], *Gravitation and Cosmology* by S. Weinberg[587], *The Membrane Paradigm* by K. S. Thorne, R. H. Price & A. MacDonald[534], *Stellar Structure and Evolution* by R. Kippenhahn and A. Weigert[295], *Introduction to General Relativity* by G. 't Hooft[527], *General Relativity* by R. M. Wald[577], *General Relativity* by H. Stephani[509], *Gravitation and Spacetime* by H. C. Ohanian and R. Ruffini[398], *A First Course in General Relativity* by Bernard F. Schutz[485], *Black Holes, White Dwarfs and Neutron Stars* by S. L. Shapiro and S. A. Teukolsky[490], *Black Hole Physics* by V. Frolov and I. D. Novikov[208], *Formation and Evolution of Black holes in the Galaxy* by H. A. Bethe, G. E. Brown and C.-H. Lee[53], and *Analysis, Manifolds and Physics* by Y. Choquet-Bruhat, C. DeWitt-Morette and M. Dillard-Bleick[120].

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Notation

The metric signature $(-, +, +, +)$ is in conformance with Misner, Thorne and Wheeler 1974[382]. The Minkowski metric is given by $\eta_{ab} = [-1, 1, 1, 1]$.

Most of the expressions are in geometrical units, except where indicated. In the case of pair creation by black holes (Appendix D), we use mixed geometrical–natural units.

Tensors are written in the so-called abstract index notation in Latin script. Indices from the middle of the alphabet denote spatial coordinates. Four-vectors and p -forms are also indicated in small boldface. Three-vectors are indicated in capital boldface.

The epsilon tensor $\epsilon_{abcd} = \Delta_{abcd}\sqrt{-g}$ is defined in terms of the totally anti-symmetric symbol Δ_{abcd} and the determinant g of the metric, where $\Delta_{0123} = 1$ which changes sign under odd permutations.

Tetrad elements are indexed by $\{(e_\mu)^b\}_{\mu=1}^4$, where μ denotes the tetrad index and b denotes the coordinate index.