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0521848628 - Combinatorial and Computational Geometry
Edited by Jacob E. Goodman, Janos Pach and Emo Welzl
Frontmatter
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During the past few decades, the gradual merger of Discrete Geometry and the newer discipline of Computational Geometry has provided enormous impetus to mathematicians and computer scientists interested in geometric problems. This volume, which contains 32 papers on a broad range of topics of current interest in the field, is an outgrowth of that synergism. It includes surveys and research articles exploring geometric arrangements, polytopes, packing, covering, discrete convexity, geometric algorithms and their complexity, and the combinatorial complexity of geometric objects, particularly in low dimension. There are points of contact with many applied areas such as mathematical programming, visibility problems, kinetic data structures, and biochemistry, as well as with algebraic topology, geometric probability, real algebraic geometry, and combinatorics.

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Combinatorial and Computational Geometry

Edited by

Jacob E. Goodman

City College, CUNY

János Pach

*City College, CUNY and
Courant Institute, NYU*

Emo Welzl

ETH Zürich



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Jacob E. Goodman: Department of Mathematics, City College, CUNY,
 New York, NY 10031 goodman@sci.ccnycuny.edu

János Pach: Courant Institute, NYU, 251 Mercer Street, New York, NY 10012
 and City College, CUNY, New York, NY 10031 pach@courant.nyu.edu

Emo Welzl: Informatik, Eidgenössische Technische Hochschule,
 Rämistrasse 101, CH-8092 Zürich, Switzerland emo@inf.ethz.ch

Silvio Levy (*Series Editor*): Mathematical Sciences Research Institute,
 17 Gauss Way, Berkeley, CA 94720, United States levy@msri.org

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Contents

| | |
|---|-----|
| Preface | xi |
| Geometric Approximation via Crests P. K. AGARWAL, S. HAR-PELED AND K. R. VARADARAJAN | 1 |
| Applications of Graph and Hypergraph Theory in Geometry IMRE BÁRÁNY | 31 |
| Convex Geometry of Orbits ALEXANDER BARVINOK AND GRIGORIY BLEKHERMAN | 51 |
| The Hadwiger Transversal Theorem for Pseudolines SAUGATA BASU, JACOB E. GOODMAN, ANDREAS HOLMSEN, AND RICHARD POLLACK | 79 |
| Isoperimetric Bounds, Applications and Algorithms SAUGATA BASU, RICHARD POLLACK, AND MARIE-FRANÇOISE ROY | 87 |
| Shelling and the h -Vector of the (Extra)ordinary Polytope MARGARET M. BAYER | 97 |
| On the Number of Mutually Touching Cylinders ANDRÁS BEZDEK | 121 |
| Edge-Antipodal 3-Polytopes KÁROLY BEZDEK, TIBOR BISZTRICZKY, AND KÁROLY BÖRÖCZKY | 129 |
| Conformal Energy for Simplicial Surfaces ALEXANDER BOBENKO | 135 |
| On the Size of Higher-Dimensional Triangulations PETER BRASS | 147 |
| The Carpenter's Ruler Folding Problem GRUIA CĂLINESCU AND ADRIAN DUMITRESCU | 155 |
| A Survey of Folding and Unfolding in Computational Geometry ERIK D. DEMAINE AND JOSEPH O'ROURKE | 167 |

| | | |
|--|----------|-----|
| viii | CONTENTS | |
| On the Rank of a Tropical Matrix | | 213 |
| MIKE DEVELIN, FRANCISCO SANTOS, AND BERND STURMFELS | | |
| The Geometry of Biomolecular Solvation | | 243 |
| HERBERT EDELSBRUNNER AND PATRICE KOEHL | | |
| Inequalities for Zonotopes | | 277 |
| RICHARD EHRENBORG | | |
| Quasiconvex Programming | | 287 |
| DAVID EPPSTEIN | | |
| De Concini-Procesi Wonderful Arrangement Models: A Discrete Geometer's Point of View | | 333 |
| EVA MARIA FEICHTNER | | |
| Thinnest Covering of a Circle by Eight, Nine, or Ten Congruent Circles | | 361 |
| GÁBOR FEJES TÓTH | | |
| On the Complexity of Visibility Problems with Moving Viewpoints | | 377 |
| PETER GRITZMANN AND THORSTEN THEOBALD | | |
| Cylindrical Partitions of Convex Bodies | | 399 |
| ALADÁR HEPPES AND WŁODZIMIERZ KUPERBERG | | |
| Tropical Halfspaces | | 409 |
| MICHAEL JOSWIG | | |
| Two Proofs for Sylvester's Problem Using an Allowable Sequence of Permutations | | 433 |
| HAGIT LAST | | |
| A Comparison of Five Implementations of 3D Delaunay Tessellation | | 439 |
| YUANXIN LIU AND JACK SNOEYINK | | |
| The Bernstein Basis and Real Root Isolation | | 459 |
| BERNARD MOURRAIN, FABRICE ROUILLIER, AND MARIE-FRANÇOISE ROY | | |
| Extremal Problems Related to the Sylvester-Gallai Theorem | | 479 |
| NIRANJAN NILAKANTAN | | |
| A Long Noncrossing Path Among Disjoint Segments in the Plane | | 495 |
| JÁNOS PACH AND ROM PINCHASI | | |
| On a Generalization of Schönhardt's Polyhedron | | 501 |
| JÖRG RAMBAU | | |
| On Hadwiger Numbers of Direct Products of Convex Bodies | | 517 |
| ISTVÁN TALATA | | |

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[More information](#)

CONTENTS

ix

| | |
|--|-----|
| Binary Space Partitions: Recent Developments | 529 |
| CSABA D. TOTH | |
| The Erdős-Szekeres Theorem: Upper Bounds and Related Results | 557 |
| GÉZA TÓTH AND PAVEL VALTR | |
| On the Pair-Crossing Number | 569 |
| PAVEL VALTR | |
| Geometric Random Walks: A Survey | 577 |
| SANTOSH VEMPALA | |

Preface

The Great Bear is looking so geometrical,
One would think that something or other could be proved.
— Christopher Fry, “The Lady’s Not for Burning”

During the past several decades, the gradual merger of the field of discrete geometry and the newer discipline of computational geometry has provided a significant impetus to mathematicians and computer scientists interested in geometric problems. The resulting field of discrete and computational geometry has now grown to the point where not even a semester program, such as the one held at the Mathematical Sciences Research Institute in the fall of 2003, with its three workshops and nearly 200 participants, could include everyone involved in making important contributions to the area. The same holds true for the present volume, which presents just a sampling of the work generated during the MSRI program; we have tried to assemble a sample that is representative of the program.

The volume includes 32 papers on topics ranging from polytopes to complexity questions on geometric arrangements, from geometric algorithms to packing and covering, from visibility problems to geometric graph theory. There are points of contact with both mathematical and applied areas such as algebraic topology, geometric probability, algebraic geometry, combinatorics, differential geometry, mathematical programming, data structures, and biochemistry.

We hope the articles in this volume — surveys as well as research papers — will serve to give the interested reader a glimpse of the current state of discrete, combinatorial and computational geometry as we stand poised at the beginning of a new century.

Jacob E. Goodman
János Pach
Emo Welzl