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# The Welfare Cost of Inflation in the Presence of Inside Money

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In this paper, we ask what role an endogenous money multiplier plays in the estimated welfare cost of inflation. The model is a variant of that used by Freeman and Kydland (2000) with inside and outside money in the spirit of Freeman and Huffman (1991). Unlike models in which the money–output link comes from either sticky prices or fixed money holdings, here prices and output are assumed to be fully flexible. Consumption goods are purchased using either currency or bank deposits. Two transaction costs affect these decisions: One is the cost of acquiring money balances, which is necessary to determine the demand for money and to make the velocity of money endogenous. The other is a fixed cost associated with using deposits. This cost is instrumental in determining the division of money balances into currency and interest-bearing deposits. Faced with these two costs and factors that may vary over time in equilibrium (such as over the business cycle), households make decisions that, in the aggregate, determine the velocity of money and the money multiplier.

The model is consistent with several features of U.S. data: (1) M1 is positively correlated with real output; (2) the money multiplier and deposit-to-currency ratio are positively correlated with output; (3) the price level is negatively correlated with output in spite of conditions (1) and (2); (4) the correlation of M1 with contemporaneous prices is substantially weaker than the correlation of M1 with real output; (5) correlations among real variables are essentially unchanged under different monetary policy regimes; and (6) real money balances are smoother than money-demand equations would predict.

A key feature of the model is that households purchase a continuum of types of goods indexed by their size. It comes from assuming a Leontief-type utility function over these types. One could argue that the distinction between nondurable and (usually larger) durable consumption goods should also be taken into account. We shall not take that step here. Instead, compared with Freeman and Kydland (2000), we consider a more flexible utility function than before, which, in equilibrium, permits the implication that households wish to consume large goods in relatively greater quantities.

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With the model economy calibrated to the usual long-run relations in the data—including the selection of values for the two transaction-cost parameters so as to make the model consistent with the empirical average deposit-to-currency ratio and the fraction of capital that is intermediated—the estimated welfare cost of inflation turns out to be rather small. An interesting finding is that the welfare cost as a function of the steady-state inflation rate is very steep for low inflation rates (well under 10%) but quite flat for higher inflation rates. Moreover, we find that the welfare cost is sensitive to the values of the transaction-cost parameters.

Beginning with Bailey (1956) and Friedman (1969), a long line of research addresses the question of the cost of inflation. Among recent contributions, the estimated gain from reducing inflation from 10% to 0% range from a consumption equivalent of 0.38% by Cooley and Hansen (1989), who address the question within a cash-in-advance model, to a consumption equivalent of around 1% by Lucas (2000), who analyzes a representative agent model with shopping time.[1]

## 1. MODEL ECONOMY

### 1.1 The Household's Problem

There is a continuum of good types of measure $c_t^*$, ordered by size and indexed by $j$ over [0,1]. The representative household has a Leontief-type instantaneous utility function over the continuum of good types,

$$\min\left[\frac{c_t(j)}{(1-\omega)j^{-\omega}}\right],$$

which gives us the parameterized distribution function for $c_t(j)$ over [0,1]

$$(1)\quad c_t(j) = (1-\omega)\, j^{-\omega} c_t^*.$$

The representative household has time-separable preferences over total consumption ($c_t^*$) and leisure ($d_t$),

$$(2)\quad \max E \sum_{t=0}^{\infty} \beta^t u(c_t^*, d_t),$$

[1] Other recent contributions include Bullard and Russell (2000), Dotsey and Ireland (1996), Gomme (1993), Imrohoroglu and Prescott (1991), Jones and Manuelli (1995), and Lacker and Schreft (1996).

where the instantaneous utility is given by

(3) $$u(c_t^*, d_t) = \frac{1}{1-\upsilon}\left[(c_t^*)^{\zeta}(d_t)^{1-\zeta}\right]^{1-\upsilon}.$$

There are three vehicles of savings available to the household: nonintermediated capital ($a_t$), nominal bank deposits ($h_t$), and currency ($m_t$). Both bank deposits ($h_t$) and currency ($m_t$) can be used to purchase consumption goods, but the use of deposits incurs an extra fixed cost, denoted by $\gamma$. Because of this fixed cost of using deposits for purchases, the deposit rate of the return net of transaction costs goes to negative infinity as purchase size ($j$) goes to zero. Therefore, some $j^*$ exists below which currency is a preferred means of payment and above which deposits are preferred.

The household's good budget constraint is given by

(4) $$c_t^* + a_t + \frac{h_t}{p_t} + \frac{m_t}{p_t} + \gamma(1 - j_t^*) = w_t\, l_t + r_t\, a_{t-1} + \tilde{r}_t \frac{h_{t-1}}{p_{t-1}} + \frac{m_{t-1}}{p_t} + \frac{x_t}{p_t},$$

where $p_t$ is the nominal price level, $w_t$ is the wage rate, $r_t$ is the real rate of return on capital, $\tilde{r}_t$ is the real rate of return on deposits, and $x_t$ is government lump-sum transfers.

Available time for the households is normalized to 1, and the time available is spent on leisure ($d_t$), labor ($l_t$), and the number of times that money balances have to be replenished each period ($n_t$) multiplied by the time each replenishment takes ($\varphi$). The time constraint is

(5) $$1 = d_t + l_t + n_t\,\varphi.$$

### 1.2 Production

Output is given by a constant-returns-to-scale production function with two inputs, capital ($k_t$) and labor ($l_t$):

$$y_t = z_t f(k_t, l_t).$$

The law of motion for the technology level $z_t$ is given by

$$z_t = p z_{t-1} + \varepsilon_t, \qquad z_t \sim N(\mu, \sigma^2),\ \mu > 0.$$

The depreciation rate is denoted by $\delta$, so the law of motion for the capital stock is

$$k_{t+1} = (1-\delta)\,k_t + i_t,$$

where $i_t$ is gross investment.

### 1.3 Government

The government controls the supply of intrinsically worthless fiat money. The law of motion for the money stock is

$$M_t = \xi M_{t-1}.$$

Net revenues from printing money are transferred to the household in a lump-sum fashion,

$$x_t = (\xi - 1)\, M_{t-1}.$$

### 1.4 Financial Intermediation

Banks accept deposits, hold the required-reserves fraction ($\theta$) as cash, and invest the proceeds in capital. Free entry ensures zero profit, and the rate of return on deposits ($\tilde{r}$), therefore, is a linear combination of the real return on capital ($r_{t+1}$) and the return on holding currency ($p_t/p_{t+1}$):

$$\tilde{r}_{t+1} = (1-\theta)\, r_{t+1} + \theta \frac{p_t}{p_{t+1}}.$$

By definition, the total stock of fiat money (the monetary base) is equal to the combined stocks of currency and reserves,

$$M_t = m_t + \theta h_t,$$

whereas the total money stock (M1) is the sum of nominal deposits and currency, which can be rewritten as the product of the monetary base and the money multiplier:

$$\mathrm{M1}_t = m_t + h_t = M_t \left[ 1 + \frac{h_t (1-\theta)}{m_t + \theta h_t} \right].$$

For the representative household, the per-period holdings of real deposits ($h_t/p_t$) are

$$(6) \quad n_t \frac{h_t}{p_t} = \int_{j^*}^{1} c_t(j)\, \mathrm{d}j = \int_{j^*}^{1} (1-\omega)\, j^{-\omega} c_t^*\, \mathrm{d}j = [j^{1-\omega} c_t^*]_{j^*}^{1} = \left(1 - (j^*)^{1-\omega}\right) c_t^*,$$

and holdings of real fiat-money balances ($m_t/p_t$) are

$$(7) \quad n_t \frac{m_t}{p_t} = \int_0^{j^*} c_t(j)\, \mathrm{d}j = \int_0^{j^*} (1-\omega)\, j^{-\omega} c_t^*\, \mathrm{d}j = [j^{1-\omega} c_t^*]_0^{j^*} = (j^*)^{1-\omega} c_t^*.$$

## 2. CALIBRATION

In the steady state, investment is one-quarter of output and the annual capital–output ratio, 2.5. The depreciation rate is then calibrated to 0.025. The parameter $\alpha$ in the production function is calibrated such that the labor share of national income is 0.64. The autocorrelation coefficient $\rho$ in the technology process is equal to 0.95, with a standard deviation of 0.0076.

Setting the average allocation of households' time (excluding sleep and personal care) to market activity equal to 0.30 restricts the value of the utility parameter $\zeta$. The risk-aversion parameter, $\upsilon$, is equal to 2, and the reserve-requirement ratio, $\theta$, is 0.10.

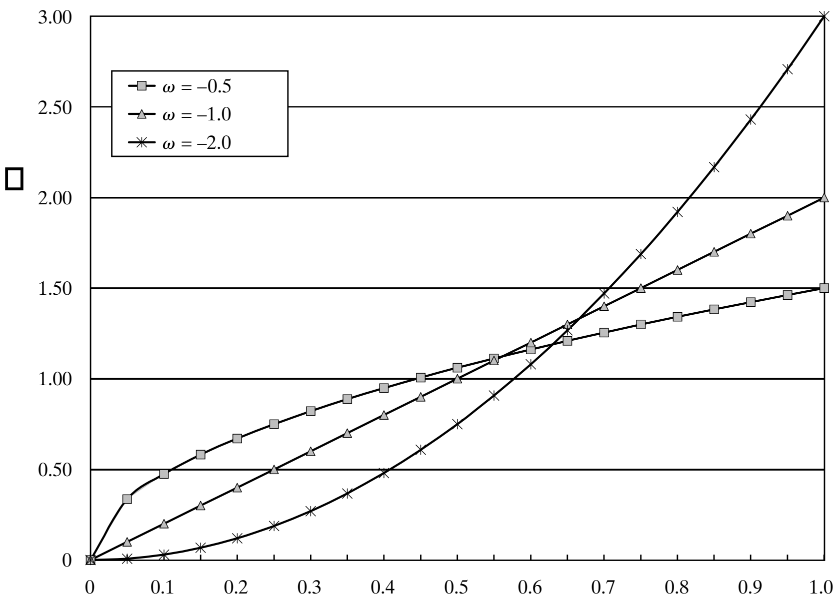
### 2.1 Utility Function

As an illustration, let the continuum of good types $c_t(j)$ be of measure $c_t^*=1$. Equation (1) can then be simplified as

$$c_t\,(j) = (1-\omega)j^{-\omega}.$$

In figure 1, $c_t\,(j)$ is plotted for three different values of $\omega$. As is apparent from the expression and visualized in the figure, for $\omega>-1$, the amount of a good that is consumed is a concave function of the size of the good, whereas for $\omega<-1$, the amount of a good that is consumed is a convex function of the size of the good.

**Figure 1: $c(j)$ for $0 \le j \le 1$, $c^* = 1$**



Combining equations (6) and (7) gives us the cutoff size for purchase, above which deposits are preferred over currency:

$$j^* = \left(1 + \frac{h_t}{m_t}\right)^{\frac{1}{\omega - 1}}. \tag{8}$$

The derivative of $j^*$ is negative, implying that, loosely speaking, the more convex $c_t(j)$, the higher $j^*$, or, conversely, the more concave $c_t(j)$, the lower $j^*$.

Note that equations (6) and (7) combined with (8) imply

$$\int_{j^*}^{1} c_t(j)\,\mathrm{d}j = \left(1 + \frac{m_t}{h_t}\right)^{-1} c_t^*$$

and

$$\int_{0}^{j^*} c_t(j)\,\mathrm{d}j = \left(1 + \frac{h_t}{m_t}\right)^{-1} c_t^*.$$

In other words, the cutoff size of purchases for which deposits are preferred over currency is a function of $\omega$, whereas the share of total consumption ($c_t^*$) for which deposits are preferred over currency (and vice versa) depends only on the deposit-to-currency ratio.

**2.2 Business Cycle Properties**

To get a sense of the reasonable values of $\omega$, we start by reexamining the business cycle findings of Freeman and Kydland (2000) with this modification of the utility function. As in Freeman and Kydland (2000), we examine the model's behavior under three different policy regimes (see figure 2): Under the first, policy A, the growth rate of fiat money is fixed at 3% in every period. Under the second, policy B, serially uncorrelated shocks have been added to the supply of fiat money, with a standard deviation of 0.5%. And under the third, policy C, the shocks to the growth rate of the monetary base are serially correlated with an autoregressive parameter of 0.7 and a standard deviation of 0.2.

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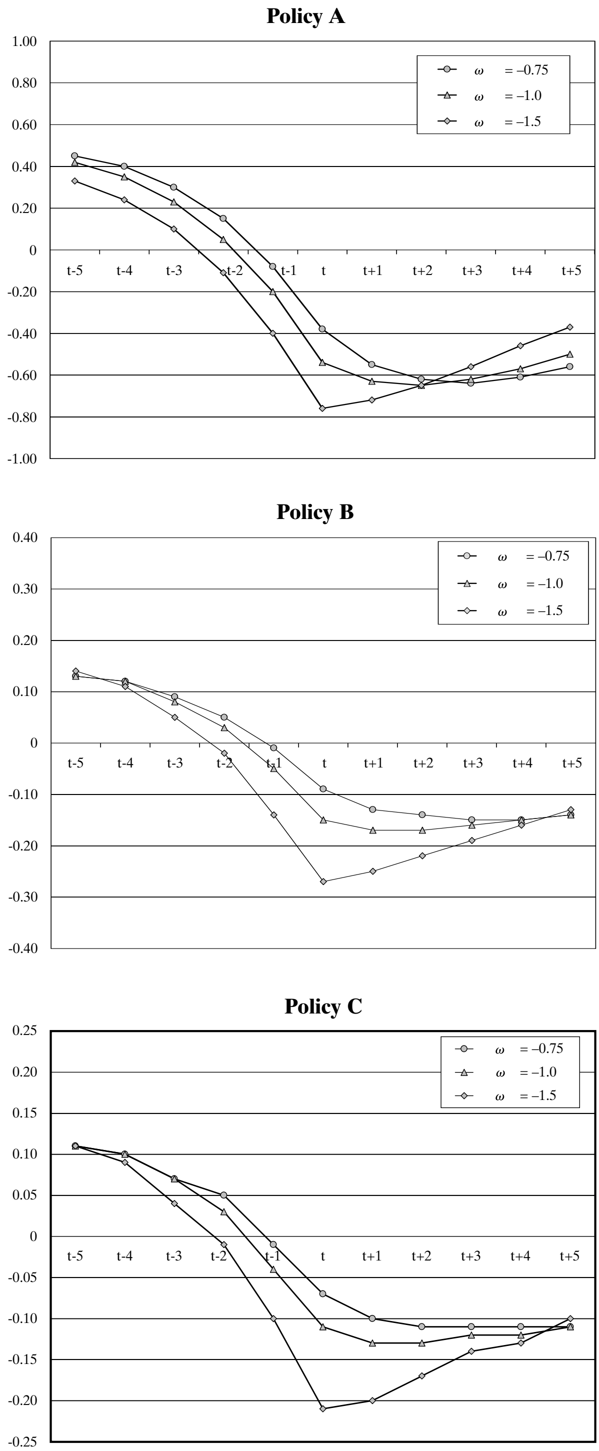
**Figure 2: Cross-Correlations: Output and Price Level**



For these three policies, we examine the business cycle properties for $\omega = \{-0.75, -1.0, -1.5\}$. Table 1 presents the contemporaneous correlations with output, which can be compared with actual data presented by Gavin and Kydland (1999).

**Table 1: Contemporaneous Correlations with Output**

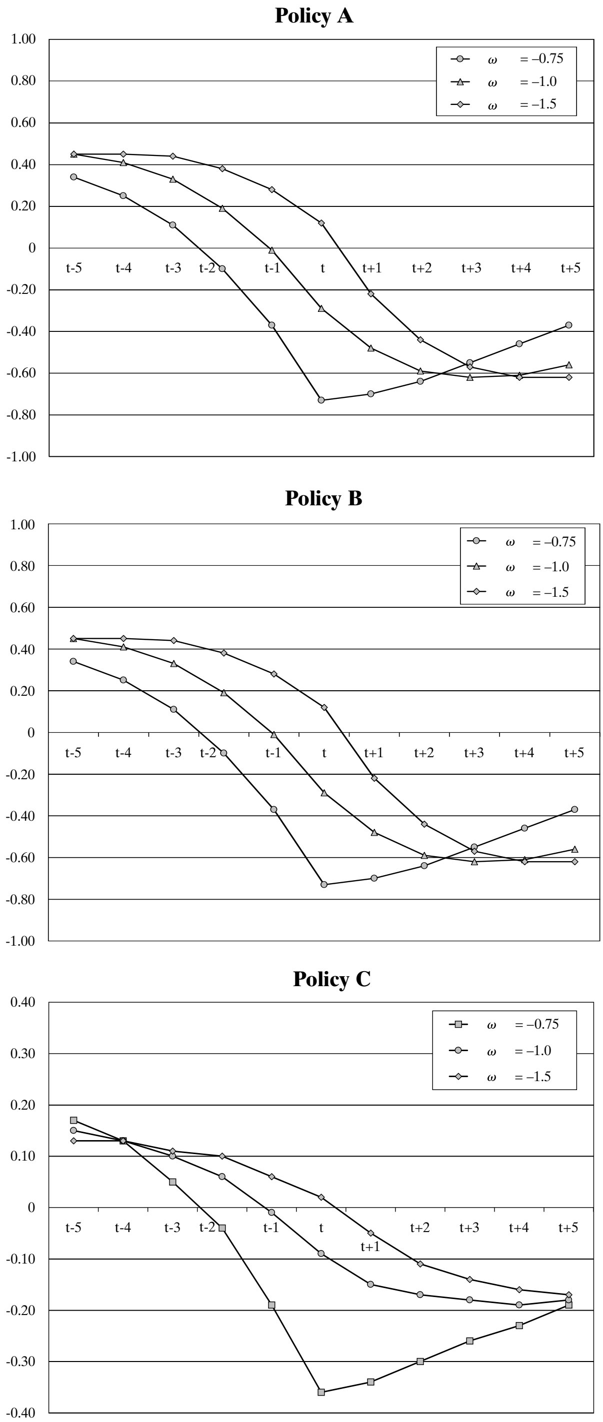
| | | M1 | $P$ | $R_{nom}$ | $C$ | $I$ | $L$ |
|---|---|---|---|---|---|---|---|
| Policy A: | $\omega = -0.75$ | 1 | –0.38 | –0.73 | 0.96 | 0.99 | 0.99 |
| | $\omega = -1.00$ | 1 | –0.54 | –0.29 | 0.96 | 0.99 | 0.99 |
| | $\omega = -1.50$ | 1 | –0.76 | 0.12 | 0.96 | 0.99 | 0.99 |
| Policy B: | $\omega = -0.75$ | 0.89 | –0.09 | –0.73 | 0.96 | 0.99 | 0.99 |
| | $\omega = -1.00$ | 0.85 | –0.15 | –0.29 | 0.96 | 0.99 | 0.99 |
| | $\omega = -1.50$ | 0.78 | –0.27 | 0.12 | 0.96 | 0.99 | 0.99 |
| Policy C: | $\omega = -0.75$ | 0.82 | –0.07 | –0.36 | 0.96 | 0.99 | 0.99 |
| | $\omega = -1.00$ | 0.78 | –0.11 | –0.09 | 0.96 | 0.99 | 0.99 |
| | $\omega = -1.50$ | 0.72 | –0.21 | 0.02 | 0.96 | 0.99 | 0.99 |

Notice that the real variables—*C, I,* and *L*—are hardly affected by changes in monetary policy or the curvature of the utility function. We also see that M1 is strongly correlated with real output. Under policy A, in which there is no randomness, the correlation is 1. Under the two other policy regimes, M1 is slightly less tightly correlated but still highly correlated.

An interesting pattern is the countercyclical behavior of the price level. We see that, for all policies, the price level is more countercyclical for $\omega = -1.5$ than for the other two values, which is consistent with the business cycle statistics reported by Gavin and Kydland (1999).

We also notice that the cyclical behavior of the nominal interest rate is closer to what is observed in the data for $\omega = -1.5$ (figure 3). For the other two values of $\omega$, the nominal rate of return ($R_{nom}$) is countercyclical, whereas for $\omega = -1.5$, the nominal interest rate is weakly procyclical. This is consistent with reported business cycle statistics.

**Figure 3: Cross-Correlations: Output and Nominal *R***



Until we have data from which we can map more directly to $\omega$, we choose $\omega = -1.5$ as our benchmark value because this value gives business cycle statistics closest to those observed.

## 3. QUANTITATIVE FINDINGS

We will begin by describing the steady-state properties of our economy under different inflation regimes. The economy is calibrated such that for an annual inflation rate equal to 0.03, the currency-to-deposit ratio is equal to 9 and the nonreserve portion of M1 divided by the capital stock is 0.05. This gives us calibrated values for $\gamma = 0.00529$ and $\varphi = 0.00060$, which implies that at this inflation rate, the fixed cost, $\gamma(1-j^*)$, is 0.36% of gross domestic product and $\varphi$ corresponds to approximately 55 minutes per quarter.

### 3.1 Steady State

Figures 4 and 5 (figure 4 is just a subset of figure 5) plot the benchmark welfare cost function $\lambda$, defined such that

$$u[\lambda c(\pi), d(\pi)] = u[c(\tilde{\pi}), d(\tilde{\pi})],$$

where $\tilde{\pi}$ equals the average inflation rate over the last 15 years, about 3%.

**Figure 4: Welfare Cost of Inflation Relative to Net Annual Inflation of 0.03**