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978-0-521-84703-2 - Applied Asymptotics Case Studies in Small-Sample Statistics

A. R. Brazzale, A. C. Davison and N. Reid

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Applied Asymptotics: Case Studies in Small-Sample Statistics

In fields such as biology, medical sciences, sociology and economics researchers often face the situation where the number of available observations, or the amount of available information, is sufficiently small that approximations based on the normal distribution may be unreliable. Theoretical work over the last quarter-century has led to new likelihood-based methods that yield very accurate approximations in finite samples, but this work has had limited impact on statistical practice. This book illustrates by means of realistic examples and case studies how to use the new theory, and investigates how and when it makes a difference to the resulting inference. The treatment is oriented towards practice and is accompanied by code in the R language which enables the methods to be applied in a range of situations of interest to practitioners. The analysis includes some comparisons of higher order likelihood inference with bootstrap and Bayesian methods.

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Preface

The likelihood function plays a central role in both statistical theory and practice. Basic results about likelihood inference, which we call first order asymptotics, were developed in fundamental work by R. A. Fisher during the 1920s, and now form an essential and widely taught part of both elementary and advanced courses in statistics. It is less well known that Fisher later proposed a more refined approach, which has been developed over the past three decades into a theory of higher order asymptotics. While this theory leads to some extremely accurate methods for parametric inference, accounts of the theory can appear forbidding, and the results may be thought to have little importance for statistical practice.

The purpose of this book is dispel this view, showing how higher order asymptotics may be applied in realistic examples with very little more effort than is needed for first order procedures, and to compare the resulting improved inferences with those from other approaches. To do this we have collected a range of examples and case studies, provided details on the implementation of higher order approximations, and compared the resulting inference to that based on other methods; usually first order likelihood theory, but where appropriate also methods based on simulation. Our examples are nearly all derived from regression models for discrete or continuous data, but range quite widely over the types of models and inference problems where likelihood methods are applied.

In order to make higher order methods accessible, we have striven for as simple an exposition as we thought feasible, aiming for heuristic explanation rather than full mathematical rigour. We do not presuppose previous knowledge of higher order asymptotics, key aspects of which are explained early in the book. The reader is assumed to have knowledge of basic statistics including some central classes of models, and some experience of standard likelihood methods in practice. We intend that the book be useful for students of statistics, practising statisticians, and data analysts, as well as researchers interested in a more applied account of the methods than has so far been available. Our effort has been made practicable by software developed by Alessandra Brazzale and Ruggero Bellio over many years, of which the `hoa` package bundle now available in `R` is the culmination. This software is extensively used throughout the book, and the ideas behind the `hoa` packages, described in Chapter 9, formed the basis for our approaches to programming

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when new software was needed for some of the examples. The `hoa` package bundle and other materials may be obtained from the book's web page

<http://statwww.epfl.ch/AA>

This book could not have been written without the help of many colleagues. We thank particularly Sven Bacher, Douglas Bates, Ruggero Bellio, Nathalie Chèvre, David Cox, Don Fraser, Torsten Hothorn, Alessandra Salvan, Ana-Maria Staicu, Charlotte Vandenberghe, and members of the R Development Core Team, for access to data, fruitful collaboration, helpful discussion, valuable comments on the material, and help with computational aspects of our work. David Tranah and Diana Gillooly of Cambridge University Press have been supportive and patient editors. We thank also the following institutions for financial and material support: the EPFL; the University of Padova; the University of Toronto; the Italian National Research Council, and in particular its Institute of Biomedical Engineering, where much of Alessandra Brazzale's work was performed; the Italian Ministry of Education, University and Research; the Canadian Natural Sciences and Engineering Research Council; and the Swiss National Science Foundation. We thank also our friends and families for their enduring encouragement and support.

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