Insurance Risk and Ruin

The focus of this book is on the two major areas of risk theory: aggregate claims distributions and ruin theory. For aggregate claims distributions, detailed descriptions are given of recursive techniques that can be used in the individual and collective risk models. For the collective model, different classes of counting distribution are discussed, and recursion schemes for probability functions and moments presented. For the individual model, the three most commonly applied techniques are discussed and illustrated. The book is based on the author's experience of teaching final-year actuarial students in Britain and Australia, and is suitable for a first course in insurance risk theory. Care has been taken to make the book accessible to readers who have a solid understanding of the basic tools of probability theory. Numerous worked examples are included in the text and each chapter concludes with a set of excercises for which outline solutions are provided.

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DAVID C. M. DICKSON

Centre for Actuarial Studies,
Department of Economics, University of Melbourne

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To Robert and Janice

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Preface

This book is designed for final-year university students taking a first course in insurance risk theory. Like many textbooks, it has its origins in lectures delivered in university courses, in this case at Heriot-Watt University, Edinburgh, and at the University of Melbourne. My intention in writing this book is to provide an introduction to the classical topics in risk theory, especially aggregate claims distributions and ruin theory.

The prerequisite knowledge for this book is probability theory at a level such as that in Grimmett and Welsh (1986). In particular, readers should be familiar with the basic concepts of distribution theory and be comfortable in the use of tools such as generating functions. Much of Chapter 1 reviews distributions and concepts with which the reader should be familiar. A basic knowledge of stochastic processes is helpful, but not essential, for Chapters 6 to 8. Throughout the text, care has been taken to use straightforward mathematical techniques to derive results.

Since the early 1980s, there has been much research in risk theory in computational methods, and recursive schemes in particular. Throughout the text, recursive methods are described and applied, but a full understanding of such methods can only be obtained by applying them. The reader should therefore by prepared to write some (short) computer programs to tackle some of the examples and exercises.

Many of these examples and exercises are drawn from materials I have used in teaching and examining, so the degree of difficulty is not uniform. At the end of the book, some outline solutions are provided, which should allow the reader to complete the exercises, but in many cases a fair amount of work (and thought!) is required of the reader. Teachers can obtain full model solutions by emailing solutions@cambridge.org.

Some references are given at the end of each chapter for the main results in that chapter, but it was not my intention to provide comprehensive references,

and readers are therefore encouraged to review the papers and books I have cited and to investigate the references therein.

Work on this book started during study leave at the University of Copenhagen in 1997 and, after much inactivity, was completed this year on study leave at the University of Waterloo and at Heriot-Watt University. I would like to thank all those at these three universities who showed great hospitality and provided a stimulating working environment. I would also like to thank former students at Melbourne: Jeffrey Chee and Kee Leong Lum for providing feedback on initial drafts, and Kwok Swan Wong who devised the examples in Section 8.6.3. Finally, I would like to single out two people in Edinburgh for thanks. First, this book would not have been possible without the support and encouragement of Emeritus Professor James Gray over a number of years as teacher, supervisor and colleague. Second, many of the ideas in this book come from joint work with Howard Waters, both in teaching and research, and I am most appreciative of his support and advice.

David C.M. Dickson
Melbourne, August 2004