Condensed Matter Field Theory

Theoretical condensed matter physics draws heavily and increasingly on the language of quantum field theory. This primer is aimed at elevating graduate students of condensed matter physics to a level where they can engage in independent research. It emphasizes the development of modern methods of classical and quantum field theory with applications of interest in both experimental and theoretical condensed matter physics. Topics covered include second quantization, path and functional field integration, mean-field theory and collective phenomena, the renormalization group, and topology. Conceptual aspects and formal methodology are emphasized and developed, but the discussion is rooted firmly in practical experimental application. As well as routine exercises, the text includes extended and challenging problems, with fully worked solutions, designed to provide a bridge between formal manipulations and research-oriented thinking. This book will complement graduate-level courses on theoretical quantum condensed matter physics.

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Preface

In the past few decades, the field of quantum condensed matter physics has seen rapid and, at times, almost revolutionary development. Undoubtedly, the success of the field owes much to ground-breaking advances in experiment: already the controlled fabrication of phase coherent electron devices on the nanoscale is commonplace (if not yet routine), while the realization of ultra-cold atomic gases presents a new arena in which to explore strong interaction and condensation phenomena in Fermi and Bose systems. These, along with many other examples, have opened entirely new perspectives on the quantum physics of many-particle systems. Yet, important as it is, experimental progress alone does not, perhaps, fully explain the appeal of modern condensed matter physics. Indeed, in concert with these experimental developments, there has been a "quiet revolution" in condensed matter theory, which has seen phenomena in seemingly quite different systems united by common physical mechanisms. This relentless "unification" of condensed matter theory, which has drawn increasingly on the language of low-energy quantum field theory, betrays the astonishing degree of *universality*, not fully appreciated in the early literature.

On a truly microscopic level, all forms of quantum matter can be formulated as a many-body Hamiltonian encoding the fundamental interactions of the constituent particles. However, in contrast with many other areas of physics, in practically all cases of interest in condensed matter the structure of this operator conveys as much information about the properties of the system as, say, the knowledge of the basic chemical constituents tells us about the behavior of a living organism! Rather, in the condensed matter environment, it has been a long-standing tenet that the degrees of freedom relevant to the low-energy properties of a system are very often not the microscopic. Although, in earlier times, the passage between the microscopic degrees of freedom and the relevant low-energy degrees of freedom has remained more or less transparent, in recent years this situation has changed profoundly. It is a hallmark of many "deep" problems of modern condensed matter physics that the connection between the two levels involves complex and, at times, even controversial mappings. To understand why, it is helpful to place these ideas on a firmer footing.

Historically, the development of modern condensed matter physics has, to a large extent, hinged on the "unreasonable" success and "notorious" failures of *non-interacting* theories. The apparent impotency of interactions observed in a wide range of physical systems can be attributed to a deep and far-reaching principle of *adiabatic continuity*: the quantum numbers that characterize a many-body system are determined by fundamental symmetries (translation, rotation, particle exchange, etc.). Providing the integrity of the symmetries is maintained, the elementary "quasi-particle" excitations of an interacting system can be usually traced back "adiabatically" to those of the bare particle excitations present in the non-interacting system. Formally, one can say that the radius of convergence of perturbation theory extends beyond the region in which the perturbation is small. For example, this *quasi-particle correspondence*, embodied in Landau's Fermi-liquid theory, has provided a reliable platform for the investigation of the wide range of Fermi systems from conventional metals to ^{3}helium fluids and cold atomic Fermi gases.

However, being contingent on symmetry, the principle of adiabatic continuity and, with it, the quasi-particle correspondence, must be abandoned at a *phase transition*. Here, interactions typically effect a substantial rearrangement of the many-body ground state. In the symmetry-broken phase, a system may – and frequently does – exhibit elementary excitations very different from those of the parent non-interacting phase. These elementary excitations may be classified as new species of quasi-particle with their own characteristic quantum numbers, or they may represent a new kind of excitation – *a collective mode* – engaging the cooperative motion of many bare particles. Many familiar examples fall into this category: when ions or electrons condense from a liquid into a solid phase, translational symmetry is broken and the elementary excitations – phonons – involve the motion of many individual bare particles. Less mundane, at certain field strengths, the effective low-energy degrees of freedom of a two-dimensional electron gas subject to a magnetic field (the quantum Hall system) appear as quasi-particles carrying a rational *fraction* (!) of the elementary electron charge – an effect manifestly non-perturbative in character.

This reorganization lends itself to a hierarchical perspective of condensed matter already familiar in the realm of particle physics. Each phase of matter is associated with a unique "non-interacting" reference state with its own characteristic quasi-particle excitations – a product only of the fundamental symmetries that classify the phase. While one stays within a given phase, one may draw on the principle of continuity to infer the influence of interactions. Yet this hierarchical picture delivers two profound implications. Firstly, within the quasi-particle framework, the underlying "bare" or elementary particles remain invisible (witness the fractionally charged quasi-particle excitations of the fractional quantum Hall fluid!). (To quote from P. W. Anderson's now famous article "More is different," (*Science* **177** (1972), 393–6), "the ability to reduce everything to simple fundamental laws does not imply the ability to start from those laws and reconstruct the universe."). Secondly, while the capacity to conceive of new types of interactions is almost unbounded (arguably the most attractive feature of the condensed matter environment!), the freedom to identify non-interacting or free theories is strongly limited, constrained by the space of fundamental symmetries. When this is combined with the principle of continuity, the origin of the observed "universality" in condensed matter is revealed. Although the principles of adiabatic continuity, universality, and the importance of symmetries have been anticipated and emphasized long ago by visionary theorists, it is perhaps not until relatively recently that their mainstream consequences have become visible.

How can these concepts be embedded into a theoretical framework? At first sight, the many-body problem seems overwhelmingly daunting. In a typical system, there exist some 10^{23} particles interacting strongly with their neighbors. Monitoring the collective dynamics, even in a classical system, is evidently a hopeless enterprise. Yet, from our discussion above, it is clear that, by focussing on the coordinates of the collective degrees of freedom, one may develop a manageable theory involving only a restricted set of excitations. The success of quantum field theory in describing low-energy theories of particle physics as a successive hierarchy of broken symmetries makes its application in the present context quite natural. As well as presenting a convenient and efficient microscopic formulation of the many-body problem, the quantum field theory description provides a vehicle to systematically identify, isolate, and develop a low-energy theory of the collective field. Moreover, when cast as a field integral, the quantum field theory affords a classification of interacting systems into a small number of universality classes defined by their fundamental symmetries (a phenomenon not confined by the boundaries of condensed matter – many concepts originally developed in medium- or high-energy physics afford a seamless application in condensed matter). This phenomenon has triggered a massive trend of unification in modern theoretical physics. Indeed, by now, several sub-fields of theoretical physics have emerged (such as conformal field theory, random matrix theory, etc.) that define themselves not so much through any specific application as by a certain conceptual or methodological framework.

In deference to the importance attached to the subject, in recent years a number of texts have been written on the subject of quantum field theory within condensed matter. It is, therefore, pertinent for a reader to question the motivation for the present text. Firstly, the principal role of this text is as a primer aimed at elevating graduate students to a level where they can engage in independent research. Secondly, while the discussion of conceptual aspects takes priority over the exposure to the gamut of condensed matter applications, we have endeavored to keep the text firmly rooted in practical experimental application. Thirdly, as well as routine exercises, the present text includes extended problems which are designed to provide a bridge from formal manipulations to research-oriented thinking. Indeed, in this context, readers may note that some of the "answered" problems are deliberately designed to challenge: it is, after all, important to develop a certain degree of *intuitive* understanding of formal structures and, sadly, this can be acquired only by persistent and, at times, even frustrating training!

With this background, let us now discuss in more detail the organization of the text. To prepare for the discussion of field theory and functional integral techniques we begin in Chapter 1 by introducing the notion of a classical and a quantum field. Here we focus on the problem of lattice vibrations in the discrete harmonic chain, and its "ancestor" in the problem of classical and quantum electrodynamics. The development of field integral methods for the many-body system relies on the formulation of quantum mechanical theories in the framework of the second quantization. In Chapter 2 we present a formal and detailed introduction to the general methodology. To assimilate this technique, and motivate some of the examples discussed later in the text, a number of separate and substantial applications are explored in this chapter. In the first of these, we present (in second-quantized form) a somewhat cursory survey of the classification of metals and insulators, identifying a canonical set of model Hamiltonians, some of which form source material for later chapters. In the case of the one-dimensional system, we will show how the spectrum of elementary collective excitations can be inferred using purely operator methods within the framework of the bosonization scheme. Finally, to close the chapter, we will discuss the application of the second quantization to the low-energy dynamics of quantum mechanical spin systems. As a final basic ingredient in the development of the quantum field theory, in Chapter 3 we introduce the Feynman path integral for the single-particle system. As well as representing a prototype for higher-dimensional field theories, the path integral method provides a valuable and recurring computational tool. This being so, we have included in this chapter, a pedagogical discussion of a number of rich and instructive applications which range from the canonical example of a particle confined to a single or double quantum well, to the tunneling of extended objects (quantum fields), quantum dissipation, and the path integral formulation of spin.

Having accumulated all of the necessary background, in Chapter 4 we turn to the formulation and development of the field integral of the quantum many-particle system. Beginning with a discussion of coherent states for Fermi and Bose systems, we develop the many-body path integral from first principles. Although the emphasis in the present text is on the field integral formulation, the majority of early and seminal works in the many-body literature were developed in the framework of diagrammatic perturbation theory. To make contact with this important class of approximation schemes, in Chapter 5 we explore the way diagrammatic perturbation series expansions can be developed systematically from the field integral. Employing the ϕ^4-theory as a canonical example, we describe how to explore the properties of a system in a high order of perturbation theory around a known reference state. To cement these ideas, we apply these techniques to the problem of the weakly interacting electron gas.

Although the field integral formulation provides a convenient means to organize perturbative approximation schemes as a diagrammatic series expansion, its real power lies in its ability to identify non-trivial reference ground states, or "mean-fields," and to provide a framework in which low-energy theories of collective excitations can be developed. In Chapter 6, a fusion of perturbative and mean-field methods is used to develop analytical machinery powerful enough to address a spectrum of rich applications ranging from metallic magnetism and superconductivity to superfluidity. To bridge the gap between the (often abstract) formalism of the field integral, and the arena of practical application, it is necessary to infer the behavior of correlation functions. Beginning with a brief survey of concepts and techniques of experimental condensed matter physics, in Chapter 7 we will highlight the importance of correlation functions and explore their connection with the theoretical formalism developed in previous chapters. In particular, we will discuss how the response of many-body systems to various types of electromagnetic perturbations can be described in terms of correlation functions and how these functions can be computed by field theoretical means.

Although the field integral is usually simple to formulate, its properties are not always easy to uncover. Amongst the armory of tools available to the theorist, perhaps the most adaptable and versatile is the method of the renormalization group. Motivating our discussion with two introductory examples drawn from a classical and a quantum theory, in Chapter 8 we will become acquainted with the renormalization group method as a concept whereby nonlinear theories can be analyzed beyond the level of plain perturbation theory. With this background, we will then proceed to discuss renormalization methods in more rigorous and general terms, introducing the notion of scaling, dimensional analysis, and the connection to the general theory of phase transitions and critical phenomena. To conclude this chapter, we will visit a number of concrete implementations of the renormalization group scheme introduced and exemplified on a number of canonical applications.

Finally, in Chapter 9, we will turn our attention to low-energy theories with non-trivial forms of long-range order. Specifically, we will learn how to detect and classify topologically non-trivial structures, and to understand their physical consequences. Specifically, we explore the impact of topological terms (i.e. θ-terms, Wess–Zumino terms, and Chern–Simons terms) on the behavior of low-energy field theories solely through the topology of the underlying field configurations. Applications discussed in this chapter include persistent currents, 't Hooft's θ-vacua, quantum spin chains, and the quantum Hall effects.

To focus and limit our discussion, we have endeavored to distill material considered "essential" from the "merely interesting" or "background." To formally acknowledge and identify this classification, we have frequently included reference to material which we believe may be of interest to the reader in placing the discussion in context, but which can be skipped without losing the essential thread of the text. These intermissions are signaled in the text as "Info" blocks.

At the end of each chapter, we have collected a number of pedagogical and instructive problems. In some cases, the problems expand on some aspect of the main text requiring only an extension, or straightforward generalization, of a concept raised in the chapter. In other cases, the problems rather complement the main text, visiting fresh applications of the same qualitative material. Such problems take the form of case studies in which both the theory and the setting chart new territory. The latter provide a vehicle to introduce some core areas of physics not encountered in the main text, and allow the reader to assess the degree to which the ideas in the chapter have been assimilated. With both types of questions, to make the problems more inclusive and useful as a reference, we have included (sometimes abridged, and sometimes lengthy) answers. In this context, Section 6.5 assumes a somewhat special role: the problem of phase coherent electron transport in weakly disordered media provides a number of profoundly important problems of great theoretical and practical significance. In preparing this section, it became apparent that the quantum disorder problem presents an ideal environment in which many of the theoretical concepts introduced in the previous chapters can be practiced and applied – to wit diagrammatic perturbation theory and series expansions, mean-field theory and collective mode expansions, correlation functions and linear response, and topology. We have therefore organized this material in the form of an extended problem set in Chapter 6.

This concludes our introduction to the text. Throughout, we have tried to limit the range of physical applications to examples which are routed in experimental fact. We have resisted the temptation to venture into more speculative areas of theoretical condensed matter at the expense of excluding many modern and more-circumspect ideas which pervade the condensed matter literature. Moreover, since the applications are intended to help motivate and support the field theoretical techniques, their discussion is, at times, necessarily superficial. (For example, the six hundred pages of text in this volume could have been invested in their entirety in the subject of superconductivity!) Therefore, where appropriate, we have tried to direct interested readers to the more specialist literature.

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